## **Contents**

Foreword	٧
Preface v	'ii
Introduction	1
Why Proof and Proving	1
Understanding Proof and Proving	
Big Ideas and Essential Understandings	
Benefits for Teaching, Learning, and Assessing	
Ready to Begin	5
Chapter 1	7
Proof and Proving: The Big Ideas and Essential Understandings	
Five Big Ideas and Related Essential Understandings	8
Proof Is an Integral Part of Mathematics: Big Idea 1	
Everyday math activities can be occasions for proving 1	11
Proof can involve many activities 1	
Evaluating others' arguments is an important proof-related activity 1	
Proof as Mathematical Argument: Big Idea 2	
Proof relies on true statements and assumptions 2	
Proofs can rely on a variety of modes of argumentation	
Proofs can take many representational forms	
Proof Shows Truth beyond Any Doubt: Big Idea 3	12
A proof is only as valid as the statements, axioms, and	
definitions that it uses	
Proof ensures that no counterexamples exist	
Not All Arguments Are Proofs: Big Idea 4	
Examples as a part of proving	
The Role of Proof: Big Idea 5	
Proof to verify the truth of a statement	
Proof as a source of insight	
Proof as an entry point 4	
Proof as a structure for communication	
Proof as an impetus for precise language	
Conclusion	
Chapter 2.	57
Connections: Looking Back and Ahead in Learning	
One Problem, Three Proofs 5	57
Argument 1—Empirical algebraic justification 5	
Argument 2—Deductive geometric justification 6	

Arguments 1 and 2 compared	65 67
Evolving forms of proof	
Changing modes of argumentation	
Advancing levels of generality	
Conclusion	72
Chapter 3	73
Challenges: Learning, Teaching, and Assessing	
Learning Goal 1: Developing Fluency in Proving	74
Introducing proof outlines	76
Making proving routine	77
Modifying problems as proving opportunities	78
Learning Goal 2: Understanding the Limitations of Examples	
Providing problems for which examples fail	
Asking for explanations why instead of demonstrations that	84
Learning Goal 3: Moving beyond Examples-Based Arguments	
to Deductive Proofs	85
Building from examples to conjectures	
Making a conjecture wall	
Clarifying the criteria for a valid proof	88
Asking for a different approach	89
Having students make sense of peers' proofs	
Assessing Students' Understanding of Proof	
Incorporating proving in quizzes and exams	
Distinguishing levels of proficiency in students' proving	
Conclusion	
References	99