Chapter 1

Stories of Success: Changing Students' Lives through Sense Making and Reasoning

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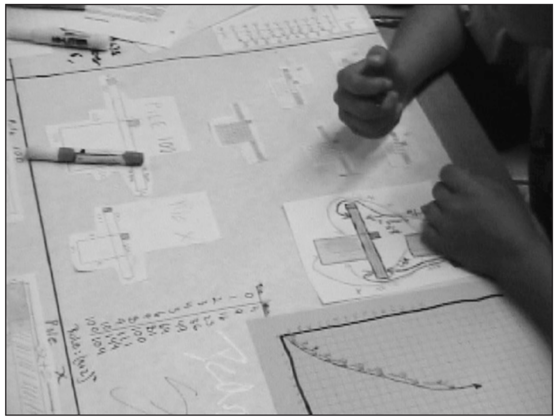
Over my years of researching mathematics classrooms in England and the United States, I have been fortunate enough to study two examples of secondary math teachers' bringing about incredible and exciting achievements for students. Both teachers focused on reasoning and sense making, and I will describe how their actions in classrooms led students to great success not only in school but also in the rest of their lives. Often researchers measure the success of different approaches by focusing on test scores. I will share the test scores of students taught in different ways, but I will also show that students' experiences in math classrooms that focused on sense making had an impact far beyond test performance. This is because learning is an experience that changes us as people; learning is not merely accumulating facts and information but also a way of shaping our beliefs, ideas, and lives. For the U.S. study, for which I followed students through four years of math classes, I will explain how teachers made reasoning and sense making a core part of their practice and how that changed their students—not only in achievement but also in how they valued other people and how they saw the world. For the UK study, I followed students through their math classes for three years and then followed up with them some eight years later to find out how their teachers' emphasis on reasoning and sense making had affected their lives. We will learn about students from a wide range of backgrounds who came to love math, to achieve at high levels, and to have a different life because of their math classrooms.

The Communicative Approach

Railside High School is an urban California high school with students from many cultures and ethnicities. The sound of speeding trains often interrupts lessons. As with many urban schools, the buildings look as though they need some repair. But Railside is not like other urban schools in all respects. Calculus classes in urban schools often have poor enrollment or are nonexistent, but at Railside, eager and successful students pack these classes. Visitors to the math classrooms were amazed, seeing all the students hard at work, engaged, and excited about math. For a research project I conducted at Stanford University, we compared the learning of the Railside students to that of a similar-sized group of students in two other high schools offering mathematics through a more typical, traditional approach. In the traditional classes the students sat in rows at individual desks; they



did not talk about mathematics or engage in sense making or reasoning. Instead the students watched the teacher demonstrate procedures at the start of lessons and then worked through textbooks filled with short, procedural questions. The two schools using the traditional approach were more suburban, and students started the schools with higher mathematics achievement levels than those of the students at Railside. But by the end of the first year of our research study, the Railside students were achieving at the same levels as the students in the more suburban schools on tests of algebra; by the end of the second year, the Railside students were significantly outperforming the other students on tests of algebra and geometry (Boaler and Staples 2008; Boaler 2008a).

In addition to the high achievement at Railside, the students enjoyed math more. In surveys administered at various times during the four years of the study, the students at Railside were always significantly more positive and more interested in mathematics than the students from the other classes. By their senior year, an incredible 41 percent of the Railside seniors were in advanced classes of precalculus and calculus, compared with only 23 percent of students from traditional classes. Further, when we interviewed 105 students (mainly seniors) at the end of the study about their future plans, almost all students from the traditional classes said that they had decided not to further pursue mathematics as a subject—even when they had succeeded. Only 5 percent of students from the traditional classes planned a future in mathematics, compared with 39 percent of Railside students. Perhaps most impressive of all: when students had started Railside, different ethnic and cultural groups had significant achievement differences, but by the time they left Railside all the differences had diminished and sometimes disappeared. The achievement differences between students of different ethnic and cultural groups remained in the other schools that taught mathematics traditionally.

I first visited Railside in 1999 because I had learned that the teachers collaborated and planned teaching ideas together, and I was interested to see their lessons. I saw enough in that visit to invite the school to be part of a new project to investigate the effectiveness of different mathematics approaches. Some four years later, after a team of doctoral students from Stanford University and I had observed, interviewed, and assessed 700 students as they progressed through different high schools, we knew that Railside's approach was both highly successful and highly unusual.

The mathematics teachers at Railside used to teach by using traditional methods, but the teachers were unhappy with the high failure rates among students and the students' lack of interest in math, so the teachers worked together to design a new approach. Teachers met together over several summers to devise a new algebra curriculum and later to improve all the courses offered. They also detracked classes and made algebra the first course that all students would take upon entering high school. In most algebra classes, students work through questions designed to give practice on mathematical techniques such as factoring polynomials or solving inequalities. At Railside the students learned the same methods, but the curriculum was organized around bigger mathematical ideas, with unifying themes such as "What is a linear function?" A focus of the Railside approach was multiple representations, which is why I have described it as communicative—the students learned about the different ways that mathematics could be communicated through words, diagrams, tables, symbols, objects, and graphs. As the students worked in their groups, they were always asked to explain work to each other, moving between different representations and communicative forms. Students worked in groups all the time, and the problems they worked on were longer and more conceptual than those the students in the traditional schools worked on (I give an example later). As the students worked in their groups, they were asked to explain their reasoning to each other and to the teacher. The students were taught to be responsible for each other's learning, and they were allowed to move on to a new problem only when everyone in the group had understood. In support of this approach, students at Railside were taught that they had two important responsibilities in math class: to ask for help if they needed it and to help anyone who needed it. After the groups had worked for a while, the teachers asked different students to come and present ideas to the class—to reason and to make sense of their methods and solutions publicly—and teachers encouraged students to ask the presenters for reasons, as third-year students Latisha and Ana explained:

Interviewer: What happens when someone says an answer?

Ana: We'll ask how they got it.

Latisha: Yeah, because we do that a lot in class. . . . Some of the students—it'll be the students that don't do their work, that'd be the ones, they'll be the ones to ask step by step. But a lot of people would probably ask how to approach it. And then if they did something else, they would show how they did it. And then you just have a little session!

The Railside classrooms were all organized in groups, and students helped each other as they worked. The teachers paid attention to how the groups worked together, and they taught students to respect the contributions of other students, regardless of prior achievement or status.

One unfortunate but common effect in such situations is that sometimes students develop beliefs about the inferiority or superiority of different students. In the traditional classes we studied, students talked about other students as smart and dumb, quick and slow. At Railside, the students did not talk in these ways; they talked instead about students who did or did not do their work (as Latisha's preceding comment shows). This did not mean that they thought all students were the same, but they came to appreciate the diversity of the classes and the various attributes that different students offered. Zane, a second-year student, said, "Everybody in there is at a different level. But what makes the class good is that everybody's at different levels, so everybody's constantly teaching each other and helping each other out."

The teachers at Railside followed an approach called complex instruction, designed to make group work more effective and to promote equity in classrooms (Cohen and Lotan 1997). The teachers continually emphasized that all children were smart and had strengths in different areas and that everyone had something important to offer when working on math. One interesting aspect of the complex-instruction approach is the creation of multidimensional classrooms. Many mathematics classrooms value one practice above all others: executing procedures correctly and quickly. The narrowness by which this system judges success means that some students rise to the top of classes, gaining good grades and teacher praise, whereas others sink to the bottom—and most students know who falls into each category. Such classrooms are unidimensional: the dimensions along which success is presented are singular. At Railside the teachers created multidimensional classes by valuing many dimensions of mathematical work. They achieved this outcome in part by having students work in groups and by giving students "group-worthy problems": open-ended problems that illustrated important mathematical concepts, allowed for multiple representations, and focused on sense making and reasoning. But the school's approach had another, rarer important aspect: the teachers enacted an expanded conception of mathematics and "smartness." The teachers at Railside knew that being good at mathematics involves many different ways of working, as mathematicians' accounts tell us. It involves asking and making sense of questions, drawing pictures and graphs, rephrasing problems, and justifying and reasoning, in addition to calculating with procedures. Instead of just rewarding the correct use of procedures, Railside teachers encouraged and rewarded all these different ways of being mathematical.

In interviews with students from both the traditional and the Railside classes, we asked students what succeeding in math class took. Students from the traditional classes were unanimous: it involved paying careful attention—watching what the teacher did and then doing the same. Students from the Railside classes talked of many different activities, including asking good questions, rephrasing problems, explaining ideas, being logical, justifying methods, representing ideas, and bringing a different perspective to a problem. Put simply: because Railside offered many more ways to succeed, many more students *did*. The following interview comments from Janet and Jasmine, both first-year students, indicate the multidimensionality of classes and the central role of reasoning and sense making.

Janet: Back in middle school, the only thing you worked on was your math skills. But here you work socially, and you also try to learn to help people and get help. Like, you improve on your social skills, math skills, and logic skills.

Jasmine: With math you have to interact with everybody and talk to them and answer their questions. You can't be just, like, "Oh, here's the book, look at the numbers and figure it out."

Interviewer: Why is that different for math?

Jasmine: It's not just one way to do it. . . . It's more interpretive. It's not just one answer. There's more than one way to get it. And then it's, like, "Why does it work?"

Hearing students describe mathematics as broader and more *interpretive* than other subjects is rare. This breadth and the teachers' continued emphasis on "why does it work?" were important to the high levels of success and participation.

The teachers also used roles in complex instruction. In groups, students had a particular role to play, such as facilitator, team captain, recorder/reporter, or resource manager (Cohen and Lotan 1997). Teachers gave students roles to ensure that everyone had something important to do and to make the group work more equal. Railside teachers often emphasized the different roles. For example, they would pause at the start of class to remind facilitators to help people check answers or explain their thinking or to ask the group "what did you get for number 1?" or "did anyone get a different answer?" Or they would ask recorder/reporters whether their group needed to go over any problem with the teacher. Students changed roles at the end of each unit of work, which usually took a few weeks. The roles contributed to the impressive ways that students interacted in the classrooms as they learned that everyone had something important to do and that all students could rely on each other. Railside teachers were also careful about identifying and talking to students about all the ways in which the students were smart. The teachers knew that students—and adults—were often severely hampered in their mathematical work by thinking that they were not smart enough. The teachers also knew that every student could contribute much to mathematics and so took it upon themselves to identify and encourage students' strengths. This approach paid off, and the motivated and eager students who believed in themselves and knew they could succeed in mathematics would have impressed any visitor to the school.

The Railside teachers brought about incredible achievements, including reductions in inequalities as evidenced by test scores, through the reasoning and justification that they required students to give. Linking the inherently mathematical practice of reasoning with the promotion of equity may seem odd, but at Railside we observed a direct link between the two, for these reasons: One of the most difficult challenges that any mathematics teacher faces is students' different levels of knowledge and understanding. At Railside the classes were heterogeneous, so different students' understanding varied widely. But the fact that students were always taught to reason and justify helped with differences in student understanding: students who didn't understand work had extra opportunities to hear explanations and justifications from other students and to understand their work. Latisha (quoted earlier) said, "it'll be the students that don't do their work, that'd be the ones, they'll be the ones to ask step by step," indicating some of the support that reasoning afforded struggling students.

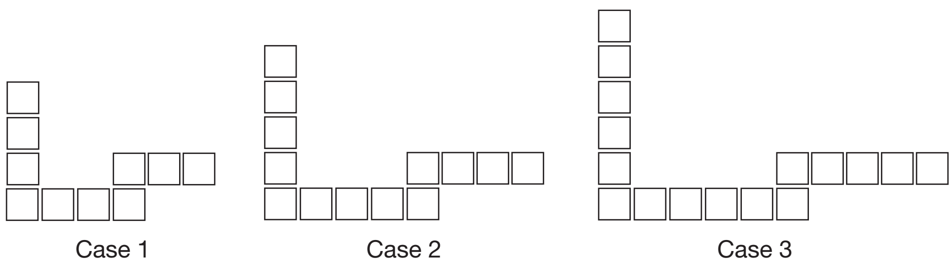
The following extract comes from second-year student Juan, one of the lower-achieving students; his description of how he copied work from someone else indicates the support that the school's focus on sense making and reasoning conveyed: "Most of them, they just, like, know what to do and everything. First you're, like, 'Why you put this?' and then, like, if I do my work and compare it to theirs . . . theirs is, like, super different 'cause they know, like, what to do. I will be, like, 'Let me copy'; I will be, like, 'Why you did this?' And then I'd be, like, 'I don't get it why you got that.' And then, like, sometimes the answer's just, like, they be, like, 'Yeah, he's right and you're wrong.' But, like, why?"

Juan had learned that it was his right to ask why and to keep asking why until he understood, which led him to encourage the student from whom he was copying to reason and justify his thinking, giving Juan more access to understanding.

In the following I give an example of one classroom activity I observed and how that teacher worked with students to encourage sense making and reasoning in their learning of algebra.

Making algebra meaningful

In one lesson I observed, students were learning about functions. The students had been given what the teachers called "pile patterns." Different students received different patterns to work with. Pedro received the following pattern, which includes the first three cases:

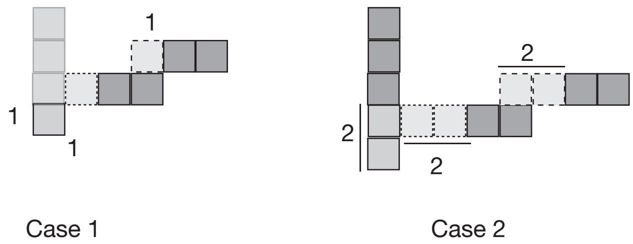


The activity was for students to work out how the pattern was growing and to represent this as an algebraic rule, a *t*-table, a graph, and a generic pattern; they also needed to show the 100th case in the sequence, having seen the first three cases.

Pedro started by working out the numbers for the first three cases, and he put these in his *t*-table:

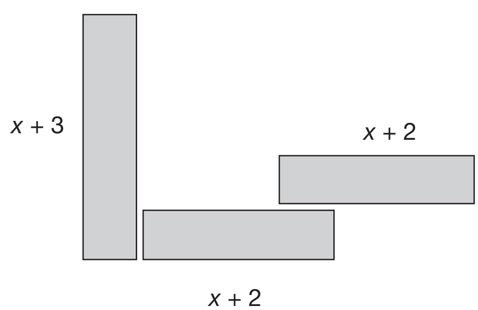
Case number	Number of tiles
1	10
2	13
3	16

He noted at this point that the pattern was "growing" by 3 each time. Next he tried to see how the pattern was growing in his shapes, and after a few minutes he saw it. He could see that each section grew by 1 each time. He represented the first two cases in the following way:



He could see that 7 tiles always stayed the same and were present in the same positions (this was how he visualized the pattern's growth, but other ways also exist). In addition to the "constant" 7, there were tiles that grew with every case number. So if we just look at the vertical line of tiles,

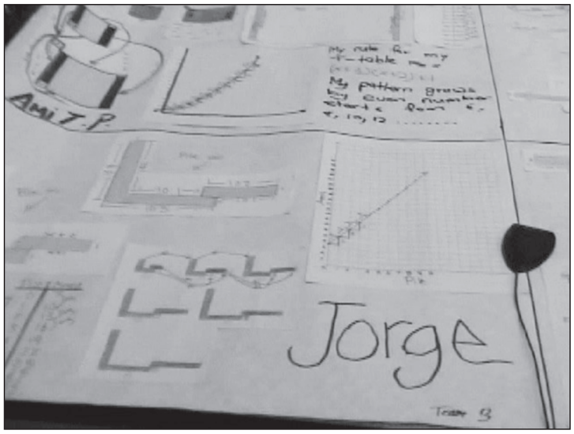
we see case 1 has 1 at the bottom, plus 3. Case 2 has 2 plus 3. Case 3 would have 3 plus 3, and case 4 would have 4 plus 3, and so on. The 3 is a constant, but each time one more is added to the lower section of tiles. We can also see that the growing section is the same size as the case number each time. When the case is 1, the total number of tiles is 1 plus 3; when the case is 2, the total is 2 plus 3; we can assume from this pattern that in the 100th case, we will get 100 + 3 tiles. This sort of work—considering, visualizing, and describing patterns—is at the heart of mathematics and its applications. Pedro represented his pattern algebraically in the following way:



Here x stood for the case number. By adding together the three sections, he could now represent the whole function as $3x + 7$.

Mathematicians, scientists, medics, computer programmers, and many other professionals use algebra so pervasively because it is a key method in describing patterns, which grow and change and are central to their work and to the world. The task in this problem—finding a way of visualizing, representing, and making sense of the pattern and using algebra to describe its changing parts—is important algebraic work.

Pedro was pleased with his work, and he decided to check his algebraic expression with his *t*-table. Satisfied that $3x + 7$ worked, he set about plotting his values on a graph. I left the group as he was eagerly reaching for graph paper and colored pencils. The next day in class I checked with him again. He was sitting with three other boys, and they were designing a poster to show their four functions. Their four desks were pushed together and covered by a large poster divided into four sections. From a distance the poster looked like a piece of mathematical artwork with color-coded diagrams, arrows connecting different representations, and large algebraic symbols.



After a while the teacher came over and looked at the boys' work, talking with them about their diagrams, graphs, and algebraic expressions; probing their thinking; and encouraging them to make sense of their work. The teacher asked Pedro where his graph represented the 7 (from $3x + 7$). Pedro showed the teacher and then decided to show the +7 in the same color on his tile patterns, on his graph, and in his algebraic expression. Communicating key features of functions by using color coding was something that all Railside students learned, to give meaning to the different representations. This technique helped the students learn something important: the algebraic expression represents something tangible, and one can see the relationships within the expression in the tables, graphs, and diagrams. This approach was one way that the teachers encouraged reasoning and sense making through algebraic work.

As well as producing posters that showed linear and nonlinear patterns, the students were asked to find and connect patterns—both in their own pile patterns and across all four teammates' patterns—and to show the patterns by using technical writing tools. One aim of the lesson was to teach students to look for patterns within and among representations and to begin to understand generalization.