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### Appendix 2

Resources for Teachers

### Appendix 3

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Sizing Up the Angle  
Finding Right Angles in Shapes  
Characterizing Obtuse Angles  
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Drawing a Square, Given Two Points  
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