Preface

Over the past seventy years or so a solid body of research evidence has been amassed pointing to the benefits of teaching for understanding in mathematics (e.g., Brownell & Moser, 1949; Brownell & Sims, 1946; Cohen, McLaughlin, & Talbert, 1993; Fuson & Briars, 1990; Hiebert & Carpenter, 1992; Hiebert & Wearne, 1993; Hiebert et al., 1996; Kilpatrick, Swafford, & Findell, 2001; Stein & Lane, 1996). In addition to "teaching for understanding," many other terms (such as *authentic instruction, ambitious instruction, higher-order instruction, meaningful instruction, problem-solving instruction,* and *sense-making instruction*) have been used in roughly synonymous ways to convey that mathematics classrooms should be places where students engage actively, deeply, and intellectually in understanding mathematical ideas rather than being treated as passive recipients of knowledge conveyed by the teacher and textbook.

Although there are many unanswered questions about precisely how specific teaching practices are linked to students' learning mathematics with understanding (see Hiebert & Grouws, 2007), and there remain other questions about how students come to understand important mathematical ideas and the obstacles they may encounter along the way, the mathematics education community has placed increasing emphasis on investigating and using teaching practices that are oriented toward the development of students' conceptual understanding, and also on studying the developmental pathways along which students develop mathematical understanding. The National Council of Teachers of Mathematics (NCTM) has played a key role in promoting the importance of teaching mathematics in ways that foster the learning of mathematics with understanding, and this volume is another resource to support the mathematics education community in this quest.

This volume also builds on the foundation laid by the pioneering work of Judith Sowder and Bonnie Schappelle to promote the integration of educational research with educational practice in mathematics education. In 2002 NCTM published *Lessons Learned from Research*, edited by Sowder and Schappelle. That book was a compendium of adapted or abstracted articles that had originally appeared in the *Journal for Research in Mathematics Education* (*JRME*) during the period of Sowder's editorship (1996–2000). It addressed in a novel way what many view as a longstanding problem in education—namely, the perceived gap between educational practice and policy on the one hand and educational research and scholarship on the other.

Although some may view the gap between research and practice in education as an inevitable consequence of the differences between the professional work of educational researchers and educational practitioners, many others view it as a problem that can and must be solved. The desire to narrow or completely close this gap is often fueled by a belief that educational practice could be more effective if it were informed by the best available knowledge from educational research. Thus, making the findings of educational research available to practitioners is viewed as an important strategy for increasing the quality of education. Yet virtually everyone agrees that the form in which research is disseminated within the research community is unlikely to be immediately comprehensible or useful to those interested in its application in classroom instruction, instructional design, or teacher preparation. As Sowder and Schappelle wrote in the introduction to their book, "Teachers rarely access original research reports, perhaps because researchers tend to write in a style that is often

not teacher-friendly. Few teachers ever open an issue of the . . . *Journal for Research in Mathematics Education* or, for that matter, any other research journal, unless they are assigned to do so for professional development or for a graduate class" (2002, p. 1).

To address both the gap between research and practice and the inaccessibility of education research reports to teachers and other mathematics education practitioners, Sowder and Schappelle undertook the creation of a compilation based on original research reports published in *JRME* but rewritten with an audience of teachers in mind. This was a novel approach to bridge the gulf between research and practice, and it was an important first step in contemporary efforts to help researchers and practitioners find common ground and learn from each other. In the decade following the publication of that book, NCTM has sponsored reports, conferences, and awards that encourage two-way traffic across the bridge that was constructed initially by Judy Sowder and Bonnie Schappelle.

With this new publication we take another step in the journey toward bridging the gap between research and practice in mathematics education. Like its companion volume (Silver & Kenney, 2015), which focused on research related to core mathematical processes and practices, such as those delineated in the Standards for Mathematical Practice in the Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers [NGA Center & CCSSO], 2010) or the NCTM Standards (NCTM 1989, 2000), this volume contains a collection of adapted or abstracted articles that originally appeared in the *Journal for Research in Mathematics Education*. All contributions to this volume are based on original research reports that appeared in *JRME* during the period 2000–2010, which includes a span of four years (2000–2004) when we edited the journal. (Edward Silver was editor, and Patricia Kenney was assistant editor.) Emulating the approach taken by Sowder and Schappelle (2002), each original article was rewritten to make the content more accessible to and useful for a teacher audience.

The twenty-four articles selected for inclusion in this volume were ones that we judged to be relevant to the theme of helping all students learn mathematics with understanding. The contributions are all based on research articles that examined the learning and teaching of key school mathematics content topics and that investigated the effects of innovative and ambitious teaching and curriculum approaches to mathematics instruction inspired by *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and *Principles and Standards for School Mathematics* (NCTM, 2000). At its core this collection of research articles addresses whether and how it might be possible to assist all students to gain proficiency with and understanding of important mathematical ideas and processes.

The book is organized into three sections. The first section includes articles related to equity and access in relation to ambitious mathematics expectations for all students, and the second and third sections include articles related to one or more topics that are known to be difficult for many students to learn, important for all students to have a chance to learn, and foundational to success in the further study of mathematics. In the second section we trace ideas and skills associated with the long-standing school mathematics trajectory from arithmetic to algebra and on to calculus. The chapters in the third section are based on research related to the topics of rational numbers and probability, which are well known to be very difficult for many students to learn with understanding.

By organizing the articles into thematically related clusters, we hope to provide readers with an opportunity to think not only about the content of each article individually but also about what the articles in a section might contribute to an understanding of what it might take to achieve the grand challenge of making high-quality mathematics teaching and learning available for all students and having students meet our lofty expectations.

As a general rule we believe that research knowledge accumulates across investigations, including those investigations that build systematically on prior work as well as those that

come from different perspectives and converge in some way on a key understanding. Although the research articles in this volume are provided individually, and we think that each has independent merit, it is by looking analytically and synthetically across these articles and other work not included here that the reader is likely to gain the most useful and usable knowledge and valuable insights that can inform and enable improvement of mathematics teaching and learning.

The preparation of this volume has increased our optimism that the gap between research and practice can be bridged. When we contacted the authors of the articles we had identified for this volume, they all agreed to participate. In fact, they agreed with enthusiasm because they saw the value of sharing their work with practitioners. Moreover, working in the spirit of this volume, many of these authors had already produced some version of their work for distribution to practitioners.

We thank the authors for their willingness to prepare adapted versions of their articles or to respond to abstracted versions that we prepared on their behalf. We also want to thank Myrna Jacobs, NCTM's former publications manager, for conveying the request from the NCTM Educational Materials Committee to one of us (Silver) to produce a sequel to Sowder and Schappelle's *Lessons Learned from Research.* We are especially grateful to Julie Schorfheide for her skillful copyediting of manuscripts and to Anita Draper for her assistance in moving the book efficiently through the NCTM production process. Finally, we acknowledge the support of the Usable Scholarship in Education (USE) Initiative at the University of Michigan.

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