Chapter 1

Focus on Number Choice

In our work, we have found that careful selection of number choices can be a powerful instrument for developing students' understanding and for meeting multiple aspects of successful mathematics teaching. Consider the Pennies problem written for a third-grade classroom from *Investigations in Number, Data, and Space* (TERC 2012, p. 71):

- Last month, Luis collected 97 pennies. This month, he collected 143 pennies. How much money does Luis have now?

What do you notice about these number choices?

We noticed a few things:

- There is a two-digit number plus a three-digit number.
- The two-digit number comes first.
- One of the numbers is close to 100.
- The answer (sum) will go over another hundred.
- The 7 (from 97) and 3 (from 143) can be combined to make a ten.
- The 7 from 97 could be added to 143 to make 150; or the 3 from 143 could be added to 97 to make 100.

You might have noted some other characteristics of this number choice. In general, we try to look for the following characteristics of number choices, including those in both the problem and the solution:

- Size of the numbers
- Complexity of the numbers
- Proximity of numbers to "landmark" numbers (e.g., decade numbers, 100, 25; TERC 2012)
- Ways in which the numbers can be combined (e.g., 7 from 97 and 3 from 103 can be combined to make a ten)

- Relationships among the numbers (Are the numbers close together or far apart? Do they lend themselves to decomposing and recomposing in particular ways?)

These various aspects of the numbers in a problem help us identify the mathematical goals that might be addressed as students work to solve the problem.

Directing our attention on number choice also helps teachers think about how students might engage with a problem. Some students might struggle with the numbers in the Pennies problem above, while others may quickly arrive at a solution—illustrating the need for alternate or additional number choices. The number choices above are interesting in that they can be manipulated in various ways to arrive at a solution. For instance, as already noted, the 7 and 3 can be combined to make a ten, or the 97 could be changed to 100. But will students notice those aspects of the numbers? In order for students to use and understand these strategies based on place value, they might first need to experience number choices that promote making a ten alone, for example, (8, 2), and those that are part of larger numbers, for instance, (14, 46). They might also need experiences for going over a hundred and for using the commutative property to solve problems.

To illustrate how a focus on productive number choice might work, we rewrote the numbers in the Pennies problem to support the development of mathematics content, strategies, and practices as described in the Common Core while meeting the needs and strengths of individual students:

- Last month, Luis collected _____ pennies. This month, he collected _____ pennies. How much money does Luis have now (TERC, 2012, p. 35)?

  (57, 43) (97, 43) (97, 143) (107, 143) (290, 357)

- The first number choice, (57, 43), works for students who are not yet comfortable crossing the hundred mark. Additionally, it gives students practice making tens. For example, students solving this number choice might first make 90 (50 + 40), and then combine it with 10 (3 + 7) to make 100. Or, they might make 10 first.

- The second number choice, (97, 43), is similar to the numbers in the original problem. Again, the number choice involves potentially making a ten in the ones column, but the total in this choice, as opposed to the first choice, is greater than 100. This is also a good number choice for supporting a compensation strategy in that students could add the 3 (from 43) to the 97 to make 100, and then add 40. In addition, this number choice provides a scaffold to the third number choice in the original problem (97, 143). If students see that 143 is 100 more than 43, they can simply add 100 to their solution to 97 + 43 to solve 97 +143.

- The third number choice, (97, 143), is from the original problem.
- The fourth number choice, (107, 143), again stresses the composition of 3 and 7 to make 10, but adds the element of seeing the relationship between 97 and 107.
- The final number choice, (290, 357), provides a challenge for students who have already learned the content and strategies supported by the original number choice. It also affords another opportunity for students to compensate. Students could change 290 to 300, add 357, and then subtract 10.

The Common Core mathematics content in this example is "adding and subtracting fluently within 1000" (3.NBT.2). Also included within that standard is the expectation that students will use "strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction" (NGA Center and CCSSO 2010, p. 24). The new number choices in the Pennies problem are designed to support the use of those strategies and algorithms. By being selective in our number choices, we have provided opportunities for students to make tens, make hundreds, and use compensation. In addition, we have provided increasingly more difficult content across the number choices.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Fig. 1.1. Thoughtful number choices for worthwhile mathematical tasks can support students' engagement in the mathematical practices.

The Standards for Mathematical Practice (NGA Center and CCSSO 2010) are also promoted by the number choices above, which provide several opportunities for students to develop and engage in the majority of the practices. In fact, the use of the multiple number choice structure, in and of itself, presents repeated situations for students to engage in and develop the mathematical practices, as will be discussed in greater detail in chapter 2. As part of the first mathematical practice, the Common Core State Standards for Mathematics (CCSSM) states that mathematically proficient students "make conjectures about the form and the meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt" (NGA and CCSSO 2010, p. 6). Through the exploration and solving of a variety of number choices that are designed to have relationships across them, students are prompted to use those relationships in their solutions, and, thus, we are supporting students in the second mathematical practice. When the teacher facilitates discussions about students' solutions across the different number choices, the number choices also support students' engagement in practices 3, 4, 7, and 8. We describe the ways in which number choices support the development of the mathematical practices in greater detail throughout the book.

To further illustrate how number choices can be optimized, we provide the Fishbowl problem written by Molly for a multiage second- and third-grade classroom:

- Sam had _____ fishbowls. He had _____ fish in each bowl. How many fish did he have?

A	B	C	D
(2, 10)	(4, 20)	(3, 11)	(4, 12)
(5, 10)	(8, 20)	(6, 11)	(8, 12)

In second grade, students are to "work with equal groups of objects to gain foundations for multiplication" (2.OA). In third grade, CCSSM calls for students to "represent and solve problems involving multiplication and division" (3.OA). This problem works well for both of these standards. We have found that multiplication problems tend to be accessible at almost all grade levels and, with appropriate number choice, can build content knowledge and strategies in several areas.

For the mathematics strategy, we chose a fourth-grade standard: "Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations" (NGA Center and CCSSO 2010, p. 29, 4.NBT.5). We chose this standard because we wanted to illustrate how work in the earlier grades can address, but also go beyond, grade-level standards. The first set of number choices (2, 10) and (5, 10) may prompt students to either draw a picture or skip-count by multiples of ten. In the last two columns, numbers were chosen to encourage students to use the distributive property in which students would break apart the tens and ones. For example, $3 \times 10 = 30$ plus 3 more ones would be 33.

In the latter three columns, numbers were chosen so that students would use what they know about one number choice to solve for another, for instance, to use the answer from 4×20 to solve for 8×20. Therefore, we maintain that the number choices promote the second mathematical practice. Additionally, the number choices may prompt students to “look for and make use of structure,” Mathematical Practice 7, by discerning a pattern in the number choices.

Chapter 2

Multiple Number Choice Structure and Differentiation

You probably noticed that our examples in chapter 1 provided multiple number choices for each problem. We call that the *multiple number choice structure*. The structure was first introduced to Molly, Jenny, and Natalie when they participated in Cognitively Guided Instruction (CGI) professional development made available through the Iowa Department of Education. The multiple number choice structure was presented as a method to differentiate instruction, but our use of it has evolved into the ways in which we use number choice progressions to support student access to problems, to promote student engagement in the mathematical practices, and to develop students' relational thinking. In other words, one clear advantage of structuring problems with multiple number choices along a progression is the promotion of equity within the mathematics classroom. Neither students who need extra support nor those who need extra challenges are singled out for "special" instruction. By using the multiple number choice structure, the needs of all students can often be met with one task without separating students based on ability. Furthermore, this structure enables all students to have access to more rigorous or challenging mathematics and to participate together in mathematical discussions—not just those students who have been identified as needing advanced mathematics content.

We do not suggest that you begin using the multiple number choice structure with as many options as we used in the Pennies and Fishbowl problems or with sets of number choices as in the Fishbowl problem. Instead, we suggest gradually introducing number choices to students. Start by using blanks in problems where numbers would typically be supplied and by providing two number choices so students can gain experience with how the process of "plugging in" the numbers works. This first step toward the multiple number choice structure also allows you, as the teacher, to develop your expectations and procedures for recording and management of students' number choices and strategies as well as to anticipate productive responses to the student who says, "What do I do now; I'm done?"

If you use a textbook, take one problem from the text and provide an additional number choice, either up or down the progression level from the number(s) provided in the textbook problem—depending on the strengths and needs of your students. If you think the numbers in the textbook will be too challenging for many of your students, formulate a number choice that will be more accessible to those students. Similarly, if the book's number choice would be too easy, pick a