

Focus in High School Mathematics: Reasoning and Sense Making

Questions and Answers

1. What do *reasoning* and *sense making* mean?

In mathematics, *reasoning* involves drawing logical conclusions based on evidence or stated assumptions. *Sense making* may be considered as developing understanding of a situation, context, or concept by connecting it with existing knowledge or previous experience. Reasoning and sense making are closely interrelated and are the foundation for a solid preparation in mathematics. Simply exposing students to mathematical topics is not enough. Nor is it enough for students to know only how to perform mathematical procedures or recall facts. They must learn to reason and make sense of mathematics so that they are able to use math in meaningful ways. Students today need to develop critical thinking skills to succeed in mathematics and in life. For example, in high school literature courses, students are often asked to analyze, interpret, or think critically about books that they are reading. Reasoning is important in fields such as literature, and it is particularly important in mathematics.

2. Why are reasoning and sense making important for high school students?

Reasoning and sense making are simultaneously the purpose for learning mathematics and the most effective means of learning it. Unless students can reason with and make sense of the mathematics that they are learning, they are likely to ask the age-old question, “Why do we need to learn this?” They need to see a purpose in studying mathematics beyond the goal of preparing for the next mathematics course or standardized test. Moreover, research shows that students are more likely to retain mathematics that has its foundation in reasoning and sense making than mathematics that is presented as a list of isolated skills.

3. What is the purpose of NCTM’s *Focus in High School Mathematics: Reasoning and Sense Making*?

Focus in High School Mathematics: Reasoning and Sense Making offers a different perspective on high school mathematics, proposing curricular emphases and instructional approaches that make reasoning and sense making foundational to the content that is taught to and learned by all high school students.

Organizing a curriculum around the goals of this publication, along with content recommendations in *Principles and Standards for School Mathematics*, can prepare students for future success as citizens and in the workplace, as well as for careers in mathematics and science.

4. Who should use *Focus in High School Mathematics: Reasoning and Sense Making*?

Focus in High School Mathematics: Reasoning and Sense Making is intended for several constituencies committed to improving the learning and teaching of mathematics:

- *Classroom teachers and instructional leaders*, as they prioritize existing lists of curriculum objectives, expectations, and instructional materials, and explore new teaching approaches that can engage students in reasoning and sense making
- *Teacher educators*, as they organize content and methods courses and help teachers and prospective teachers develop their own mathematical reasoning and sense making and experience instructional methods that will promote the development of those processes in their students
- *Administrators*, as they actively work with teachers to develop and maintain curriculum standards and programs
- *Families*, as they help their children succeed in learning mathematics and prepare for their futures
- *Policymakers*, as they take the next steps in the development of future curricula, instruction, and assessment materials

Collaboration among all these constituencies is essential to ensure that all students receive the background that they need in mathematical reasoning and sense making for future success.

5. Will students who focus on reasoning and sense making learn the skills that they need for future success?

To be well prepared for their future lives, students need to have mathematical competence, which includes knowing not only how to carry out basic mathematical procedures but also which procedures to choose, when to choose them, and for what purpose. Being successful in our fast-paced, economically competitive society will increasingly require innovation and creativity. Such success most often depends on hard work and builds on a firm foundation of usable knowledge. Mathematical reasoning and sense making are keys to such a foundation and consequently belong at the core of a high school mathematics education.

Organizations that devote attention to college mathematics are increasingly calling for the same kind of reasoning, problem solving, and other critical thinking skills that *Focus in High School Mathematics: Reasoning and Sense Making* advocates. Moreover, students who develop a deep understanding of the mathematics that they study are more likely to remember it and to be able to use it in the future, thus ensuring that they will do well in college-level courses.

6. What does a curriculum focused on reasoning and sense making look like?

Many groups have offered recommendations for what topics to include in high school mathematics. Among these, the Standards for grades 9–12 recommended by NCTM in

Principles and Standards for School Mathematics have been influential. However, *Focus in High School Mathematics* goes one step further, recommending that no matter what curriculum is adopted, reasoning and sense making be made the central foci of high school mathematics. To ensure that this occurs, teachers must incorporate worthwhile tasks that engage all students in thinking about and making sense of mathematics, not just practicing concepts and procedures they have already been taught.

Developing strong reasoning habits will of course take instructional time. However, such an approach also promises compensating efficiencies. If students truly understand what they are learning, they are more likely to retain the concepts and skills, reducing the need for reteaching. Also, organizing the curriculum around central ideas or themes developed through reasoning and sense making introduces coherence that may ultimately allow a streamlining of the curriculum.

7. What can teachers do in their classrooms to ensure that reasoning and sense making are paramount?

Teachers can make reasoning and sense making a focus in any mathematics class. A crucial step is to determine how reasoning and sense making serve as integral components of the material that they teach.

Even with topics traditionally transmitted through procedural approaches, teachers can present the material in ways that allow students to reason about what they are doing. Although procedural fluency is important in high school mathematics, it should not be sought in the absence—or at the expense—of reasoning and sense making.

The focus of every mathematics class should be on helping students make sense of the mathematics for themselves. Bringing this focus to instruction depends on—

- selecting worthwhile tasks that engage and develop students' mathematical understanding, skills, and reasoning;
- creating a classroom environment in which serious engagement in mathematical thinking is the norm;
- effectively orchestrating purposeful discourse aimed at encouraging students to reason and make sense of what they are doing;
- using a range of assessments to monitor and promote reasoning and sense making, both in identifying student progress and in making instructional decisions; and
- constantly reflecting on teaching practice to be sure that the focus of the class is on reasoning and sense making (based on recommendations in *Mathematics Teaching Today*, published by NCTM in 2007).

8. What should high school students be expected to be able to do?

Focus in High School: Reasoning and Sense Making describes reasoning habits that should become routine and fully expected in all mathematics classes at all levels of high school. Approaching these reasoning habits as new topics to be taught is not likely to have the desired effect. The crowded high school mathematics curriculum affords little

room for introducing them in this way. Instead, reasoning habits should be integrate into the existing curriculum to ensure that students both understand and can use what they are taught.

Focus in High School Mathematics: Reasoning and Sense Making organizes reasoning habits into four broad categories:

- Analyzing a problem
- Implementing a strategy
- Seeking and using connections
- Reflecting on a solution

Many reasoning habits fit in more than one category, and students should move naturally and flexibly among them as they solve problems and think about mathematics. *Focus in High School: Reasoning and Sense Making* offers examples of ways to promote these habits in the high school classroom.

9. Are all students capable of reasoning and sense making?

Focus in High School Mathematics: Reasoning and Sense Making takes the strong stand that “reasoning and sense making must be evident in the mathematical experiences of all students.” All too often, inequity is caused by well-meaning teachers and administrators through practices that unintentionally create biases. The consequences of all practices and policies must be closely examined. For example, students in “informal” geometry need to receive the same engaging experiences with reasoning and sense making as those in “honors” geometry. Likewise, students from all racial and socioeconomic groups, as well as those with perceived learning or behavioral problems, must be offered rich experiences in mathematics.

It is essential to hold and communicate high expectations for all students. This can motivate students to perform at high levels, just as low expectations can reduce students’ confidence in their mathematical abilities and inhibit their performance. Schools and teachers that operate on the assumption that all students can engage in reasoning and sense making will design programs that can make equitable, successful learning a reality.

10. How can districts and schools use assessments to promote mathematical reasoning and sense making?

Regardless of our educational aims, students, teachers, administrators, and many others equate what we test with what we value. This means that assessment instruments need to include items that call for reasoning and sense making. Such items should, for instance, require students to explain their thinking and reasoning or show that they can use mathematics flexibly in nonroutine situations. Students need to experience these kinds of assessments regularly in high school mathematics classrooms.

Moreover, items requiring reasoning and sense making should be incorporated in high-stakes and accountability measures. Adding such items will mean modifying,

strengthening, and improving the assessment instruments in use in high-stakes assessments and accountability measures. Evidence indicates that this can be done, and we cannot settle for less. Together, we can address the limitations of current practices, which may involve the use of single, short-answer end-of-year examinations.

Some important types of assessments that measure reasoning and sense making may call for more than a single response to each item, and so they may require more time to administer and to grade. To accommodate these needs, accountability assessments might be incorporated into actual classroom work throughout a semester in place of a separate examination at the end of the term. A collection (portfolio) of students' work during the semester, for example, might be used as justification for an accountability score.

11. Should reasoning and sense making be incorporated into professional development, and if so, how?

Professional development will be necessary to ensure that teachers have the tools that they need to build a classroom culture that promotes student engagement in reasoning and sense making. Simply providing teachers with isolated workshops will not achieve this goal. Rather, teachers need intensive, long-term experiences to help them transform their practice. Professional development needs to engage them in thinking deeply about the practices that they use and the impact of these practices on their students, to explore new approaches that may be more effective, and to have classroom support as they begin to implement new methodologies that promise to enhance reasoning and sense making.

Effective professional development must incorporate collaboration among mathematics teachers. Improvement of the high school curriculum cannot be the responsibility of a single individual or a small group but rather must be a shared responsibility of the entire mathematics department. Research suggests that teachers who engage in joint efforts to improve their practice make much more progress. Moreover, schedules should be designed to facilitate collaboration, incorporating, for example, joint planning periods that are organized to enhance collaboration among teachers of a particular course, as well as professional development experiences that are embedded in the school day and focused workshops on in-service days to address identified areas of concern.

As high school mathematics programs move toward the goal of focusing on reasoning and sense making, new teachers coming into the profession must be given adequate support to join the emergent community of practice. Schools should establish programs to provide this support. New mathematics teachers need mentoring by strong mathematics teacher leaders who will help them embark on successful careers focused on continued growth in promoting mathematical reasoning and sense making in their students.

12. What can families do to support the mathematics learning of their high school students?

Families need to be involved in the mathematical preparation of their high school students. Each year, families should be sure that their students are enrolled in challenging

mathematics classes—courses that will move the students toward their long-term goals. Families should show interest in what is happening in the mathematics classroom and look at their students’ mathematics work. They should encourage their students to persevere, even when the material is difficult, and urge them to ask questions in class when the material is not clear.

As parents help students complete their homework or prepare for a test, they should ask the students to explain what they are doing and why it makes sense. Family members should remind their students that simply finding the answer to a problem is not enough; being able to explain how they found the answer and why their method works is also essential.

If a family’s student is having serious difficulties, family members should encourage him or her to set up sessions with the teacher for extra help. Families should emphasize to students that effort is the major determinant of long-term success. By the same token, if students are doing well, families should encourage them to explore what is going on in class more deeply, doing additional research, or solving extended problems provided by the teacher.

Finally, families should stay in contact with their students’ mathematics teacher. If the teacher raises any concerns or asks for assistance, families should try to help. Likewise, if at any point families have questions about any aspect of their students’ mathematical preparation, they should not hesitate to contact the teacher. Families should recognize—and teachers and administrators should demonstrate—that their students’ success is a shared goal, with many people working together to achieve it.

13. What needs to occur to ensure that the vision of *Focus in High School Mathematics: Reasoning and Sense Making* becomes a reality?

NCTM has a number of follow-up projects in progress, including “topic books” that present more detail on how to implement reasoning and sense making in classrooms. Furthermore, reasoning and sense making will be a continuing focus in NCTM’s activities in the coming years. NCTM will also seek to partner with other organizations concerned with high school mathematics. Although NCTM can take a leadership role, all stakeholders must work together in meaningful ways to ensure that the continuing story of missed opportunities to improve high school mathematics across the United States will not be told five years from now, let alone in three decades. We simply cannot afford to wait any longer to address the large-scale changes that are necessary. The success of our students—and of our nation—depends on it.

To learn more about *Focus in High School Mathematics: Reasoning and Sense Making*, visit www.nctm.org/hsfocus.