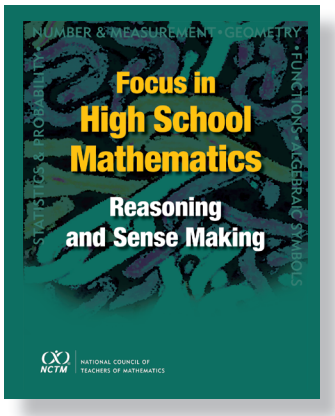
Executive Summary

Focus in High School Mathematics: Reasoning and Sense Making

Building on three decades of advocacy for Standards-based mathematics learning of the highest quality for all students, a new publication of the National Council of Teachers of Mathematics recommends that all high school mathematics programs focus on reasoning and sense making. In recent years, a number of documents have provided detailed analyses of the topics that should be addressed in each course of high school mathematics (see, for example, American Diploma Project [2004]; College Board [2006, 2007]; ACT [2007]; Achieve [2007a, 2007b]). NCTM's *Focus in High School Mathematics: Reasoning and Sense Making* offers a different perspective, proposing curricular emphases and instructional approaches that make reasoning and sense making foundational to the content that is taught and learned in high school.

A high school mathematics program based on reasoning and sense making will prepare students for citizenship, the workplace, and further study.

High school students face major challenges in their mathematics preparation. U.S. students lag in basic mathematical literacy—the knowledge and skills that prepare them to apply mathematics in a variety of contexts, including their future lives as responsible citizens (see, for example, the Programme for International Student Assessment [2007]). They are not prepared to face the economic and workforce challenges of an increasingly global, technological society. This inadequate preparation is contributing to the decline of U.S. leadership in many technical fields (see Tapping America's Potential [2008]). *Focus in High School Mathematics: Reasoning and Sense Making* argues that focusing on reasoning and sense making in the context of strong mathematical content will help high school students meet these challenges.



Reasoning involves drawing conclusions on the basis of evidence or assumptions. Although reasoning is an important part of all disciplines, it plays a special role in mathematics. In addition to formal reasoning or proof, reasoning in mathematics often begins with explorations, conjectures, or false starts. As students progress through the high school years, they should develop increasingly sophisticated standards for explanations. Sense making involves developing an understanding of a situation, context, or concept by connecting it with existing knowledge. Reasoning and sense making are closely intertwined and interdependent.

Reasoning and sense making are the foundations for the processes of mathematics—problem solving, reasoning and proof, connections, communication, and representation (see NCTM [2000]). Moreover, reasoning and sense making help students develop connections between new learning and their existing knowledge, increasing their likelihood of understanding and retaining the new information. (As this volume uses the term *reasoning*, mathematical reasoning encompasses statistical reasoning.)

Reasoning and sense making should be a part of the mathematics classroom every day.

Focus in High School Mathematics: Reasoning and Sense Making describes "reasoning habits," which are productive ways of thinking that should become customary in the processes of mathematical inquiry and sense making. In addition to "covering" mathematical topics, high school mathematics programs must give attention to developing these reasoning habits on a continuing basis—not as a set of new topics to be taught but as an integral part of the curriculum. The publication offers a list of sample reasoning habits, which it emphasizes are not experienced in isolation or sequentially. To help their students progress to higher levels of reasoning, teachers must judiciously select tasks that require them to figure things out for themselves and ask probing questions. Both teachers and students should ask and answer such questions as "What's going on here?" and "Why do you think that?"

Reasoning and sense making are inherent in developing the components of mathematical competence (Kilpatrick, Swafford, and Findell 2001). Conceptual understanding is interrelated with sense making as defined in this volume. Procedural fluency includes not only knowing how to carry out procedures, but also

understanding why they work, how they might be used, and how their results should be interpreted, all of which are grounded in reasoning and sense making. Other aspects of mathematical competence directly draw on reasoning and sense-making skills, including the ability to formulate, represent, and solve mathematical problems and the capacity for logical thought and explanation. Finally, students who view mathematics as a reasoning and sense-making enterprise may be more likely to develop a productive disposition toward mathematics.

Mathematics should help students understand and operate in the physical and social worlds. They should be able to connect mathematics with a real-world situation through the use of mathematical models. The connections between mathematics and real-world problems developed in mathematical modeling add value to, and provide incentive and context for, studying mathematical topics.

Technology is an integral part of the world in which students live, and high school mathematics classrooms must reflect that reality. Technology can advance the goals of reasoning and sense making—facilitating students' searches for patterns, reducing the load of burdensome calculations so that they can focus on thinking strategically, and providing them with multiple ways of representing a mathematical situation. However, the use of technology should not be allowed to overshadow the development of procedural proficiency. Students who have opportunities to reflect on how to use technological tools effectively will be less likely to use them as a crutch.

Reasoning and sense making are integral to the experiences of all students across the high school mathematics curriculum.

Reasoning and sense making should be pervasive in all areas of the high school mathematics curriculum. Although formal reasoning is often emphasized in geometry, students are less likely to experience reasoning in other areas of the curriculum, such as algebra. When reasoning and sense making are infused everywhere in the curriculum, they allow students to discover coherence across the domains of mathematics and help them see how new concepts connect with existing knowledge. Although "reasoning" should not be viewed as a set of new topics but rather a stance toward mathematics learning, a focus on reasoning and sense making will inevitably require instructional time. However, developing strong reasoning habits may yield compensating efficiencies. Students who make reasoned connections with existing knowledge may be more likely to retain what they have learned in previous courses, thus reducing the need for reteaching. Furthermore, instruction that emphasizes underlying connections among ideas may provide coherence that allows for streamlining the curriculum and eliminating lists of particular skills that teachers must help students to master.

Focus in High School Mathematics highlights reasoning opportunities in five specific content areas of the high school mathematics curriculum:

- Reasoning with Numbers and Measurements
- Reasoning with Algebraic Symbols
- Reasoning with Functions
- Reasoning with Geometry
- Reasoning with Statistics and Probability

Within each content area, the publication identifies a number of key elements that provide a broad structure for considering possible ways of focusing on reasoning and sense making. These key elements are not intended to be an exhaustive list of specific topics to be addressed but rather a lens through which to view the potential of high school programs for promoting mathematical reasoning and sense making. A separate chapter on each content area focuses on how reasoning and sense making can be promoted within the key elements of that area. The chapters also include a series of examples intended to provide idealized illustrations of how reasoning and sense making might develop.

The task of creating a curriculum that realizes the goals of this document will be challenging. Although such a curriculum must address important content, its creation requires much more than developing lists of topics to be taught in particular courses. Moreover, students must have experiences with reasoning and sense making within a broad curriculum that meets a wide range of their future needs, preparing them for future success as citizens and in the workplace, as well as for careers in mathematics and science.

Mathematical reasoning and sense making must be evident in the mathematical experiences of all students.

Essential to realizing the vision for high school mathematics outlined in this publication is ensuring that all students—no matter their mathematical background or the mathematics class in which they are enrolled—have full access to opportunities for reasoning and sense making in their mathematics classes. *Focus in High School Mathematics: Reasoning and Sense Making* provides high school teachers, administrators, and staff with some considerations for ensuring that their schools are enacting equitable learning for all their students. In particular, *Focus in High School Mathematics* communicates the message that high schools can monitor equity by attending to phenomena that potentially pose barriers to, or have a significant impact on, the opportunities for engaging every student in the activities of reasoning and sense making. These phenomena include the following:

- ***Courses.*** It is very important that high schools look critically at their practices involving tracking or grouping of students by ability. The courses that students take have an impact on the opportunities that they have for reasoning and sense making. Students in all levels of mathematics—from prealgebra to calculus and from low-track to Advanced Placement—must have mathematical experiences that offer rich opportunities to build reasoning habits as well as to make sense of what they are doing mathematically.
- ***Students' demographics.*** Mathematics educators continue to be concerned about discrepancies in achievement among demographic groups on the basis of race, ethnicity, socioeconomic status, and other variables. Students from some groups receive fewer opportunities for reasoning and sense making than students in other groups. As a result, it is important that high schools do everything they can to promote success among all students—for example, encouraging enrollment by students from all demographic groups in advanced mathematics courses. Also, providing students with opportunities to see that mathematics is important for their lives and future career is a must for high schools.
- ***Expectations, beliefs, and biases.*** The expectations, beliefs, and biases of others can significantly affect the mathematical opportunities provided for students. Building on *Principles and Standards for School Mathematics* (NCTM 2000), *Focus in High School Mathematics: Reasoning and Sense Making* emphasizes the need for teachers, administrators, and school staff to hold high expectations for all students. Teachers' beliefs about students' mathematical capabilities can have serious implications for the opportunities that students are afforded in high school mathematics. Teachers must believe that students will benefit from and can engage in reasoning and sense making, and they must work to help students succeed in this endeavor.

Curriculum, instruction, and assessment form a coherent whole to support reasoning and sense making.

To achieve the vision of reasoning and sense making as the focus of high school students' mathematical experience, all the components of the educational system—curriculum, instruction, and assessment—must work together and be designed to support students' reasoning and sense making. A coherent and cohesive mathematics program requires strong alignment of these three elements.

This publication's recommendations, in combination with the more detailed content recommendations in *Principles and Standards for School Mathematics*, provide a critical filter for examining any curriculum arrangement to ensure the achievement of the goals of high school mathematics. Although curriculum is undeniably crucial to reaching the goal set out in this publication, it cannot stand alone. Mathematical instruction and a classroom environment promoting and valuing students' reasoning and sense making are essential as well. Teachers must select worthwhile tasks that engage students in reasoning and sense making.

As students move through mathematics from prekindergarten through college, coherence in curriculum and instruction is crucial to their success. Too often, as students progress, they fall victim to differing mathematical expectations. To achieve the goal of curricular coherence, an open dialogue is essential among prekindergarten through grade 8 teachers, secondary school mathematics teachers, mathematics teacher educators, and mathematics and statistics faculty in higher education, as well as others in client disciplines, to ensure continuing support and development of students' mathematical abilities.

Schools, parents, policymakers, and others need to see evidence that the development of reasoning and sense-making abilities is a shared goal at all levels of mathematics teaching, including elementary school mathematics, high school mathematics, and the undergraduate curriculum. The time is right to build strong partnerships and recognize the benefits of a mathematics curriculum that focuses on reasoning and sense making from prekindergarten through grade 16.

Finally, realizing any goal for students' learning involves assessment. Schools are currently under tremendous pressure to demonstrate success as measured by high-stakes tests. It is important to assess what we value. Assessments that support the goals of this publication will probe students' development of mathematical reasoning and sense making and contribute to students' progress in mathematics. This endeavor is essential for at least two reasons. First, we will not be able to determine whether we are meeting our goals if we do not measure our progress. Second, high-stakes testing that concentrates primarily on procedural skills *without* assessing reasoning and sense making sends a message that is contrary to the vision of *Focus in High School Mathematics* and can adversely influence instruction and learning. Assessment that focuses primarily on students' abilities to do algebraic manipulations, apply geometric formulas, and perform basic statistical computations will lead students to believe that reasoning and sense making are not important.

Formative assessment—which involves providing students with learning activities and, on the basis of feedback from those activities, adjusting teaching to meet the students' needs—is important in helping teachers ensure

that their students' reasoning and sense making are progressing. Formative assessments rely on a variety of tools, including teacher observations, classroom discussions, student journals, student presentations, homework, and in-class tasks, as well as tests and quizzes that ask students to explain their thinking.

All stakeholders must work together to ensure that reasoning and sense making are the focus of high school mathematics programs.

Focus in High School Mathematics: Reasoning and Sense Making presents an ambitious vision for the improvement of high school mathematics. Its refocusing of the high school curriculum on reasoning and sense making is not a minor tweaking but a substantial rethinking of the high school mathematics curriculum and requires the engagement of all involved in high school mathematics.

Significant effort will be needed to realign the curriculum. To develop new understanding, teachers will need long-term professional development and support, including opportunities for reflection on their practice and guidance in improving it. Students must be offered the resources needed to prepare them for our rapidly changing world and must recognize that studying mathematics in high school is important to their future careers. Families should understand which mathematics classes are important for their students to take and should help them develop good study and homework habits. Teachers must believe in—and communicate their conviction about—the importance of reasoning and sense making for every student in every mathematics topic.

Together, school districts, schools, departments, and teachers must ensure that high school students are exposed to a high-quality mathematics curriculum that promotes reasoning and sense making. State and local assessments policies should emphasize the need for and importance of items that examine students' abilities to reason and make sense of mathematical situations. In addition, policymakers must secure adequate resources to assist schools and districts in efforts to implement an effective curriculum based on reasoning and sense making.

This publication provides a framework for considering necessary changes to the high school mathematics curriculum and how those changes might be made. However, many issues beyond those addressed in this publication remain to be answered. Future publications, including an initial set of topic books that set forth additional guidance in particular content areas, will offer resources that build on this framework. Although NCTM is taking a leadership role, all stakeholders must join forces and work together in meaningful ways to ensure that the story of missed opportunities to improve high school mathematics across the United States does not continue, to be told five years from now, let alone in three decades. We simply cannot afford to wait any longer to address the large-scale changes that are needed. The success of our students and of our nation depends on it.

(August 14, 2009)

References

ACT. *ACT National Curriculum Survey, 2005–2006*. Iowa City, Iowa: ACT, 2007.

Achieve. *High School Model Three-Year Integrated Course Sequence*. American Diploma Project, 2007a. http://www.achieve.org/node/969.

———. *High School Model Three-Year Traditional Plus Course Sequence*. American Diploma Project, 2007b. http://www.achieve.org/node/969.

American Diploma Project. *Ready or Not: Creating a High School Diploma That Counts*. Washington, D.C.: Achieve, 2004.

College Board. *College Board Standards for College Success: Mathematics and Statistics*. New York: College Board, 2006.

———. *College Board Standards for College Success: Mathematics and Statistics: Adapted for an Integrated Curricula*. New York: College Board, 2007.

Kilpatrick, Jeremy, Jane Swafford, and Bradford Findell, eds. *Adding It Up: Helping Children Learn Mathematics*. Washington, D.C.: National Academy Press, 2001.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, Va.: NCTM, 2000.

Programme for International Student Assessment (PISA). *PISA 2006: Science Competencies for Tomorrow's World*. Paris: Organisation for Economic Co-operation and Development, 2007. http://www.pisa.oecd.org/dataoecd/30/17/39703267.pdf.

Tapping America's Potential. *Gaining Momentum, Losing Ground*. 2008. http://www.tap2015.org/news/tap_2008_progress.pdf.

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