

Essential Concepts in High School Mathematics

Catalyzing Change identifies a set of Essential Concepts from the content domains of number, algebra and functions, statistics and probability, and geometry and measurement. These concepts represent the most critical content from these content domains—the deep understandings that are important for students to remember long after they have forgotten how to carry out specific techniques or apply particular formulas.

Essential Concepts in Number

- Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line.
- Quantitative reasoning includes, and mathematical modeling requires, attention to units of measurement.

Essential Concepts in Algebra and Functions

Algebra

- Expressions can be rewritten in equivalent forms by using algebraic properties, including properties of addition, multiplication, and exponentiation, to make different characteristics or features visible.
- Finding solutions to an equation, inequality, or system of equations or inequalities requires the checking of candidate solutions, whether generated analytically or graphically, to ensure that solutions are found and that those found are not extraneous.
- The structure of an equation or inequality (including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.
- Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts—in particular, contexts that arise in relation to linear, quadratic, and exponential situations.

Connecting Algebra to Functions

- Functions shift the emphasis from a point-by-point relationship between two variables (input/output) to considering an entire set of ordered pairs (where each first element is paired with exactly one second element) as an entity with its own features and characteristics.
- Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities—including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology).

Functions

- Functions can be described by using a variety of representations: mapping diagrams, function notation (e.g., $f(x) = x^2$), recursive definitions, tables, and graphs.
- Functions that are members of the same family have distinguishing attributes (structure) common to all functions within that family.
- Functions can be represented graphically, and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change, and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation.
- Functions model a wide variety of real situations and can help students understand the processes of making and changing assumptions, assigning variables, and finding solutions to contextual problems.

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Essential Concepts in Statistics and Probability

Quantitative Literacy

- Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks.
- Making and defending informed data-based decisions is a characteristic of a quantitatively literate person.

Visualizing and Summarizing Data

- Data arise from a context and come in two types: quantitative (continuous or discrete) and categorical. Technology can be used to “clean” and organize data, including very large data sets, into a useful and manageable structure—a first step in any analysis of data.
- Distributions of quantitative data (continuous or discrete) in one variable should be described in the context of the data with respect to what is typical (the shape, with appropriate measures of center and variability, including standard deviation) and what is not (outliers), and these characteristics can be used to compare two or more subgroups with respect to a variable.
- The association between two categorical variables is typically represented by using two-way tables and segmented bar graphs.
- Scatterplots, including plots over time, can reveal patterns, trends, clusters, and gaps that are useful in analyzing the association between two contextual variables.
- Analyzing the association between two quantitative variables should involve statistical procedures, such as examining (with technology) the sum of squared deviations in fitting a linear model, analyzing residuals for patterns, generating a least-squares regression line and finding a correlation coefficient, and differentiating between correlation and causation.
- Data-analysis techniques can be used to develop models of contextual situations and to generate and evaluate possible solutions to real problems involving those contexts.

Statistical Inference

- Study designs are of three main types: sample survey, experiment, and observational study.
- The role of randomization is different in randomly selecting samples and in randomly assigning subjects to experimental treatment groups.
- The scope and validity of statistical inferences are dependent on the role of randomization in the study design.
- Bias, such as sampling, response, or nonresponse bias, may occur in surveys, yielding results that are not representative of the population of interest.
- The larger the sample size, the less the expected variability in the sampling distribution of a sample statistic.
- The sampling distribution of a sample statistic formed from repeated samples for a given sample size drawn from a population can be used to identify typical behavior for that statistic. Examining several such sampling distributions leads to estimating a set of plausible values for the population parameter, using the margin of error as a measure that describes the sampling variability.
- Simulation of sampling distributions by hand or with technology can be used to determine whether a statistic (or statistical difference) is significant in a statistical sense or whether it is surprising or unlikely to happen under the assumption that outcomes are occurring by random chance.

Probability

- Two events are independent if the occurrence of one event does not affect the probability of the other event. Determining whether two events are independent can be used for finding and understanding probabilities.
- Conditional probabilities—that is, those probabilities that are “conditioned” by some known information—can be computed from data organized in contingency tables. Conditions or assumptions may affect the computation of a probability.

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Essential Concepts in Geometry and Measurement

Measurement

- Areas and volumes of figures can be computed by determining how the figure might be obtained from simpler figures by dissection and recombining.
- Constructing approximations of measurements with different tools, including technology, can support an understanding of measurement.
- When an object is the image of a known object under a similarity transformation, a length, area, or volume on the image can be computed by using proportional relationships.

Transformations

- Applying geometric transformations to figures provides opportunities for describing the attributes of the figures preserved by the transformation and for describing symmetries by examining when a figure can be mapped onto itself.
- Showing that two figures are congruent involves showing that there is a rigid motion (translation, rotation, reflection, or glide reflection) or, equivalently, a sequence of rigid motions that maps one figure to the other.
- Showing that two figures are similar involves finding a similarity transformation (dilation or composite of a dilation with a rigid motion) or, equivalently, a sequence of similarity transformations that maps one figure onto the other.
- Transformations in geometry serve as a connection with algebra, both through the concept of functions and through the analysis of graphs of functions as geometric figures.

Geometric Arguments, Reasoning, and Proof

- Proof is the means by which we demonstrate whether a statement is true or false mathematically, and proofs can be communicated in a variety of ways (e.g., two-column, paragraph).
- Using technology to construct and explore figures with constraints provides an opportunity to explore the independence and dependence of assumptions and conjectures.
- Proofs of theorems can sometimes be made with transformations, coordinates, or algebra; all approaches can be useful, and in some cases one may provide a more accessible or understandable argument than another.

Solving Applied Problems and Modeling in Geometry

- Recognizing congruence, similarity, symmetry, measurement opportunities, and other geometric ideas, including right triangle trigonometry in real-world contexts, provides a means of building understanding of these concepts and is a powerful tool for solving problems related to the physical world in which we live.
- Experiencing the mathematical modeling cycle in problems involving geometric concepts, from the simplification of the real problem through the solving of the simplified problem, the interpretation of its solution, and the checking of the solution's feasibility, introduces geometric techniques, tools, and points of view that are valuable to problem solving.

