

plus 3 is 18, so $35 - 17 = 18$.” Another student might think about the same problem in this way: “I changed the 35 to 37 and first solved $37 - 17$, which is 20. Then I have to subtract the 2 I added, which gives me 18.” Such strategies as these, built on decomposing and recomposing numbers, allow students to become flexible in their thinking and solve problems mentally in addition to using and understanding the standard paper-and-pencil algorithm.

Instruction to Support a Focused Curriculum

Questions to Reflect On

- What characterizes instruction that supports depth of understanding and connections among mathematical ideas?
- How can questioning be used to support the development of depth of understanding and connections in a focused curriculum?
- What is the role of practice in a focused curriculum?
- What impact does instruction that supports a focused curriculum have on time management?

Although NCTM’s Curriculum Focal Points can help prioritize and organize mathematics content, teachers and the instruction they provide are crucial to using Focal Points to improve students’ learning. Focusing mathematics instruction on a few central ideas at each grade requires skilled teachers who know the content well and can connect mathematical ideas and teach for depth of understanding.

The Use of the Process Standards

Teachers must incorporate the Process Standards of Problem Solving, Reasoning and Proof, Communication, Connections, and Representation as described in *Principles and Standards for School Mathematics* (NCTM 2000) into classroom instruction. Teachers should create a climate that supports mathematical thinking and communication. In this kind of classroom, students are accustomed to reasoning about a mathematical problem and justifying or explaining their results, representing mathematical ideas in multiple ways, and building new knowledge, as well as applying knowledge through problem solving. Brief descriptions of the Process Standards can be found in the table that follows. More detailed descriptions can be found in *Principles and Standards for School Mathematics* (NCTM 2000).

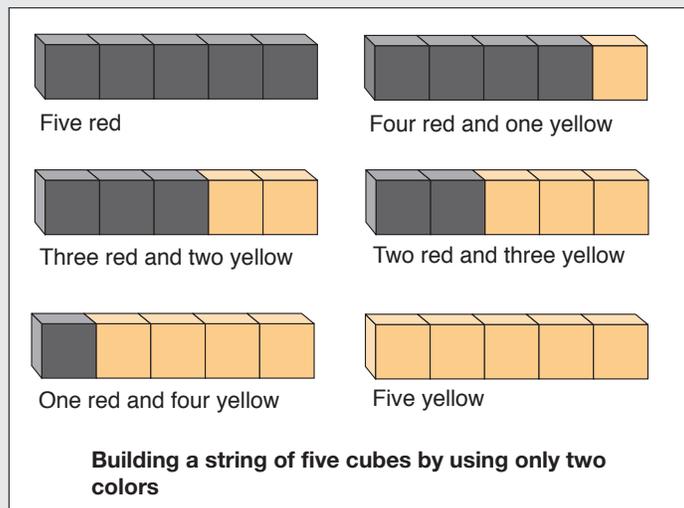
Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.

—The Teaching Principle,
*Principles and Standards for
School Mathematics*

Focusing mathematics instruction on a few central ideas at each grade requires skilled teachers who know the content well and can connect mathematical ideas and teach for depth of understanding.

The NCTM Process Standards

Problem Solving. Through problem solving, students can not only apply the knowledge and skills they have acquired but can also learn new mathematical content. Problem solving is not a specific skill to be taught, but should permeate all aspects of learning. Teachers should make an effort to choose “good” problems—ones that invite exploration of an important mathematical concept and allow students the chance to solidify and extend their knowledge. For example, suppose a first-grade class is working on learning the basic addition facts. Instead of simply giving the students a list of addition problems to solve, such as $3 + 2$, the teacher could ask the students to build as many different towers of five cubes using only two different colors of cubes (see figure below). Such an activity not only helps students understand and learn basic addition facts but also explores the commutative property as well as connections to other areas, such as patterns. A subsequent activity could involve building towers of ten cubes using only two different colors. The instructional strategies used in the classroom should also promote collaborative problem solving. Students’ learning of mathematics is enhanced in a learning environment that is a community of people collaborating to make sense of mathematical ideas (Hiebert et al. 1997).



Reasoning and Proof. For students to learn mathematics with understanding, it must make sense to them. Teachers can help students make sense of the mathematics they are learning by encouraging them to always explain and justify their solutions and strategies, as well as to evaluate other students’ ideas. Questions such as “Why?” and “How do you know?” should be a regular part of classroom discussions. The teacher should respond in ways that focus on thinking and reasoning rather than only on getting the correct answer. Incorrect answers should not

simply be judged wrong. Instead, teachers can help students identify the parts of their thinking that may be correct, often leading to new ideas and solutions that are correct.

Communication. Reasoning and proof go hand in hand with the process of communication. Students should have plenty of opportunities and support for speaking, writing, reading, and listening in the mathematics classroom. Communicating one’s ideas orally and in writing helps solidify and refine learning. Listening to others’ explanations can also sharpen learning by providing multiple ways to think about a problem. The teacher plays an important role in developing students’ communication skills by modeling effective oral and written communication of mathematical ideas as well as giving students regular opportunities to communicate mathematically. Precise mathematical vocabulary and definitions are important, and teachers need to help students articulate these ideas and ensure that students understand these ideas during class discussions.

Connections. As students move through the grades, they should be presented with new mathematical content. Students’ abilities to understand these new ideas depend greatly on connecting the new ideas with previously learned ideas. Mathematics is an integrated field of study and should be presented in this way instead of as a set of disconnected and isolated concepts and skills. Instruction should emphasize the interconnectedness of mathematical ideas both within and across grade levels and should be presented in a variety of contexts.

Representation. Mathematical ideas can be represented in a variety of ways: pictures, concrete materials, tables, graphs, numerical and alphabetical symbols, spreadsheet displays, and so on. Such representations should be an essential part of learning and doing mathematics and should serve as a tool for thinking about and solving problems. Teachers should model representing mathematical ideas in a variety of ways and discuss why some representations are more effective than others in particular situations.

Facilitating Classroom Discourse

The Process Standards, especially the Communication Standard and the Reasoning and Proof Standard, are related to the discourse in the mathematics classroom. “The discourse of a classroom—the way of representing, thinking, talking, agreeing, and disagreeing—is central to what and how students learn mathematics” (NCTM 2007, p. 46). The teacher plays an important role in initiating and facilitating this discourse and can do so by—

- posing questions and tasks that elicit, engage, and challenge each student’s thinking;
- listening carefully to students’ ideas and deciding what to pursue in depth from among the ideas that students generate during a discussion;
- asking students to clarify and justify their ideas orally and in writing and by accepting a variety of presentation modes;
- deciding when and how to attach mathematical notation and language to students’ ideas;
- encouraging and accepting the use of multiple representations;
- making available tools for exploration and analysis;
- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let students wrestle with a difficulty; and
- monitoring students’ participation in discussions and deciding when and how to encourage each student to participate. (NCTM 2007, p. 45)

The following classroom vignette depicts a kindergarten teacher facilitating a discussion about counting.

Kindergarten Vignette

Ms. Nakamura has done a lot of number work with her kindergarten class this year, and she is pleased with the results. Now, near the end of the year, the class has been investigating patterns in the number of various body parts in the classroom—how many noses or eyes, for example, are present among the children in the class.

Earlier that week, each child had made a nose out of clay. Ms. Nakamura opens the discussion by revisiting that project. She asks, “And how many noses did we make?”

Becky (points to her nostrils): Two of these.

Teacher: But how many actual noses?

Anne: Twenty-nine.

Teacher: Why? Why were there twenty-nine noses?

Adam: Because every kid in the class made one clay nose, and that is the same number as kids in the class.

Teacher (pointing to her nostrils): Now Becky just said—remember what these are called?

Children: Nostrils!

Teacher: So were there twenty-nine nostrils?

Pat: No, there were more.

- Gwen:* Fifty-eight! We had fifty-eight nostrils!
Teacher: Why fifty-eight?
Gwen: I counted.
Felice: If we had thirty kids, we would be sixty. So it is fifty-nine 'cause it should be one less.
Teacher: Can you explain that again?

The teacher probes Felice's answer even though her approach goes beyond what many of the children are trying to do at this point.

- Felice:* It's fifty-nine because we don't have thirty kids, we have twenty-nine, so it is one less than sixty.
Teacher: What does anyone else think?
Adam: I think it is fifty-eight. Each kid has two nostrils. So if sixty would be for thirty kids, then it has to be two less: fifty-eight.

The teacher solicits other students' reactions instead of showing them the right answer. Her tone of voice and her questions show the students that she values their thinking.

- Lawrence:* But Felice says thirty kids makes sixty ...
Felice: No! Adam makes sense. Fifty-eight.

The teacher moves on, asking, "What else do you think we have on our bodies that would be more than twenty-nine?"

The teacher's question challenges students to think it is open-ended; more than one right answer exists.

- Graham:* More than twenty-nine fingers.
Teacher: More than twenty-nine fingers? Why do you think so?
Graham: Because each kid, we have ten fingers.
Ricky: More than twenty-nine shoes.
Teacher: More than twenty-nine shoes. And what are those shoes covering?
Ricky: Your feet.
Sarah: Ears.
Beth: More than twenty-nine legs.

Ms. Nakamura tells the children, "You did some good thinking today!" The teacher chooses to comment on the children's thinking instead of their behavior.

Ms. Nakamura tells the children that they now are to work on a picture: "Choose some body part, and draw a picture of how many of those we have in our class and how you know that." She directs the children back to their tables, where she has laid out paper and cans of crayons.

Source: *Mathematics Teaching Today* (Martin 2007, pp. 48–50).

Most students in prekindergarten–grade 2 naturally share their ideas and thinking and are comfortable talking aloud as they solve problems. The teacher’s role is to help sustain and advance these discussions, particularly with the intent of highlighting essential mathematical ideas. In the classroom example in the preceding vignette, the teacher’s use of questioning and asking students to explain and justify their solutions invites many students to participate in the discussion. The dialogue clearly shows that the students in this classroom are accustomed to sharing their ideas and not relying on the teacher as the sole authority for solutions to problems. They are learning to be mathematical thinkers and to listen to and question others’ approaches to solving problems, behaviors that help solidify their own mathematical knowledge.

The Use of Questioning to Focus Learning and Promote Connections

As described in the introduction and the “Focusing Curriculum” section of this guide, using Focal Points to organize instruction does not mean teaching less or more content, but instead means directing the majority of instruction at a smaller number of core areas with the goal of students’ gaining a deeper mathematical understanding of those mathematical ideas and the connections among them. To teach for depth of understanding, teachers need to understand what their students are thinking and be able to support and extend that thinking. A teacher’s use of questioning plays a vital role in focusing learning on foundational mathematical ideas and promoting mathematical connections. Such reasoning questions as “Why?” and “How do you know that?” posed during a lesson are great starters, but teachers also need to incorporate questioning techniques into their planning by thinking about specific questions to ask related to the particular topic being studied. When planning instruction, teachers must also anticipate the kinds of answers they might get from students in response to the questions posed.

Let us look at the following classroom example to show a teacher’s use of questioning as well as students’ questioning in a second-grade lesson that focuses on place-value concepts and multidigit addition.

A teacher’s use of questioning plays a vital role in focusing learning on foundational mathematical ideas and promoting mathematical connections.

Second-Grade Vignette

Teacher: Here’s our first problem for today (points to a horizontal $76 + 58$ written on the board). Who can give me a word problem for this? Doug?

Doug: There were 76 girls and 58 boys waiting for the buses. How many children are waiting?

Teacher: So we’re all going to solve this in our own ways. Then three of the people at the board will explain their methods and how they relate to their drawings. [Pause as everyone solves,