Part 3

Number and Operations—Fractions

Number and Operations—Fractions

Domain Overview

GRADE 3

Students use visual models, including area models, fraction strips, and the number line, to develop conceptual understanding of the meaning of a fraction as a number in relationship to a defined whole. They work with unit fractions to understand the meaning of the numerator and denominator. They build equivalent fractions and use a variety of strategies to compare fractions. In Grade 3, denominators are limited to 2, 3, 4, 6, and 8.

GRADE 4

Fourth graders extend understanding from third grade experiences, composing fractions from unit fractions and decomposing fractions into unit fractions, and apply this understanding to add and subtract fractions with like denominators. They begin with visual models and progress to making generalizations for addition and subtraction fractions with like denominators. They compare fractions that refer to the same whole using a variety of strategies. Using visual models and making connections to whole number multiplication supports students as they begin to explore multiplying a whole number times a fraction. In Grade 4, denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100. Students build equivalent fractions with denominators of 10 and 100 and connect that work to decimal notation for tenths and hundredths.

GRADE 5

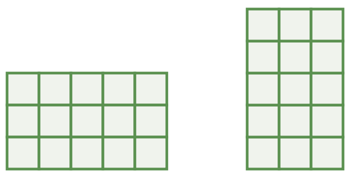
Fifth graders build on previous experiences with fractions and use a variety of visual models and strategies to add and subtract fractions and mixed numbers with unlike denominators. Problem solving provides contexts for students to use mathematical reasoning to determine whether their answers make sense. They extend their understanding of fractions as parts of a whole to interpret a fraction as a division representation of the numerator divided by the denominator. Students use this understanding in the context of dividing whole numbers with an answer in the form of a fraction or mixed number. They continue to build conceptual understanding of multiplication of fractions using visual models and connecting the meaning to the meaning of multiplication of whole numbers. The meaning of the operation is the same; however, the procedure is different. Students use visual models and problem solving contexts to develop understanding of dividing a unit fraction by a whole number and a whole number by a unit fraction. Once conceptual understanding is established, students generalize efficient procedures for multiplying and dividing fractions.

SUGGESTED MATERIALS FOR THIS DOMAIN

3	4	5	
	✓	✓	Decimal models (base-ten blocks) (Reproducible 4)
✓	✓	✓	Fraction area models (circular) (Reproducible 5)
✓	✓	✓	Fraction area models (rectangular) (Reproducible 6)
✓	✓	✓	Fraction strips/bars (Reproducible 7)
✓	✓	✓	Grid paper (Reproducible 3)
✓	✓	✓	Objects for counting, such as beans, linking cubes, two-color counter chips, coins
✓	✓	✓	Place value chart (Reproducible 8)

KEY VOCABULARY

3	4	5	
✓	✓	✓	**area model** a concrete model for multiplication or division made up of a rectangle. The length and width represent the factors, and the area represents the product.



3×5 5×3

3	4	5	
✓	✓	✓	**benchmark** a number or numbers that help to estimate or determine the reasonableness of an answer. Sample benchmarks for fractions include 0, $\frac{1}{2}$, 1.
	✓	✓	**decimal fraction** a fraction whose denominator is a power of 10, written in decimal form (for example, 0.4, 0.67)
✓	✓	✓	**denominator** the number of equal-sized pieces in a whole, the number of members of a set with an identified attribute. The bottom number in a fraction.
	✓	✓	**equivalent fractions** fractions that name the same amount or number but look different (*Example:* $\frac{2}{3}$ and $\frac{6}{9}$ are equivalent fractions)
	✓	✓	**hundredth** one part when a whole is divided into 100 equal parts
	✓	✓	**like denominator (common denominator)** having the same denominator
	✓	✓	**like numerator (common numerator)** having the same numerator

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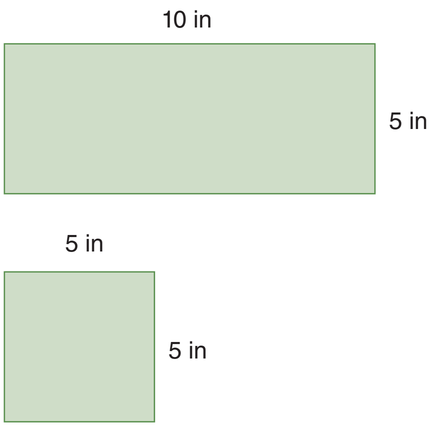
KEY VOCABULARY

3	4	5	
✓	✓	✓	**measurement division (equal groups model)** a division model in which the total number of items and the number of items in each group is known. The number of groups that can be made is the unknown. *Example:* I have 3 yards of ribbon. It takes $\frac{1}{6}$ of a yard to make a bow. How many bows can I make? (How many groups of $\frac{1}{6}$ yards can I make from 3 yards?)
	✓	✓	**mixed number** a number that is made up of a whole number and a fraction (for example, $2\frac{3}{4}$)
✓	✓	✓	**numerator** the number in a fraction that indicates the number of parts of the whole that are being considered. The top number in a fraction.
✓	✓	✓	**partitive division (fair share model)** a division model in which the total number and the number of groups is known and the number of items in each group is unknown. *Example:* Erik has $\frac{1}{2}$ of a gallon of lemonade. He wants to pour the same amount in 5 glasses. How much lemonade will he pour into each glass if he uses all of the lemonade?
		✓	**scale (multiplication)** compare the size of a product to the size of one factor on the basis of the size of the other factor *Example:* Compare the area of these rectangles. When you double *one* dimension, the area is doubled. 
	✓	✓	**tenth** one part when one whole is divided into 10 equal parts
✓	✓	✓	**unit fraction** a fraction with a numerator of one, showing one of equal-sized parts in a whole (for example, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$)

Number and Operations—Fractions[1]

3.NF.A*

[1] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

Cluster A

Develop understanding of fractions as numbers.

STANDARD 1

3.NF.A.1: Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.

STANDARD 2

3.NF.A.2: Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

b. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.

STANDARD 3

3.NF.A.3: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

Examples: Express 3 in the form $3 = \frac{3}{1}$*; recognize that* $\frac{6}{1} = 6$*; locate* $\frac{4}{4}$ *and 1 at the same point of a number line diagram.*

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

*Major cluster

Number and Operations—Fractions[1] 3.NF.A

Cluster A: Develop understanding of fractions as numbers.

[1] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

Grade 3 Overview

As students begin to develop understanding of fractions as a special group of numbers, they work with area models (circles, rectangles, and squares), fraction strips and fraction bars, and the number line to explore the meaning of the denominator and the meaning of the numerator. Unit fractions, fractions with a numerator of 1, form the foundation for initial fraction work. Students extend work with unit fractions to comparing fractions and finding simple equivalent fractions. Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8, which provides an opportunity to develop deep understanding of these foundational concepts.

Standards for Mathematical Practice
SFMP 2. Use quantitative reasoning.
SFMP 3. Construct viable arguments and critique the reasoning of others.
SFMP 4. Model with mathematics.
SFMP 5. Use appropriate tools strategically.
SFMP 6. Attend to precision.
SFMP 7. Look for and make use of structure.
SFMP 8. Look for and express regularity in repeated reasoning.

As third graders begin formal work with fractions, first and foremost they understand that fractions are numbers. They reason with physical models including area models, fraction strips, and number lines to understand unit fractions, such as $\frac{1}{4}$, as one part of a defined whole cut into four equivalent parts. They begin to develop an understanding of the meaning of the numerator and the denominator. Students extend their understanding of the structure of fractions beyond unit fractions, using visual representations to explain their thinking. They use repeated reasoning to compose other fractions from unit fractions including fractions equal to or greater than 1. Connecting area models to fraction strip models and to number lines provides a meaningful progression of models. This helps students to make generalizations as they build understanding of the meaning of common fractions extended to fractions greater than one. They use this understanding to compare and find equivalent fractions.

Related Content Standards

1.G.A.3 2.G.A.3 3.G.A.2 4.NF.A.1 4.NF.A.2 4.NF.B.3 4.NF.C.5

Notes

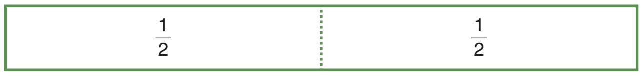
STANDARD 1 (3.NF.A.1)

Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.

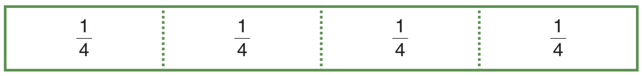
Note: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

A fundamental goal throughout work across fraction clusters is for students to understand that fractions are numbers. They represent a quantity or amount that happens to be less than, equal to, or greater than 1. Too often we project the notion of fractions as parts of a whole without emphasizing that they are special numbers that allow us to count pieces that are part of a whole. Fractions in third grade are about a whole being divided (partitioned) into equal parts. Suggested models for Grade 3 include area models (circles, squares, rectangles), strip or fraction bar models, and number line models. Set models (parts of a group) are not models used in Grade 3. This Standard is about understanding unit fractions (fractions with a numerator of 1) and how other fractions are composed of unit fractions.

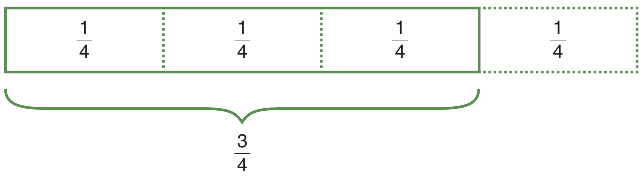
Folding a strip into 2 equal parts (one fold), each part or section would be $\frac{1}{2}$.



Folding a strip into 4 equal parts (three folds), each part or section would be $\frac{1}{4}$.

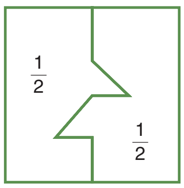


The fraction $\frac{3}{4}$ is the quantity formed by 3 parts that are each $\frac{1}{4}$ of the whole.

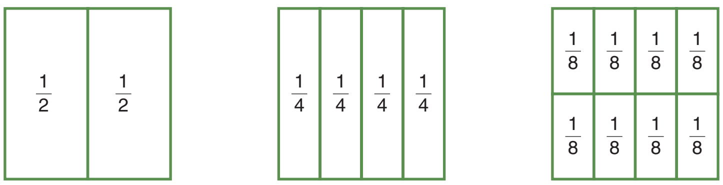


Important ideas for students to consider as they begin their work with fractional parts include:

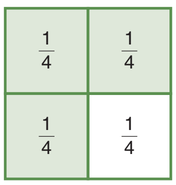
- When working with any type of area model (circles, squares, rectangles) or strip models, fractional parts must be of equal size (but not necessarily equal shape). Using grid paper or geoboards can help students to determine when two pieces are the same size even if they are not the same shape.



- The denominator represents the number of equal size parts that make a whole.
- The more equal pieces in the whole (greater denominator), the smaller the size of the piece.



- The numerator of a fraction represents the number of equal pieces in the whole that are counted.



$\frac{3}{4}$ are shaded.

What the TEACHER does:

- Begin with strip models. These can simply be strips of construction paper about 2 inches by 11 inches. It is important that students understand that one strip represents one whole. If it is possible to use different colors it will help students to identify and compare fractions.
- Have students fold one strip into 2 equal parts and label each part $\frac{1}{2}$.
    - Ask students to make a conjecture about the meaning of the 2 in $\frac{1}{2}$ (the number of equal-size parts the whole strip).
    - Ask students to make a conjecture of the meaning of the 1 in $\frac{1}{2}$ (each piece is one part of the whole).
- Repeat the process folding and labeling strips for fourths, eighths, thirds, and sixths.
- Introduce the terms *numerator* and *denominator*. Ask students to explain what each term means based on this activity.
- Show students $\frac{3}{4}$ of a strip. Ask them what part (fraction) of one whole strip that amount represents. Students should use the terminology *numerator* and *denominator* in justifying their reasoning (that is, I know it is $\frac{3}{4}$ because it is made up of $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$).
- Prepare other activities in which students name parts of a whole and describe them as the sum of unit fractions.
- Use a variety of concrete representations for activities in which students compare the size of various unit fractions and then develop an understanding that the larger the denominator, the smaller the size of the piece. Using the same size whole is an important part of this understanding.
- When students are ready, use two different size wholes to have them talk about when $\frac{1}{4}$ might be greater than $\frac{1}{2}$. (When $\frac{1}{4}$ is part of a larger whole than $\frac{1}{2}$.)
- Give examples of fraction models that are equal size but not equal shape. Use area models or geoboards to have students represent unit fractions that are equal sized but not equal shape.

What the STUDENTS do:

- Make models of fractions (with denominators of 2, 3, 4, 6, and 8) using fraction strips. Label each part with the correct unit fraction.
- Describe the meaning of the denominator and the numerator using pictures, numbers, and words.
- Name various parts of the whole using fractions and explain that the fraction is made up of that number of unit pieces.
  $\frac{5}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$
- Demonstrate an understanding that given the same size whole, the larger the denominator the smaller the size of the pieces because there are more pieces in the whole. Students demonstrate understanding by explaining their reasoning using concrete materials, pictures, numbers, and words.
- Identify and demonstrate fractional parts of a whole that are the same size but not the same shape using concrete materials.

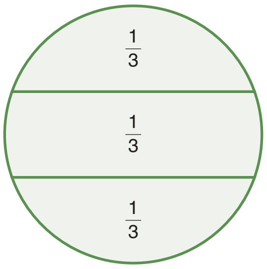
Addressing Student Misconceptions and Common Errors

There are many foundational fraction ideas in this Standard, and it is important to take the time necessary to develop student understanding of each idea. This is best accomplished through extensive use of concrete representations, including fraction strips, area models, fraction bars, geoboards, and similar items. Do not work with too many representations at the same time. Begin with activities that use area models and reinforce those idea with fraction strips and then number lines. For most students one experience with a concept will not be adequate to develop deep understanding.

Students who demonstrate any of the following misconceptions need additional experiences connecting concrete representations to fraction concepts:

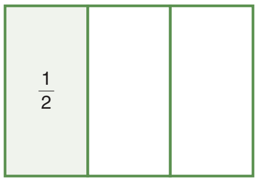
- Given the same size whole, the smaller the denominator, the smaller the piece.
- Fraction pieces must be the same shape and size.

- Students write a fraction numeral based on the number of pieces in a whole even if they are not the same sized pieces.



Misconception: Student considers the number of pieces in the whole but does not understand they must be the same size.

- Student label fractions as $\frac{\text{part}}{\text{part}}$ rather than as $\frac{\text{part}}{\text{whole}}$.



Misconception: Student writes the fraction as a part to part relationship rather than $\frac{1}{3}$ (part to whole).

Notes

GRADE 3

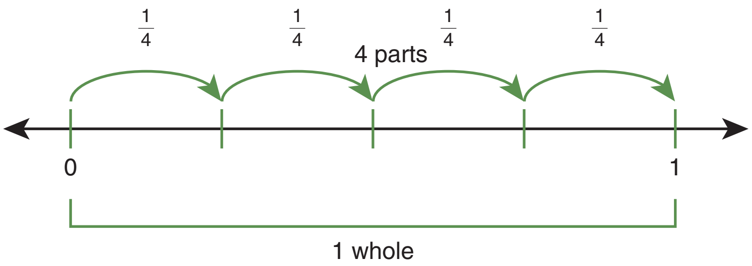
STANDARD 2 (3.NF.A.2)

Understand a fraction as a number on the number line; represent fractions on a number line diagram.

Note: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

a. *Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.*

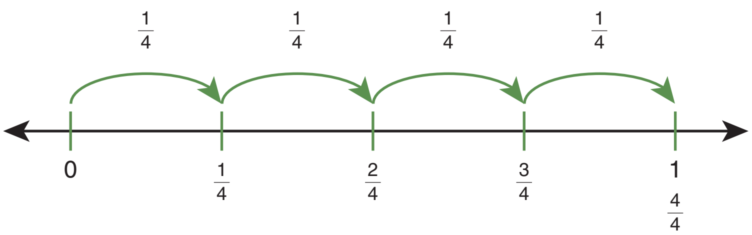
Students have had previous experience with whole numbers on the number line. They extend this understanding by focusing on subdividing the distance from 0 to 1. Representing $\frac{1}{4}$ on the number line requires students to understand the distance from 0 to 1 represents one whole. When they partition this distance, the whole, into 4 equal parts, each part has the size of $\frac{1}{4}$. They also reason and justify the location of unit fractions by folding strips or on the number line. Previous work with fraction strips or fraction bars can be extended to developing parts on the number line.



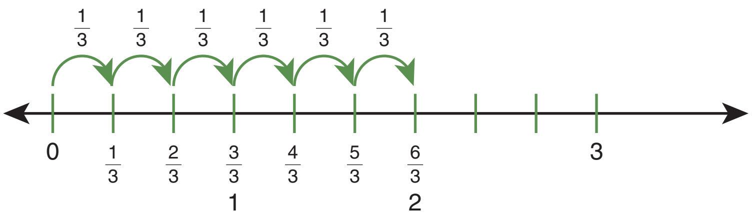
b. *Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a length $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.*

As students develop conceptual understanding of unit fractions they extend this to work counting unit fractions to represent and name other fractions on the number line.

For example, represent the fraction $\frac{3}{4}$ on a number line by marking off lengths of $\frac{1}{4}$ starting at 0. They can explain that 3 pieces of $\frac{1}{4}$ ($\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$) or that the distance from 0 to that point represents $\frac{3}{4}$ on the number line.

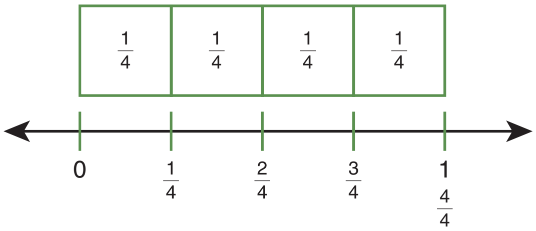


This Standard also includes work with improper fractions, not as a special group of fractions but as a continuation of counting unit fractions. By extending the number line, students develop the understanding that fractions equal to 1 have the same numerator and denominator and fractions greater than 1 have a numerator that will be greater than the denominator. They develop this understanding by counting on the number line using unit factions and recognizing patterns with fractional numbers.



What the TEACHER does:

- Provide students with fraction strips (Reproducible 7) and number lines and ask students to transfer the parts from the fraction strip to the number line.
- Model labeling unit fraction intervals on the number line.
- Ask students to use the unit fraction intervals to "count" and label the fraction name for each division from zero to one.



- Facilitate discussions in which students explain their reasoning as they label the number line.
- Repeat this process for fractions with denominators of 2, 3, 4, 6, and 8.
- Extend the number line to numbers greater than 1 using the same rationale for naming points on the number line.
- Provide students with many opportunities to describe patterns they see as they label number lines.

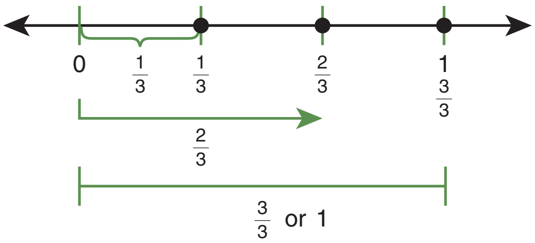
What the STUDENTS do:

- Use fraction strips to find fractional parts on the number line.
- Label intervals and points on the number lines. Intervals are unit fractions. Points on the number line represent the distance from 0 to that specific point and are made up of the number of unit fraction intervals.
- Demonstrate how they labeled the number line and explain their thinking.
- Extend number lines and activities to include fractions greater than 1.

GRADE 3

Addressing Student Misconceptions and Common Errors

Although it is not critical for students to differentiate between the intervals between points and actual points on the number line, you want to be careful not to cause any misconceptions. The fraction that names a point on the number line describes the distance of that point from 0 and not the point itself.



Notes

STANDARD 3 (3.NF.A.3)

Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

Note: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

a. *Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.*

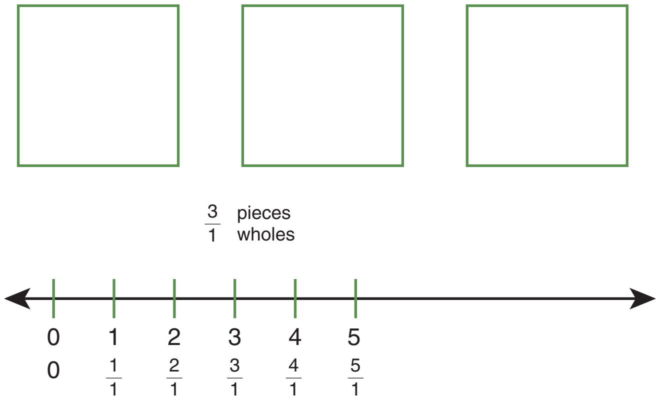
The number line is one of several models such as area models and fraction bar models that can help students to develop conceptual understanding of equivalent fractions. Concrete experiences drawing area models and folding fraction strips should gradually transition to equivalent fractions on the number line.

b. *Recognize and generate simple equivalent fraction, e.g.,* $\frac{1}{2} = \frac{2}{4}, \frac{4}{6} = \frac{2}{3}$*. Explain why the fractions are equivalent, e.g., by using a visual fraction model.*

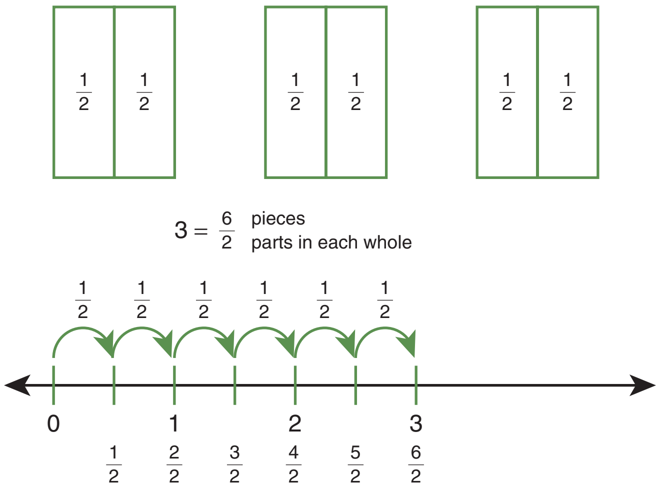
Patterns with visual models help students to reason and justify why two fractions are equivalent. The use of procedures or algorithms is not a third grade expectation.

c. *Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.* Examples: Express 3 in the form $3 = \frac{3}{1}$; recognize that $\frac{6}{1} = 6$; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.

The foundational understanding of this Standard is established by providing experiences for students to recognize that any whole number can be expressed as a fraction with a denominator of 1. Previous experiences developing the understanding that the denominator tells the number of pieces into which one whole has been partitioned now extends to situations in which the whole is not divided and remains in 1 piece, resulting in a denominator of 1.



Students extend this understanding to dividing a number of area models that are wholes into parts and determining the resulting fraction.

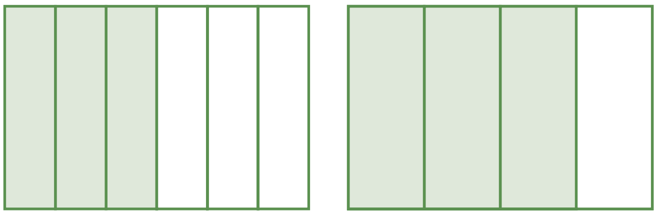


Classroom discussions and visual representations lead students to make the connection between fraction representations and division. For example, the fraction $\frac{6}{2}$ represents 6 pieces that are each $\frac{1}{2}$ of one whole. Two pieces are needed to make one whole. Modeling by putting the wholes back together with each whole representing one group shows that I can make 3 wholes or groups, each of which is $\frac{2}{2}$. Therefore $\frac{6}{2}$ is the same as $6 \div 2$. Note that students are just beginning to make this connection, and multiple activities will help students to develop this understanding rather than teaching it by simply giving them a rule.

d. *Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.*

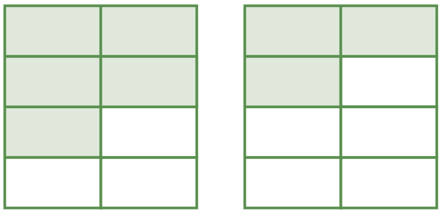
Students work with models and the number line to compare fractions with the same numerator. Models should refer to the same whole and include examples of how different size wholes impact the size of the fraction. Students explain their reasoning using pictures, words, and numbers, focusing on the meaning of the denominator as describing the number of pieces in one whole or size of the pieces and the numerator as the number of pieces or count. If the pieces are the same size (denominator), then the number of pieces (numerator) will determine which fraction is greater.

$\frac{3}{6} < \frac{3}{4}$ because sixths are smaller than fourths



Students extend their reasoning to compare fractions with different denominators and the same numerator using models and the number line, and explain their reasoning using pictures, words, and numbers. They generalize that when the number of pieces (numerator) is the same, the number of pieces in a whole (denominator) will determine which fraction is greater. The larger the denominator, the smaller the size of the piece.

$\frac{5}{8} > \frac{3}{8}$



because 5 pieces that are eighths are more than 3 pieces that are eighths

What the TEACHER does:

- Provide a variety of activities with visual models, including area models, fraction strips, and the number line, to give students experience developing conceptual understanding that
  - Many fractions can describe the same quantity or point on a number line.
  - Fractions that represent the same amount are called *equivalent fractions*.
- Use purposeful questions to help students recognize patterns in equivalent fractions.
  - What do you notice about the numerators in equivalent fractions?
  - What do you notice about the denominators in equivalent fractions?
  - What patterns do you see in the numerators and denominators in two equivalent fractions?
- Connect concrete experiences to building sets of equivalent fractions using numerals. This should not be based on a procedure or algorithm, rather by looking for patterns and having students describe what is happening to the visual representation and numbers as they find equivalent fractions.
- Provide activities and experiences in which students use visual representations to express whole numbers as fractions.

(continued)

What the TEACHER does (continued):

- ○ Cutting one whole into fourths shows that $\frac{4}{4}$ equals one whole.
- ○ Generating fractions from more than one whole. Cutting 4 wholes into thirds will result in 12 pieces. Because each piece is $\frac{1}{3}$ of a whole, the resulting fraction is $\frac{12}{3}$. Therefore $\frac{12}{3}$ is equivalent to four wholes.
- ○ Leaving several wholes intact shows that 4 can be represented as $\frac{4}{1}$ since there are 4 pieces that are each 1 whole piece.

- Provide concrete experiences for students to compare parts of the same size whole with the same numerator and different denominators. Ask questions that will help students to generalize that when the size of the piece (denominator) is the same, the number of pieces (numerator) will determine which is the greater fraction.
- Provide concrete experiences for students to compare fractions of the same size whole with the same denominator and different numerators and generalize that when the number of pieces in the whole is the same (denominator), the number of pieces (numerator) will determine which fraction is greater. The larger the denominator, the smaller the size of the piece.

What the STUDENTS do:

- Use visual representations including rectangular and circular area models, fraction bars, and the number line to find various (equivalent) fractions that name the same quantity or point.
- Build sets of equivalent fractions from visual models and by recognizing patterns.
- Explain their reasoning in building sets of equivalent fractions. For example, $\frac{3}{4}$ is equivalent to $\frac{6}{8}$ because doubling the number of pieces in the whole (denominator) then will also double the count of pieces (numerator).
- Use visual representations to find fractional names for 1.
- Use visual representations to find fractional names for several wholes that are not partitioned (denominator is 1).
- Use visual representations to find fractional names for several wholes that are partitioned into pieces.
- Explain patterns they see as they are working with wholes and their equivalent fractions.
- Provide experiences that help students to make the following generalizations:
  - ○ When the numerator and denominator are the same, the value of the number is one whole.

    $\frac{6}{6} = 1 \qquad 1 = \frac{8}{8} \qquad \frac{4}{4} = 1$

  - ○ When the denominator is 1, the fraction represents wholes. The number of wholes is the same as the numerator.

    $\frac{8}{1} = 8 \qquad 7 = \frac{7}{1} \qquad 3 = \frac{3}{1}$

  - ○ When the numerator is a multiple of the denominator, the number of wholes is their quotient.
  - ○ $\frac{12}{4} = 3 \qquad \frac{10}{2} = 5 \qquad 6 = \frac{18}{3}$

Addressing Student Misconceptions and Common Errors

As students work with equivalent fractions, it is important that they understand that different fractions can name the same quantity and there is a multiplicative relationship between equivalent fractions. Students need multiple experiences using concrete materials as they explore each of these important concepts. They need to explain their reasoning and explicitly connect visual representations (concrete and pictorial) to numerical representations. It is important that students have time to make these connections, describe patterns, and make generalizations rather than by practicing rote rules.

The following misconceptions indicate that students need more work with concrete and then pictorial representations:

- The numerator cannot be greater than the denominator.
- The larger the denominator, the larger the piece.
- Fractions are a part of a whole; therefore, you cannot have a fraction that is greater than 1 whole.
- In building sets of equivalent fractions, students use addition or subtraction to find equivalent fractions.

Notes

Sample PLANNING PAGE

Standard: 3.NF.A.1. *Understand a fraction* $\frac{1}{b}$ *as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction* $\frac{a}{b}$ *as the quantity formed by a parts of size* $\frac{1}{b}$.

Mathematical Practice or Process Standards:

SFMP 4. Model with mathematics.

Students make fraction strips to use as they begin to explore the meaning of fractional parts of a whole.

SFMP 6. Attend to precision.

Initial experiences with fractions emphasize that a fraction is a number. Students develop fraction-related vocabulary, starting with *numerator* and *denominator*.

Goal:

Students use physical models as they begin to work with fractions, focusing on the meaning of fractions as a number as well as the meanings of the numerator and the denominator.

Planning:

Materials: 3 inch by 12 inch construction paper strips for each student. If possible provide each student with five strips that are different colors. Be sure to have extra strips on hand for students who make a mistake. Color marking pens.

Sample Activity:

Begin with one strip. Designate that strip as one whole and label it 1 WHOLE.

Have students take a strip of a color (for example, red) and fold it into two parts that are the same size. Talk about the pieces. Have students describe the pieces. Have students label each piece $\frac{1}{2}$. Talk about the meaning of the 1 (it is 1 part) and the meaning of the number 2 (there are 2 parts in the whole strip).

Introduce the terms *whole*, *fraction*, *unit fraction*, *numerator*, and *denominator*. Add them to your mathematics word wall.

Continue with another color, asking students to fold the piece into four equal parts. Have a similar discussion about the pieces. Proceed with eighths, thirds, and sixths.

Notes

Sample PLANNING PAGE (Continued)

Questions/Prompts:

Ask questions that directly relate new vocabulary to the work students are doing.

Be sure to give students plenty of time to talk about what they noticed. Important ideas that should come out of the discussion include:

- The whole is the same size for each fraction.
- A fraction is a part of the whole.
- The smaller the denominator the larger the piece (thirds are greater than fourths).
- The numerator indicates it is one part of the whole. These are called unit fractions.
- The denominator indicates the number of equal-size pieces in the whole.

Save these fraction strips for future work with comparing fractions.

Differentiating Instruction:

Struggling Students: Watch for students who may struggle with figuring out how to fold the fractions, particularly thirds and sixths.

Students need to label each part with a unit fraction. Give struggling students the opportunity to talk about the size of unit fractions. It may help these students to cut the pieces apart after labeling them. Ask them to reconstruct the whole.

Have extra prepared strips for students who are not successful in folding the fraction strips into equal parts. It is important to let them try—several times.

Extension: Although it is not expected at this grade level, some students may want to experiment folding fractions with other denominators.

Notes

PLANNING PAGE

Standard:

Mathematical Practice or Process Standards:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

Number and Operations—Fractions[1]
4.NF.A*

[1] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Cluster A

Extend understanding of fraction equivalence and ordering.

STANDARD 1 **4.NF.A.1:** Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{(n \times a)}{(n \times b)}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

STANDARD 2 **4.NF.A.2:** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

*Major cluster

Number and Operations—Fractions[1] 4.NF.A

Cluster A: Extend understanding of fraction equivalence and ordering.

[1] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Grade 4 Overview

Fourth graders continue to work with equivalence beginning with models and using those models to generalize a pattern and eventually a rule for finding equivalent fractions. They justify their reasoning using pictures numbers and words. In Grade 3, students compared fractions with like numerators or like denominators. They now extend that understanding to comparing fractions with different numerators and denominators reinforcing the important comparison concept that fractions must refer to the same whole.

Standards for Mathematical Practice
SFMP 2. Use quantitative reasoning.
SFMP 3. Construct viable arguments and critique the reasoning of others.
SFMP 4. Model with mathematics.
SFMP 5. Use appropriate tools strategically.
SFMP 7. Look for and make use of structure.
SFMP 8. Look for and express regularity in repeated reasoning.

Fourth graders extend their understanding of equivalent fractions reasoning with visual models. They look for patterns both physical (when I double the number of pieces in the whole pizza, I double the number of pieces that I ate.) and think about these patterns in terms of the meaning of the numerator and the denominator. Providing experiences with appropriate visual models will help students to develop understanding rather than just following a rule that has no meaning. Through finding and discussing patterns students construct mathematical arguments to explain their thinking as they build sets of equivalent fractions. All of this work supports the fundamental structure of fractional numbers that is critical to all future work with fractions in this domain.

Related Content Standards

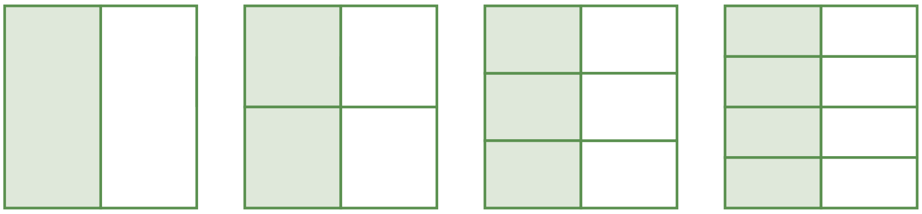
3.NF.A.2 3.NF.A.3

STANDARD 1 (4.NF.A.1)

Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{(n \times a)}{(n \times b)}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Previous work in Grade 3 included exploring to find equivalent fractions using area models, fraction strips, and the number line. Although students looked for patterns, a formal algorithm for finding equivalent fractions was not developed. Fourth graders build on prior experiences, beginning with area models, to formally describe what happens to the number of pieces in the whole and the number of pieces shaded when they compare $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$ and $\frac{4}{8}$ using models, pictures, words and numbers.



$\frac{1}{2}$ $\quad$ $\frac{2}{4}$ $\frac{1 \times 3}{2 \times 2}$ $\quad$ $\frac{3}{6}$ $\frac{1 \times 3}{2 \times 3}$ $\quad$ $\frac{4}{8}$ $\frac{1 \times 4}{2 \times 4}$

Students should be able to explain that when the number of pieces in the whole is doubled, the number of pieces in the count (the numerator) also doubles. This is true when multiplying by any factor.

Note that the Standards do not require students to simplify fractions although students may find fractions written in simpler form easier to understand. For example, if they recognize that $\frac{50}{100}$ is equivalent to $\frac{1}{2}$, they may choose to use $\frac{1}{2}$ since the two fractions are equivalent. Having students find equivalent fractions "in both directions" may help students to realize that fractions can be written in simpler form without formally simplifying fractions.

What the TEACHER does:

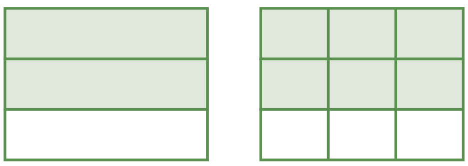
- Provide students with different models to use in building sets of equivalent fractions for visual representations and then write the fractions as numerals.
- Facilitate student discussions about patterns they see in sets of equivalent fractions.
- Expect students to use models and written numerals to generate a rule for finding equivalent fractions.
- Provide a variety of activities to help students build and recognize equivalent fractions.

What the STUDENTS do:

- Connect visual representations of equivalent fractions to numerical representations.
- Use pictures, words, and numbers to explain why fractions are equivalent.
- Generate a rule for finding equivalent fractions and follow that rule.
- Recognize equivalent fractions.

Addressing Student Misconceptions and Common Errors

Students who use addition or subtraction instead of multiplication to develop sets of equivalent fractions need additional experiences with visual representations including fraction bars, areas models, and the number line. Explanations of why one multiplies or divides to find an equivalent fraction should begin with visual representations and eventually connect to the rule/algorithm.



$\frac{2}{3}$ $\quad$ $\frac{2 \times 3}{3 \times 3} = \frac{6}{9}$

If I triple the number of pieces in the whole, that triples the number of pieces in my count.

STANDARD 2 (4.NF.A.2)

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Students compare two fractions with different denominators by creating equivalent fractions with a common denominator or with a common numerator. Using benchmarks such as 0, $\frac{1}{2}$, or 1 will help students to determine the relative size of fractions.

Students justify their thinking using visual representations (fraction bars, area models, and number lines), numbers, and words. It is important for students to realize that size of the wholes must be the same when comparing fractions.

Use benchmark fractions,

Compare $\frac{5}{6}$ and $\frac{7}{12}$.

think

$\frac{5}{6}$ is almost 1

So $\frac{7}{12}$ is a little more than $\frac{1}{2}\left(\frac{6}{12}\right)$

$$\frac{5}{6} > \frac{7}{12}$$

Use common denominators,

Compare $\frac{5}{6}$ and $\frac{3}{4}$.

think

So $\frac{5}{6} = \frac{10}{12}$ and $\frac{3}{4} = \frac{9}{12}$ and $\frac{10}{12} > \frac{9}{12}$

$$\frac{5}{6} > \frac{3}{4}$$

Use common numerators.

Compare $\frac{3}{6}$ and $\frac{5}{8}$.

think

$\frac{3}{6} = \frac{15}{25}$ $\frac{5}{8} = \frac{15}{24}$

Since 25ths are less than 24ths, $\frac{15}{25} < \frac{15}{24}$

So

$$\frac{3}{5} < \frac{5}{8}$$

Students should have opportunities to justify their thinking as well as which strategy is the most efficient to use.

What the TEACHER does:

- Provide a variety of concrete materials for students to use in comparing fractions.
  - ○ Use 0, $\frac{1}{2}$, 1 as benchmarks to compare fractions.
  - ○ Find common denominators to compare fractions.
  - ○ Find common numerators to compare fractions.

    Note: Students should determine which method makes the most sense to them, realizing that they will use different methods for different situations.
- Engage students in a variety of activities and problem solving situations in which they compare fractions and justify their reasoning using pictures, words, and numbers.

What the STUDENTS do:

- Use a variety of representations to compare fractions including concrete models, benchmarks, common denominators, and common numerators.
- Determine which method makes the most sense for a given situation and justify their thinking.
  - ○ Louisa and Linda went to the movies. Each bought a small box of popcorn. Linda ate $\frac{5}{6}$ of her popcorn and Louisa at $\frac{5}{8}$ of her popcorn. Who ate more?
  - ○ Linda ate more. Because sixths are larger than eighths, $\frac{5}{6} > \frac{5}{8}$.
  - ○ Mrs. Multiple made two pans of brownies. One pan had nuts and the other was plain. Each pan was the same size. The pan of brownies with nuts has $\frac{5}{12}$ left. The pan of plain brownies has $\frac{5}{8}$ left. Which pan has less left?

    I know that $\frac{5}{12}$ is less than $\frac{1}{2}$ (which is $\frac{6}{12}$). I know that $\frac{5}{8}$ is more than $\frac{1}{2}$ (which is $\frac{4}{8}$). Therefore the pan of brownies with nuts has less than the pan with the plain brownies because $\frac{5}{12} < \frac{5}{8}$.
  - ○ Terri has collected $\frac{2}{3}$ of the money she needs to buy her mom's birthday present. Her brother Timmy has collected $\frac{5}{6}$ of the money he needs to buy his gift. Who is closer to their goal?

    I know that $\frac{2}{3}$ is equivalent to $\frac{4}{6}$. Timmy has $\frac{5}{6}$, Terry has $\frac{4}{6}$. Timmy is closer to his goal because $\frac{5}{6} > \frac{2}{3}$ ($\frac{4}{6}$).

GRADE 4

Addressing Student Misconceptions and Common Errors

It is important for students to use reasoning and number sense to compare fractions and justify their thinking. Students who forget that the larger the number in the denominator, the smaller the piece, may base their comparisons on incorrect notions. These students need additional practice with concrete models and making connections to the written numerals. When comparing fractions, students must consider the size of the whole. One-half of a large box of popcorn is greater than $\frac{1}{2}$ of a small box of popcorn. Take time to provide a variety of experiences for students to make sense of these important concepts.

Number and Operations—Fractions[1]
4.NF.B*

[1] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Cluster B

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

STANDARD 3

4.NF.B.3: Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:*

$$\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$$

$$2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$$

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

STANDARD 4

4.NF.B.4: Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. *For example, use a visual fraction model to represent $\frac{5}{4}$ as the product $5 \times \frac{1}{4}$, recording the conclusion by the equation $\frac{5}{4} = 5 \times \frac{1}{4}$.*

b. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express $3 \times \frac{2}{5}$ as $6 \times \frac{1}{5}$, recognizing this product as $\frac{6}{5}$. (In general, $n \times \frac{a}{b} = \frac{(n \times a)}{b}$.)*

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

*Major cluster

Number and Operations—Fractions[1] 4.NF.B

Cluster B: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

[1] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

GRADE 4

Grade 4 Overview

Fourth graders continue to develop understanding of fractions as numbers composed of unit fractions (for example, $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$). They also extend their understanding that fractions greater than 1 can be expressed as mixed numbers (for example, $\frac{12}{5} = \frac{5}{5} + \frac{5}{5} + \frac{2}{5} = 2\frac{2}{5}$). They connect their understanding of addition and subtraction of whole numbers as adding to/joining and taking apart/separating to fraction contexts using fractions with like denominators. They begin with visual representations, including area models, fraction strips, and number lines, and connect these representations to written equations.

First experiences with multiplication of a fraction by a whole number begin with connecting the meaning of multiplication of whole numbers to multiplication of a fraction by a whole number (for example, $5 \times \frac{1}{4}$ means 5 groups of $\frac{1}{4}$) using visual representations. Following many experiences modeling multiplication with unit fractions by whole numbers, students continue to work with other fractions. They solve problems by modeling using area models, fraction strips, and number lines and explain their reasoning to others.

Standards for Mathematical Practice

SFMP 1. Make sense of problems and persevere in solving them.
SFMP 2. Use quantitative reasoning.
SFMP 3. Construct viable arguments and critique the reasoning of others.
SFMP 4. Model with mathematics.
SFMP 5. Use appropriate tools strategically.
SFMP 6. Attend to precision.
SFMP 7. Look for and make use of structure.
SFMP 8. Look for and express regularity in repeated reasoning.

Students extend their work with unit fractions to composing and decomposing non-unit fractions. In doing so, they reason about fractions as numbers (quantitatively) and understand that fractions, like whole numbers, represent a "count" of something. The main difference is the "something" includes part of a whole. Problem solving contexts reinforce the meaning of addition and subtraction, presenting opportunities for students to relate previous work with addition and subtraction situations with whole numbers to adding and subtracting fractions. They use models including area models, fraction strips, and number lines, and connect those visual models to written equations when they are ready. They build on previous understandings of the meaning of the numerator and denominator (precision) to see the structure of addition and subtraction and explain what is happening when they add and subtract fractions (for example, why they add or subtract numerators but keep the same denominator).

Related Content Standards

1.OA.A.1 2.OA.A.1 3.NF.A.2 3.G.A.2 5.NF.A.1 5.NF.A.2

Notes

STANDARD 3 (4.NF.B.3)

Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$.

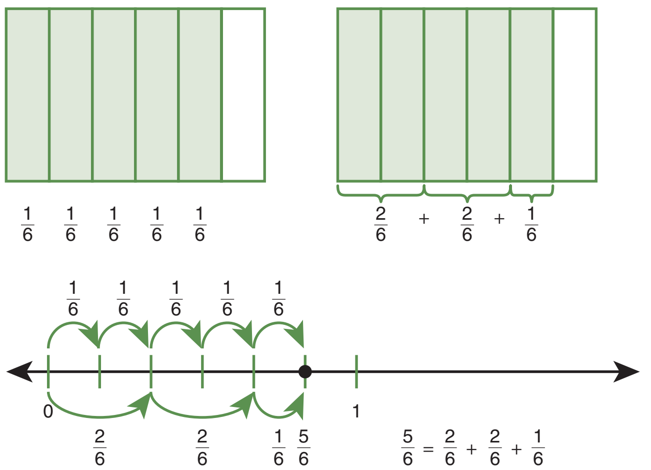
Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Unit fractions are fractions with a numerator of 1. Third graders' experiences with fractions focused on unit fractions. Their work with non-unit fractions was limited to using visual models such as fraction strips and number lines to see that fractions such as $\frac{3}{4}$ are composed of three jumps of $\frac{1}{4}$ on the number line. This is an important concept as students prepare to add and subtract fractions. Fourth grade experiences extend to composing and decomposing fractions greater than 1 (improper fractions) and mixed numbers into unit fractions Students use prior knowledge of using concrete fraction representations for whole numbers to move between mixed numbers and fractions.

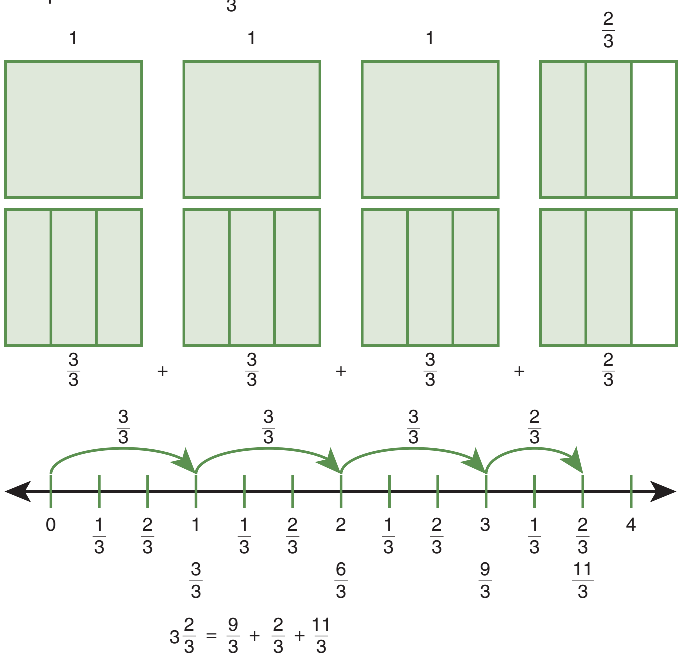
What the TEACHER does:

- Provide a variety of experiences for students to compose and decompose fractions, including fractions greater than 1 and mixed numbers, into unit fractions using concrete and pictorial representations, words, and numbers.

Representations for $\frac{5}{6}$



Representations for $3\frac{2}{3}$



What the STUDENTS do:

- Compose and decompose fractions, including fractions greater than 1 and mixed numbers, into unit fractions using concrete and pictorial representations including the number line.
- Explain their reasoning using pictures, words, and numbers.

Addressing Student Misconceptions and Common Errors

Although students may be able to decompose a fraction into unit fractions (that is, $\frac{4}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$), when given the unit fractions to compose into a fraction, they may think they need to add denominators as well as numerators. This misconception can be avoided by giving students multiple opportunities with various concrete models, pictures, and the number line and making explicit connections to written equations.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

This Standard begins with an understanding that addition and subtraction of fractions has the same meaning as addition and subtraction of whole numbers, although the process of adding and subtracting is different with fractions. Remember expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. Addition and subtraction work is limited to examples with like denominators.

What the TEACHER does:

- Give students activities that relate the meaning of addition and subtraction of fractions to addition and subtraction of whole numbers.
- Use problem solving situations with addition and subtraction of fractions relating to the same whole, and have the students determine which operation should be used to solve the problem. (See Table 1, page 254.)

What the STUDENTS do:

- Use a variety of materials to model and describe various problem situations that require adding and subtracting fractions.

Addressing Student Misconceptions and Common Errors

Students need not actually add or subtract fractions at this point, although many of them will be ready. Students who struggle with identifying a situation as an addition situation or a subtraction situation need more experience solving problems that require addition or subtraction. Modeling such situations using fraction pieces will help them to relate these operations to previous work with whole numbers (Table 1, page 254).

Notes

b. *Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.*

Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$

$2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$

This Standard includes work with improper fractions and mixed numbers.

What the TEACHER does:

- Provide a variety of activities in which students must decompose a fraction into fractions with the same denominator. Use a variety of denominators.
    - Begin with decomposing a fraction into unit fractions.

      $\frac{5}{12} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12}$
    - Ask students to combine the unit fractions to show other addends that compose the fraction.

      $\frac{5}{12} = \frac{1}{12} + \frac{1}{12} + \frac{3}{12}$

      $\frac{5}{12} = \frac{2}{12} + \frac{3}{12}$
- Facilitate discussions in which students use visual models, including area models and the number line, to justify their thinking.
- As students demonstrate understanding with fractions less than one, extend to activities with fractions greater than 1 and mixed numbers.

  $\frac{5}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

  $\frac{5}{4} = \frac{2}{4} + \frac{3}{4}$

  $\frac{5}{4} = \frac{4}{4} + \frac{1}{4}$

  $2\frac{3}{8} = \frac{8}{8} + \frac{8}{8} + \frac{3}{8}$

  $2\frac{3}{8} = \frac{16}{8} + \frac{3}{8}$
- Encourage students to find many different ways to decompose fractions and explain their reasoning.

What the STUDENTS do:

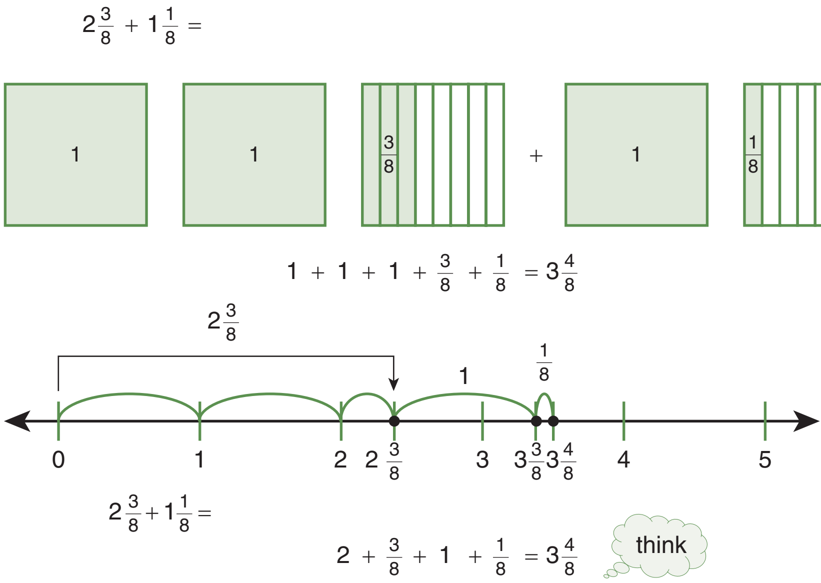
- Decompose fractions less than 1 into fractional parts with the same denominator using models, pictures, words, and numbers.
- Explain their reasoning using visual models.
- Decompose fractions greater than 1 into fractional parts with the same denominator using models, pictures, words, and numbers.
- Explain their reasoning using visual models and equations.
- Decompose mixed numbers into fractional parts with the same denominator using models, pictures, words, and numbers.
- Explain their reasoning using visual models and equations.

Addressing Student Misconceptions and Common Errors

Although this work may seem obvious to some students, it is important to take the time to build this concept because it lays the foundation for adding and subtracting fractions. Students who see fractions as composed of smaller parts develop the understanding that when they add or subtract fractions, the numerator describes the count of pieces and the denominator describes the piece. Carefully developing this concept now will avoid misconceptions many students have when adding two fractions with unlike denominators.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

After students have had ample experience composing and decomposing various fractions and mixed numbers, they work with adding and subtracting fractions with like denominators. This Standard includes work with fractions less than one, fractions greater than one, and mixed numbers. At this point students do not need to regroup or decompose mixed numbers. When adding and subtracting mixed numbers, students should use concrete materials and develop strategies that make sense to them.



Note that this Standard and 4.NF.B.3.d should be taught simultaneously so that students have contexts in which to build understanding and determine whether their answers make sense.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

(Refer to Table 1, page 254.) Students have experienced all of these situations using whole numbers in earlier grades. Use similar situations that involve fractions and mixed numbers with like denominators for students to solve as they continue to add and subtract fractions using pictures, words, and numbers. Be sure to give students many opportunities to consider the reasonableness of their answers.

What the TEACHER does:

- Provide students with fraction models, including area models, fraction bars, and number lines, to use as they solve addition and subtraction of fraction problems.
- Scaffold examples and problems.
  - Adding and subtracting fractions less than 1.
  - Adding and subtracting fractions greater than 1.
  - Adding and subtracting mixed numbers (with no regrouping).
- Expect students to solve problems using visual representations and provide opportunities to have them make explicit connections to numerical representations.
- Facilitate discussions in which students explain their thinking using materials, pictures, words, and numbers.

What the STUDENTS do:

- Use concrete materials and pictures to solve a variety of problems involving addition and subtraction of fractions and mixed numbers.
- Connect visual models to addition and subtraction equations.
- Explain their thinking using models, pictures, numbers, and words.

Addressing Student Misconceptions and Common Errors

Watch for students who may add or subtract denominators when adding and subtracting fractions. These students need additional concrete experiences and specific questions about whether their answer is reasonable. For example, if a student adds $\frac{2}{3} + \frac{3}{3}$ and gets a sum of $\frac{5}{6}$, talk about the value of the addends and the value of the sum to realize that the answer should be greater than 1.

Number lines and visual models will also reinforce correct thinking. It is important that students understand that the numerator tells the count (how many pieces) and the denominator describes the piece. Since the pieces are the same size, the numerator (count) is added and the description of the pieces does not change. When I add 2 pieces that are thirds to 3 pieces that are thirds I will get 5 pieces that are thirds.

Notes

STANDARD 4 (4.NF.B.4)

Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

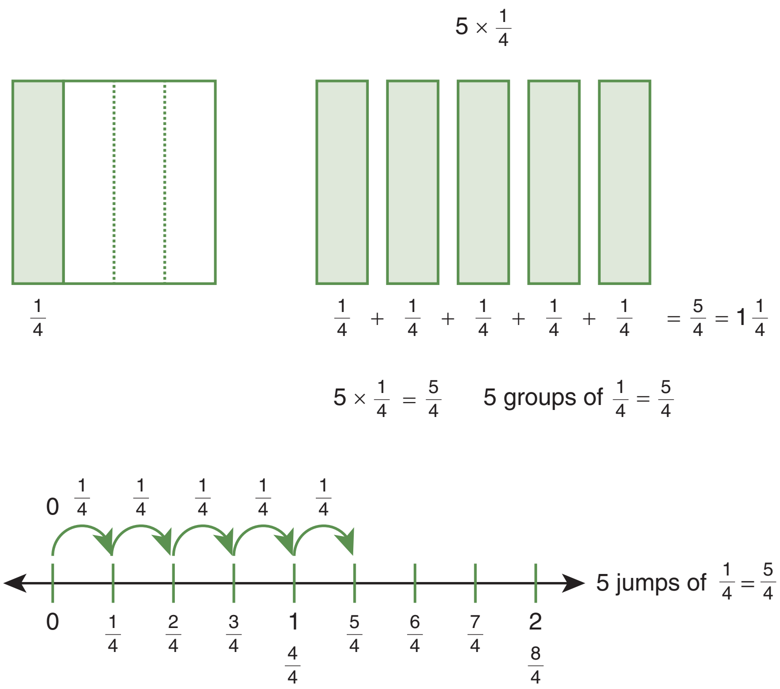
Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Students need a variety of experiences to understand that the meaning of multiplication with fractions is the same as the meaning of multiplication with whole numbers. They begin by thinking about a whole number of fractional pieces or the number of groups of a given fraction. Note that Standard 4.NF.B.4.d should be taught at the same time as Standards 4.NF.B.4.a and 4.NF.B.4.b, using appropriate numbers so that students have contexts in which to build understanding rather than focusing only on the numbers.

a. *Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$.* For example, use a visual fraction model to represent $\frac{5}{4}$ as the product $5 \times \frac{1}{4}$, recording the conclusion by the equation $\frac{5}{4} = 5 \times \frac{1}{4}$.

This Standard builds on experiences with decomposing fractions into unit fractions and connecting that understanding to multiplication.

$\frac{5}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ or 5 groups of $\frac{1}{4}$, which can be represented as $5 \times \frac{1}{4}$.



At this point, some students may find a pattern and a more efficient algorithm (procedure) for multiplying a whole number times a fraction (that is, multiply the whole number times the numerator of the fraction) but it is not an expectation for all students. The critical focus of this Standard is to develop an understanding of what is happening when multiplying a whole number times a unit fraction by relating the process to the meaning of multiplication.

Note: The language of this Standard can be confusing. When we multiply a fraction by a whole number we are thinking a whole number of groups of a given fraction (for example, $5 \times \frac{1}{4}$). Students will multiply whole numbers by fractions in Grade 5 (for example, $\frac{1}{4} \times 5$).

What the TEACHER does:

- Review the meaning of multiplication of whole numbers as one factor representing the number of "groups" and the other factor representing the number of items in a group using physical representations and the number line.
  - ○ 3×4 means I have 3 groups of 4.
  - ○ 3×4 means 3 jumps of 4 on the number line.
- Extend this meaning to physical representations of a unit fraction multiplied by a whole number using problem solving contexts.

  I bought 3 boxes of crackers. Each box had $\frac{1}{4}$ lb. What is the total weight of the crackers?
  - ○ $3 \times \frac{1}{4}$ means I have three groups of $\frac{1}{4}$.
  - ○ $3 \times \frac{1}{4}$ means 3 jumps of $\frac{1}{4}$ on the number line.
- Provide students with many experiences to model a whole number times a unit fraction.
- Facilitate student discussions in which students explain their thinking using pictures, words, and numbers.
- Watch for students who see a pattern and may generalize a "rule" for multiplication. Be certain they understand why their rule works.

What the STUDENTS do:

- Model and explain the meaning of whole number multiplication.
- Extend the model to examples in which they multiply a fraction by a whole number.
- Explain their thinking using pictures, words, and numbers.

Addressing Student Misconceptions and Common Errors

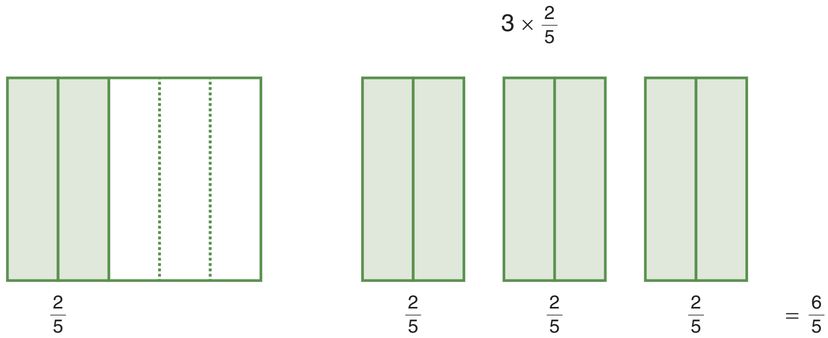
Students may see "the rule" without really understanding the connection to the meaning of multiplication. It is especially important to expect these students to model and explain their thinking rather than simply using the rule.

Some students may want to put a denominator on the whole number to relate this work to previous work with addition and subtraction of fractions. These students need additional opportunities to solve problems that provide a context for the meaning of multiplication as it relates to fractions. Once they can model the situation, help them connect the model to a written equation. Ask questions about what is happening and give them opportunities to explain what they are doing.

Notes

b. *Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number.* For example, use a visual fraction model to express $3 \times \frac{2}{5}$ as $6 \times \frac{1}{5}$, recognizing this product as $\frac{6}{5}$. (In general, $n \times \frac{a}{b} = \frac{(n \times a)}{b}$.)

Once students can multiply a whole number times a unit fraction, they extend that understanding to multiplying a whole number times any fraction first with visual models and then connecting those models to numerical representations.



Give enough practice with models and connecting those models to written equations to help students see the structure of multiplication working with fractions. The whole number represents the number of groups and the fraction represents the number of items in each group. Because the numerator gives the count of how many pieces and the denominator describes the pieces, multiplying the number of groups times the count of items in each group (the numerator) will tell the total number of pieces. Because the denominator describes the piece it does not change.

What the TEACHER does:

- Provide a variety of problem contexts for students to model multiplication of any fraction by a whole number.
- Ask questions to facilitate student explanations of their reasoning.
  - How many groups do you have?
  - How many are in each group?
  - What would this look like if you model it with fraction pieces or on the number line?
- Scaffold to problems that include fractions greater than 1 and mixed numbers.
- Help students make explicit connections between models and written equations.
- Watch for students who are able to generalize a rule for multiplying a whole number times any fraction to be certain they understand why it works as well as how it works.

What the STUDENTS do:

- Solve a variety of problems involving multiplication of a fraction by a whole number using models, including area models, fraction strips, and number lines.
- Explain their reasoning using pictures, words, and numbers.

Addressing Student Misconceptions and Common Errors

Watch for students who are rewriting the whole number as a name for 1 (for example, writing 4 as $\frac{4}{4}$ rather than $\frac{4}{1}$). In these situations students should be thinking of the whole number as the number of groups and therefore they do not need to rewrite it as a fraction.

c. *Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.* For example, if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

This Standard should be taught as the same time as parts a and b. Students should have a variety of contexts in which to solve multiplication of whole number and fraction examples so they can estimate, model, and determine whether their answers are reasonable.

What the TEACHER does:

- Provide students with a variety of multiplication problem situations (Table 2, page 256) using multiplication of fractions and mixed numbers by a whole number as contexts.
- Scaffold student experiences.
  - Begin with a fraction times a whole number. $3 \times \frac{4}{5}$
  - Multiply a fraction greater than 1 by a whole number. $3 \times \frac{14}{4}$
  - Multiply a mixed number by a whole number. $9 \times 1\frac{7}{10}$
- Expect students to model and explain their solutions using concrete and pictorial representations, words, and numbers.

What the STUDENTS do:

- Use models to solve a variety of problem situations involving multiplying a whole number times a fraction or mixed number.
- Explain their reasoning using models, pictures, words, and numbers.
- Talk about any patterns they see when multiplying a fraction or mixed number times a whole number in relation to the meaning of the whole number as the number of groups, the numerator and denominator of the fraction, and the meaning of multiplication.

Addressing Student Misconceptions and Common Errors

Students who struggle with identifying and modeling multiplication situations from Table 2 (page 256) need more experience with these situations and using appropriate models. Use fractions of reasonable size so that students can focus both on the situation and why it is a multiplication situation as well as deal with the numbers they need to use to solve the problem. Do not teach students to look for key words (such as *of*) because this does not support making sense of the situation and what is happening with the fractions.

Notes

Number and Operations—Fractions[1]
4.NF.C*

Cluster C

[1] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

GRADE 4

Understand decimal notation for fractions, and compare decimal fractions.

STANDARD 5

4.NF.C.5: Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[2] *For example, express* $\frac{3}{10}$ *as* $\frac{30}{100}$, *and add* $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$.

[2] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

STANDARD 6

4.NF.C.6: Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as* $\frac{62}{100}$; *describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

STANDARD 7

4.NF.C.7: Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

*Major cluster

Number and Operations—Fractions[1] 4.NF.C

Cluster C: Understand decimal notation for fractions, and compare decimal fractions.

[1] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Grade 4 Overview

As students continue to work with fractions, they make explicit connections between building equivalent fractions with tenths and hundredths. They use decimal notation as another way to write these numerical values and build an understanding of tenths and hundredths as an extension of the place value system to numbers less than 1. They compare decimals using physical models. Models for this cluster include base-ten blocks and the number line.

Standards for Mathematical Practice
SFMP 1. Make sense of problems and persevere in solving them.
SFMP 2. Use quantitative reasoning.
SFMP 3. Construct viable arguments and critique the reasoning of others.
SFMP 4. Model with mathematics.
SFMP 5. Use appropriate tools strategically.
SFMP 6. Attend to precision.
SFMP 7. Look for and make use of structure.
SFMP 8. Look for and express regularity in repeated reasoning.

As fourth graders begin to explore decimal notation for a special group of fractions (those with denominators that are powers of 10) and connect decimal numbers to previous experiences with place value, they should have opportunities to find and share examples of where they see decimals used in their everyday life (money, sports statistics). They connect their experiences with equivalent fractions to work with a specific group of fractions, those with denominators that are powers of 10 (tenths and hundredths). They extend their previous work with the structure of our place value system to write these special fractions as decimals, explaining the value of tenths and hundredths as related to the ones place and one whole.

Related Content Standards

3.NF.A.3 4.NF.A.1 4.NF.A.2 5.NBT.A.1 5.NBT.A.3

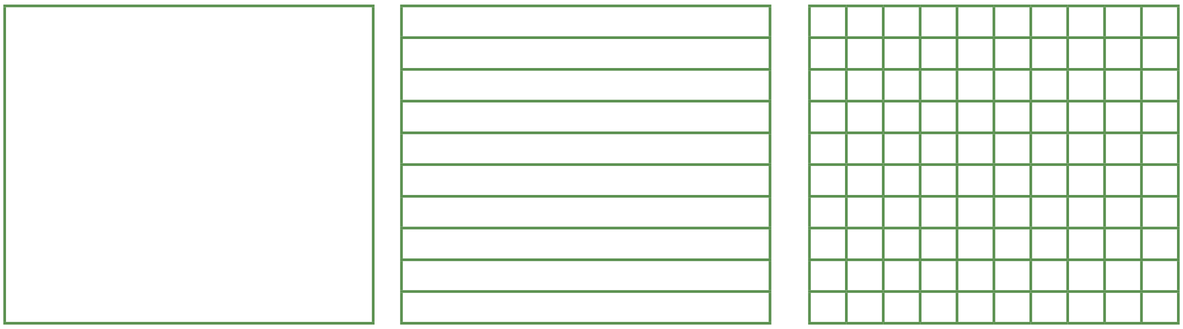
STANDARD 5 (4.NF.C.5)

Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[2] For example, express $\frac{3}{10}$ as $\frac{30}{100}$, and add $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$.

[2] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Students use their previous experience with finding equivalent fractions to change fractions with a denominator of 10 to equivalent fractions with a denominator of 100. This is fourth graders' first and only required experience adding fractions with unlike denominators. They are also preparing for initial work with decimal numbers. Previous experience with building equivalent fractions should support students in the initial stages of working with changing tenths to an equivalent number of hundredths.



One whole   Ten tenths   One hundred hundredths

What the TEACHER does:

- Provide opportunities for students to find equivalent fractions for tenths.
- Extend activities and instruction to focus on finding equivalent fractions with a denominator of 100. Using base-ten blocks as models will provide a visual model of equivalence and can be used later as students extend this work to decimals.
- Explore adding tenths and hundredths as fractions using models, pictures, words, and numbers. Keep in mind this is exploratory and students' first experience adding fractions with unlike denominators.

What the STUDENTS do:

- Build equivalent fractions that are tenths to fractions that are hundredths using models and pictures.
- Explain their thinking using models, words, and numbers.
- Look for patterns and make generalizations about equivalent fractions that are tenths and hundredths.
- Explore adding and subtracting tenths plus hundredths using models and verbal explanations.

Addressing Student Misconceptions and Common Errors

Remember that at this point students are not expected to develop an algorithm for adding fractions with unlike denominators. This is an important opportunity for students to think about and explore situations in which adding two fractions with unlike denominators necessitates finding a common denominator, and why. Students who add numerators and denominators need more explicit experiences with models and to talk about why the denominator needs to be the same. Experiences should also focus on why they do not add denominators when adding fractions. Reinforcing the meaning of the numerator as the count of the number of pieces and the denominator as a descriptor telling the number of pieces in the whole supports future experiences adding fractions with unlike denominators in Grade 5.

STANDARD 6 (4.NF.C.6)

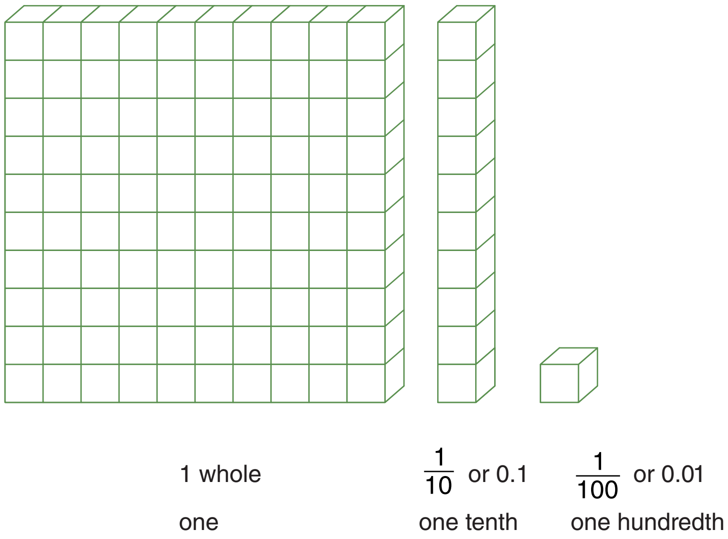
Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as $\frac{62}{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

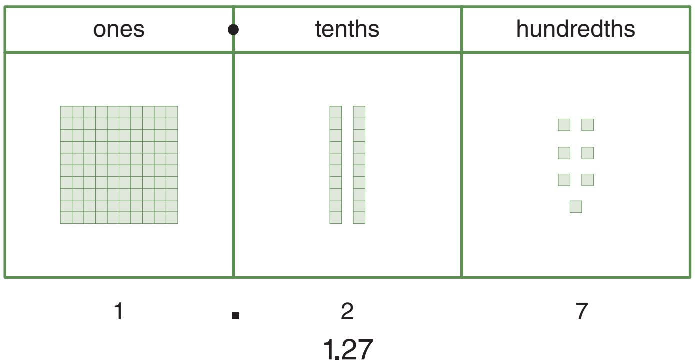
Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

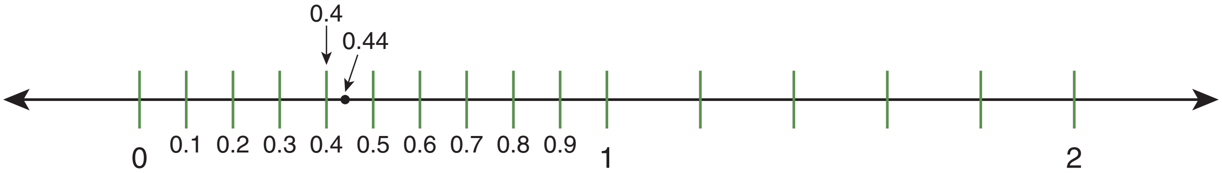
This is the first formal experience with decimals for students. Include models of decimals using base-ten blocks and grid paper (Reproducibles 4 and 3) as students practice visualizing or shading in a number of tenths or hundredths and relate these models to fraction and decimal notation. Their work should focus around understanding that 0.8 and $\frac{8}{10}$ are different representations for the same number. Students extend their work with whole number place value to include decimal places on the place value chart (Reproducible 8). Base-ten blocks, grids, and number lines are primary models for developing conceptual understanding within this Standard. Students should have many opportunities to make connections from previous experiences with whole numbers to showing concrete representations and numerical representations of decimals and reading decimal numbers to hundredths.

Connecting fractions to decimal representations

Base-ten blocks







What the TEACHER does:

- Introduce decimal notation as an extension of whole number place value, emphasizing that as we move one place to the right of 1, the place value is $\frac{1}{10}$ of a whole.
- Connect those experiences to writing a number as a fraction and as a decimal.
- Give students many opportunities to work with tenths using models, pictures, words, and numbers.
- Progress from models to words and written representations of decimal numbers in tenths, emphasizing the meaning of a tenth as one part of ten equal parts in the whole and that $\frac{1}{10}$ and 0.1 are different ways to represent the same number.

One whole Ten tenths

- Once students demonstrate understanding of tenths, extend activities to hundredths with models including base-ten blocks (Reproducible 4), grids, and the number line.
  - $\frac{1}{100}$ or 0.01 means one out of one hundred equal pieces that make the whole.
  - $\frac{1}{100}$ also represents dividing one tenth into ten equal pieces.
- Model appropriate reading and writing of decimal numbers to reinforce the meaning of decimals.
  - 0.3 is read as "three tenths" (not as "point three").
  - 0.45 is read as "forty-five hundredths" (not as "point four five").
  - 3.07 is read as "three and seven hundredths" (not as "three point zero seven").
- Facilitate discussions in which students relate decimals to real-life contexts such as money, metric measures, and sports statistics.
- Give students experiences with locating fractions and decimals on a number line to reinforce the fact that they are equivalent values.

What the STUDENTS do:

- Model fractions with denominators of 10 using base-ten blocks and grid paper models.
- Represent fractions in decimal notation and understand that they are two different ways of writing the same quantity. ($\frac{1}{10}$ and 0.1 mean the same amount when referring to the same whole.)
- Model, read, and write decimal numbers in the tenths place using base-ten blocks, extended place value charts, grids, and number lines.
- Model fractions with denominators of 100 using base-ten blocks and grid paper models.
- Model, read, and write decimal numbers to the hundredths place using base-ten blocks, extended place value charts, grids, and number lines.
- Demonstrate understanding that $\frac{1}{100}$ is one of 100 equal pieces in one whole or 1 of ten equal parts of a tenth.
- Explain their reasoning.
- Connect understandings to real life situations that use decimal notation.

Addressing Student Misconceptions and Common Errors

One of the most important understandings of decimal numbers is the relationship of a decimal to one whole as well as a decimal number to other decimal numbers. Just as students need to understand that 100 is the same as 100 ones, they should also understand that it is also the same as 10 tens. Similarly, when working with decimal numbers less than one whole, a foundational understanding that needs to be developed is that 0.01 represents one out of 100 parts of the whole, and it is also one of 10 parts of a tenth (a tenth of a tenth). Student need many activities using concrete models to understand this concept. Similarly one tenth (0.1) is equivalent to ten hundredths (0.10).

Using money as a familiar context will also help to reinforce this understanding.

$.10 is one tenth of a dollar. Ten dimes make 1 dollar.

$.01 is one hundredth of a dollar. One hundred pennies make 1 dollar. Additionally, ten pennies make a dime, so we can think of $.10 as 10 hundredths or one tenth.

STANDARD 7 (4.NF.C.7)

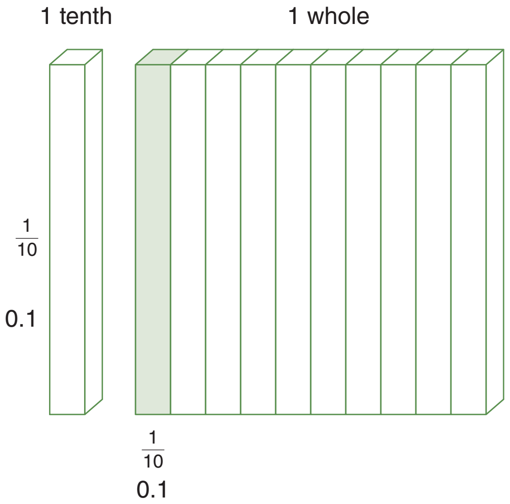
Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

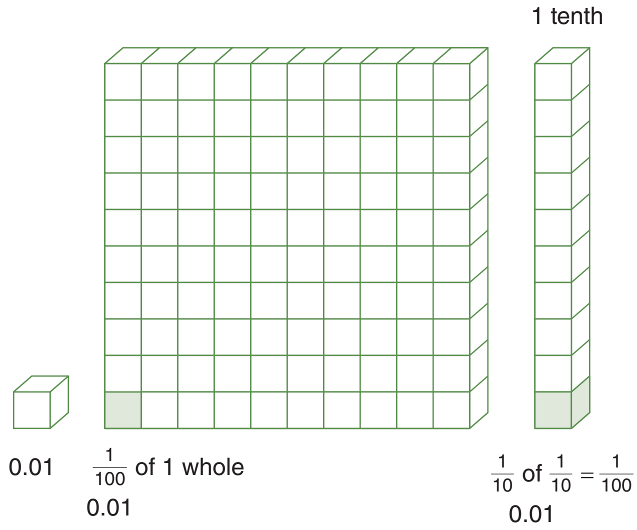
Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Students need to understand the relationship among decimal places just as they understand the relationship among places when considering whole number place value. This will support students as they begin to compare decimals that refer to the same whole. Using visual models, including base-ten and grid models as well as number line models, and fractions that are equivalent to the decimals will help students to make sense of this concept rather than learning it by rote rules. Scaffolding examples will help students develop conceptual understanding of decimal place value when comparing numbers. Students should justify their reasoning using models and explanations.

What the TEACHER does:

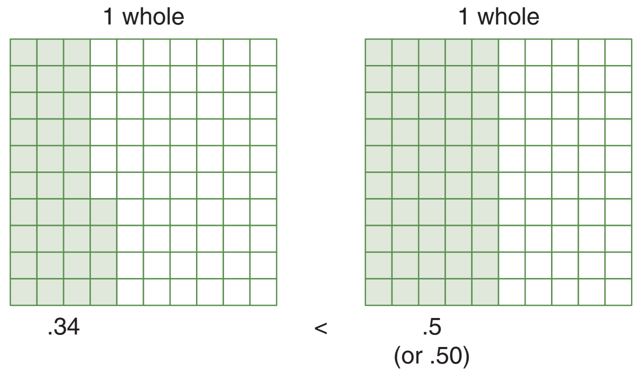
- Ask students to compare tenths written as fractions with tenths written as decimals to understand that they are different ways to write the same number.
- Give students activities in which they compare two decimals in the tenths place. Expect students to work flexibly among representations (base-ten blocks, grid models, number line, fraction representations) to compare tenths, using representations that make sense to them.





- Ask students to compare hundredths written as fractions with hundredths written as decimals to understand that they are different ways to write the same number.
- Give students activities in which they compare two decimals in the hundredths place. Expect students to work flexibly among representations (base-ten blocks, grid models, number line, fraction representations) to compare tenths, using representations that make sense to them.

- Provide activities for students to compare tenths and hundredths (for example, compare 0.3 and 0.32) using models and pictures.
- Connect these activities to including numerical representations.
- Ask students to compare decimal numbers in tenths and hundredths and justify their solutions.



What the STUDENTS do:

- Compare decimals that are tenths using a strategy that makes sense (using fraction numbers, base-ten blocks or the number line).
- Explain their reasoning using models, pictures, numbers, or words.
- Compare decimals that are hundredths using a strategy that makes sense (using fraction numbers, base-ten blocks, or the number line).
- Explain their reasoning using models, pictures, numbers, or words.
- Compare decimals that are hundredths with decimals that are tenths using a strategy that makes sense (using fraction numbers, base-ten blocks, or the number line).
- Explain their reasoning using models, pictures, numbers, or words.

Addressing Student Misconceptions and Common Errors

Watch for students who think that 0.54 is greater than 0.8 because 54 is greater than 8. These students do not understand the relationship between tenths and hundredths and need more experience with modeling decimals. Key to their understanding is the fact that 0.8 is equivalent to 0.80. Writing decimals in terms of equivalent fractions (comparing tenths with tenths and hundredths with hundredths) will help students develop an understanding of the relationship between tenths and hundredths and use this relationship to accurately compare decimals.

Notes

Sample PLANNING PAGE

Standard: 4.NF.C.6. *Use decimal notation for fractions with denominators 10 or 100.* For example, rewrite 0.62 as $\frac{62}{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

Mathematical Practice or Process Standards:

SFMP 2. Use quantitative reasoning.

Students use reasoning to understand the relationship between tenths and hundredths (0.5 =0 .50).

SFMP 3. Construct viable arguments and critique the reasoning of others.

Students explain their thinking as they describe the value of decimal models and find similarities and differences among their thinking and that of their classmates.

SFMP 4. Model with mathematics.

Students use base-ten blocks to model decimal numbers in order to relate decimal place value to whole number place value and to determine the value of a decimal number.

SFMP 6. Attend to precision.

Students model, read, and write decimal numbers accurately.

SFMP 7. Look for and make use of structure.

Students extend the structure of the place value system with whole numbers to decimal numbers.

SFMP 8. Look for and express regularity in repeated reasoning.

Concrete models provide students with beginning ideas around the relationship among places. Moving a place to the left increased the value of a digit ten times. Moving a place to the right decreased the value of the number by one tenth.

Goal:

Students will model decimal fractions using base-ten blocks.

Planning:

Materials: Decimal place value chart (Reproducible 8), base-ten blocks or cut-outs of base-ten blocks for each student (Reproducible 4)

Sample Activity:

Begin with a discussion identifying the 10 × 10 flat block as one whole. Place it on the place value chart. Ask students to identify the "rod." (tenth) Ask questions that help them to see that 10 rods make a flat, so each rod is $\frac{1}{10}$, also written as 0.1. Place the rod on the place value chart. Continue with the small cube, asking questions that help students identify the 100 small cubes make 1 whole, so it would represent $\frac{1}{100}$, written as 0.01 as a decimal.

Once students demonstrate understanding of the value of each type of block, give them a variety of decimal numbers, first in writing and later orally, for them to model on the place value chart. Then provide them with models and ask them to read and write the decimal.

Sample PLANNING PAGE (Continued)

Questions/Prompts:

Scaffold student experiences so they are progressing from tenths to hundredths. Ask questions that support seeing the relationship between the tenths and hundredths places and between the decimal places and one whole. As students model, reinforce conceptual understanding with questions such as:

- In the decimal 3.42, how many wholes (ones) are there?
- How many tenths?
- How many hundredths?
- Why do we read the decimal as "three and forty-two hundredths" when there are only 2 hundredths?

As students begin work with decimal numbers, be sure they understand that decimals are another way to write fractions with denominators that are powers of ten (10ths, 100ths, 1000ths).

Differentiating Instruction:

Struggling Students: Students who are still shaky on the relationship among places with whole numbers may find decimals more confusing. Give them many concrete experiences modeling ones and tenths before moving to hundredths.

Extension: Include the large block (ten) in the decimals you are modeling. For students who demonstrate understanding, consider extending to thousandths, although that is not a requirement for this grade level.

Notes

PLANNING PAGE

Standard:

Mathematical Practice or Process Standards:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

PLANNING PAGE

Standard:

Mathematical Practice or Process Standards:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

PLANNING PAGE

Standard:

Mathematical Practice or Process Standards:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

Number and Operations—Fractions
5.NF.A*

Cluster A

Use equivalent fractions as a strategy to add and subtract fractions.

STANDARD 1 **5.NF.A.1:** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,* $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. *(In general,* $\frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}$.)

STANDARD 2 **5.NF.A.2:** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result* $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$, *by observing that* $\frac{3}{7} < \frac{1}{2}$.

*Major cluster

Number and Operations—Fractions 5.NF.A

Cluster A: Use equivalent fractions as a strategy to add and subtract fractions.
Grade 5 Overview

In fourth grade students added and subtracted fractions with like denominators and began to explore adding tenths plus hundredths in preparation for work with decimals. Students in Grade 5 extend this work to adding and subtracting fractions with unlike denominators using visual representations, reasoning, and equations.

Standards for Mathematical Practice
SFMP 1. Make sense of problems and persevere in solving them.
SFMP 2. Use quantitative reasoning.
SFMP 3. Construct viable arguments and critique the reasoning of others.
SFMP 4. Model with mathematics.
SFMP 5. Use appropriate tools strategically.
SFMP 6. Attend to precision.
SFMP 7. Look for and make use of structure.
SFMP 8. Look for and express regularity in repeated reasoning.

Problem solving provides the context students use to develop conceptual understanding of addition and subtraction of fractions and mixed numbers with unlike denominators. Students use quantitative reasoning to determine whether their answers make sense. Because a common error when adding or subtracting fractions with unlike denominators is to add or subtract the numerators and denominators, using benchmark fractions to reason about the value of the fractions will help students realize that if they add or subtract denominators their answer will not be reasonable. Using appropriate models including area models, fraction bars, and the number line will help students to develop efficient strategies for adding and subtracting fractions and mixed numbers.

Related Content Standards

3.NF.A.2 3.G.A.2 4.NF.B.3

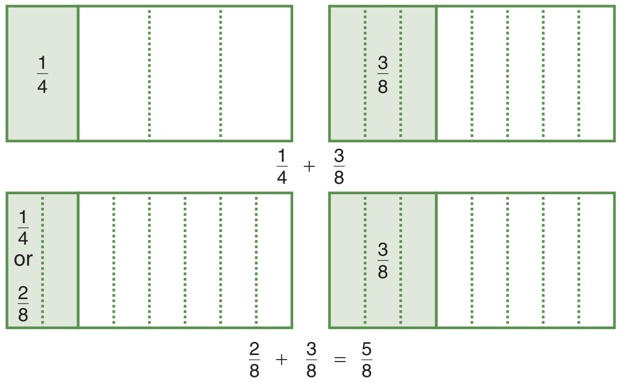
STANDARD 1 (5.NF.A.1)

Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $\frac{2}{3}+\frac{5}{4}=\frac{8}{12}+\frac{15}{12}=\frac{23}{12}$. (In general, $\frac{a}{b}+\frac{c}{d}=\frac{(ad+bc)}{bd}$.)

As fifth graders begin to add fractions with unlike denominators, they use visual models, including area models, fraction strips, and number lines. They understand the need for like denominators in addition and subtraction by examining situations using concrete models. Students should explore a variety of strategies for finding a common denominator, including examples in which one denominator is a multiple of the other, finding common multiples, and multiplying the denominators. No matter which strategy students use, it is important for students to have many experiences to understand why a strategy works. Note that this Standard does not call for students to use the least common denominator. Once a common denominator is determined, students apply their previous work with equivalent fractions to rewrite the fractions as equivalent fractions with a common denominator.

What the TEACHER does:

- Provide students with opportunities to add fractions with unlike denominators using concrete models, progressing to pictorial models, and making explicit connections to writing it with numerals. Begin with examples in which one denominator is a multiple of the other using visual models.



- Facilitate discussions in which students explain why they need to find like denominators to add fractions and their strategies for finding like denominators. Student explanations should include why their strategy works in addition to what they did to find like denominators. Teacher questions should promote student understanding.
    - What makes adding halves and fourths difficult?
    - Why did you choose 4 as a common denominator?
    - How can you show that you can change halves to fourths using models?
    - What patterns do you see in the examples we have solved so far?
- Continue to give students examples with a variety of denominators. Before using any algorithm, give students ample opportunities to work with visual models to see and explain how like denominators are related. Once students understand the need for like denominators and can identify appropriate denominators for addition and subtraction examples, they should apply their understanding of equivalent fractions to rewrite the given fractions so they can add or subtract.
- Expect students to justify their thinking as they use efficient strategies to add and subtract fractions.

What the STUDENTS do:

- Use a variety of visual representations to understand the need for common denominators when adding and subtracting fractions and mixed numbers.
- Practice finding common denominators for a variety of fraction addition and subtraction examples.
- Explain their reasoning as they find like denominators for any given addends.
- Apply their understanding of equivalent fractions to change given fractions in an addition or subtraction example to fractions with like denominators.
- Use reasoning to determine if their answer makes sense.

Addressing Student Misconceptions and Common Errors

Watch for students who have surface understanding of the necessity for finding common denominators when adding and subtracting fractions and mixed numbers. Consistent practice in the form of number talks or using formative assessment tasks coupled with students explaining their thinking and considering the reasonableness of their solutions will help students to see the importance of thinking about the value of the numbers rather than using random calculations (add the numerators, add the denominators). Relating the fractions to benchmark numbers (0, $\frac{1}{2}$, 1) will help students to determine whether their answer is reasonable.

Two areas that should be explicit in providing meaningful situations include considering the size of the piece (that is, how many pieces make one whole or the denominator) and that the fractions must refer to the same size whole. Students must always consider that adding $\frac{1}{2}$ of a small candy bar with $\frac{1}{2}$ of a large candy bar will not produce 1 whole candy bar.

STANDARD 2 (5.NF.A.2)

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $\frac{2}{5}+\frac{1}{2}=\frac{3}{7}$, by observing that $\frac{3}{7}<\frac{1}{2}$.

Working with addition and subtraction of fractions should include solving problems with various situations, as shown in the Resources, Table 1 on page 254. Students continue to work with visual representations to understand the need for like denominators in addition and subtraction of fractions. They explain their thinking to classmates as they demonstrate their solution strategies. Important to this Standard is developing number sense with fractions. Using benchmarks (0, $\frac{1}{2}$, 1) to determine whether an answer is reasonable using comparisons, mental addition, or subtraction will help students to justify their thinking with oral and written explanations.

What the TEACHER does:

- Provide a context for all addition and subtraction of fraction and mixed number problems by using various problem situations with fractions from Table 1 on page 254.
- Facilitate class discussions in which students model and explain their reasoning in finding like denominators and setting up an equation with equivalent fractions using models, pictures, words, and numbers.
- Facilitate class discussions in which students justify why their answer is reasonable—especially when misconceptions such as adding unlike denominators as part of the solution process need to be addressed.

Example:

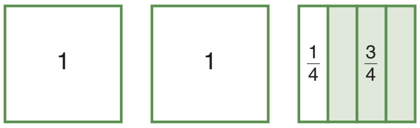
Anna needs 3 cups of flour for the cookies she is making. She has already measured $\frac{3}{4}$ of a cup of flour. How much more flour does she need?

Note a common error in solving this problem:

Student error

$$\begin{array}{r} 3 \\ -\ \frac{3}{4} \\ \hline 3\frac{3}{4} \end{array}$$

Conceptual model



$3 - \frac{3}{4} = 2\frac{1}{4}$

Anna needs $2\frac{1}{4}$ cups of flour.

Using estimation, students should explain that the answer must be somewhere between 2 and 3 cups of flour because she needs 3 cups and has already measured $\frac{3}{4}$ of a cup. So the answer $3\frac{3}{4}$ is not reasonable.

What the STUDENTS do:

- Use visual models including area models, fraction strips, number lines to solve addition and subtraction problems with fractions and mixed numbers.
- Explain their solution process using models, pictures, words, and numbers.
- Analyze results using models and benchmark fractions to determine whether an answer is reasonable.

Example:

Charlie got the following addition problem wrong on his mathematics quiz. Write him a note explaining why the sum doesn't make sense.

$$\frac{3}{5} + \frac{7}{9} = \frac{8}{14}$$

A student could use benchmark fractions to reason that $\frac{3}{5}$ is a little more than $\frac{1}{2}$ and $\frac{7}{9}$ is very close to 1 whole, so the answer should be around $1\frac{1}{2}$. Charlie's answer of $\frac{8}{14}$ is only a little more than $\frac{1}{2}$, so it is not reasonable.

Addressing Student Misconceptions and Common Errors

Students who struggle to determine the appropriate operation to solve a problem need more experience with the problem situations for addition and subtraction (see Table 1 on page 254). They need to use strategies such as act it out, draw a picture, write an equation, or make a model to determine how to best approach a problem. Give students opportunities to explain their thinking as they read the problem and use models to determine the correct operation. Make connections to earlier experiences with whole numbers that will help students to think of addition and subtraction in a particular situation. Once students determine the correct operation, they can use fractions and mixed numbers to solve the problem.

Number and Operations—Fractions
5.NF.B*

Cluster B

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

STANDARD 3

5.NF.B.3: Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

STANDARD 4

5.NF.B.4: Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $\frac{a}{b} \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $\frac{2}{3} \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$. (In general, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.)*

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

STANDARD 5

5.NF.B.5: Interpret multiplication as scaling (resizing), by:

a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{(n \times a)}{(n \times b)}$ to the effect of multiplying $\frac{a}{b}$ by 1.

STANDARD 6

5.NF.B.6: Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

STANDARD 7

5.NF.B.7: Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]

[1] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

(continued)

GRADE 5

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for $\frac{1}{3} \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $\frac{1}{3} \div 4 = \frac{1}{12}$ because $\frac{1}{12} \times 4 = \frac{1}{3}$.*

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for $4 \div \frac{1}{5}$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div \frac{1}{5} = 20$ because $20 \times \frac{1}{5} = 4$.*

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share $\frac{1}{2}$ lb of chocolate equally? How many $\frac{1}{3}$-cup servings are in 2 cups of raisins?*

*Major cluster

Number and Operations—Fractions 5.NF.B

Cluster B: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
Grade 5 Overview

Students worked with concrete models for multiplying a fraction by a whole number in Grade 4. They continue to extend this work to additional situations of multiplying a whole number by a fraction. They use area models to connect their understanding of multiplication of whole numbers to multiplication of fractions. Fifth graders make generalizations about multiplying fractions and whole numbers through using scaling as a model for multiplication and reasoning about the size of the product based on the size of the factors. The procedure for multiplying fractions is developed by making sense of what multiplication of fractions means rather than simply presenting students with a rule to follow. They solve a variety of multiplication problems applying their understanding to real-life situations.

Students explore division of a whole number by a fraction and a fraction by a whole number through visual models and contexts in order to make sense of what division fractions entails. They use concrete models and explain their reasoning as they work to apply previous understandings of division to fraction situations.

Standards for Mathematical Practice
SFMP 1. Make sense of problems and persevere in solving them.
SFMP 2. Use quantitative reasoning.
SFMP 3. Construct viable arguments and critique the reasoning of others.
SFMP 4. Model with mathematics.
SFMP 5. Use appropriate tools strategically.
SFMP 6. Attend to precision.
SFMP 7. Look for and make use of structure.
SFMP 8. Look for and express regularity in repeated reasoning.

Students use a variety of problem solving situations to develop understanding of multiplication of fractions and mixed numbers. They solve and write problems that include dividing a fraction by a whole number and a whole number by a fraction using models and verbal explanations. They make connections between the structure of whole number multiplication and division and the structure of multiplication and division of fractions and use these connections to solve problems. Students explain using models and how the meaning of these operations is the same with whole numbers and fractions but the actual procedures are quite different. Following many opportunities to model, explain, and solve problems, students use their experiences to recognize patterns and develop efficient strategies.

Related Content Standards

3.OA.A.1 3.NF.A.2 5.NF.B.4 5.NF.B.5 5.NF.B.6

STANDARD 3 (5.NF.B.3)

Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

Students have had some experience considering a fraction as a division situation in Grade 4. They extend their previous work to expressing the quotient of a division problem as a fraction or mixed number. Real-life problems present situations and contexts in which expressing the remainder as a fraction makes sense. Using concrete materials such as fraction pieces and area models will help students to understand what the "leftover" fraction represents.

Division examples with the remainder expressed as a fraction:

Example 1:

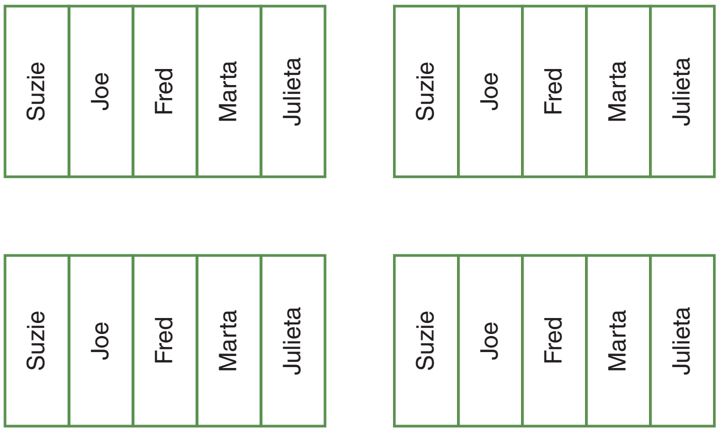
Josh and Jean are packing bags of cookies for the bake sale. They have 152 cookies and want to put one dozen cookies in each bag. How many bags can they fill? What part of a bag will be left?

$$\begin{array}{r} 12 \\ 12\overline{)152} \\ \underline{12} \\ 32 \\ \underline{24} \\ 8 \end{array}$$

They can fill 12 bags and they will have 8 out of the 12 cookies they need to fill another bag, so they will fill 12 bags. There will be $\frac{8}{12}$ (or $\frac{2}{3}$) of a bag left.

Example 2:

Liz has 4 candy bars. She wants to split them among 5 friends: Suzie, Joe, Fred, Marta, and Julieta. If each person should get the same amount, what part of a candy bar will each friend get?



Liz can break each candy bar into five parts and give each friend one of those parts. So each friend will receive $\frac{4}{5}$ of a candy bar, because they get 4 out of the 5 pieces (or $\frac{4}{5}$) of the whole candy bar.

What the TEACHER does:

- Provide students with a variety of division problems to model interpreting the remainder as a fraction.
- Give students the opportunity to model problems and determine the meaning of the remainder when it is expressed as a fraction. It is important for students to understand that the fraction tells what part of a whole is left over.
- Facilitate student discussions about the meaning of the remainder and why a fraction makes sense in given situations.
- Combine a variety of division problems and ask students to determine what to do with the remainder.
    - The remainder is the answer.
    - Drop the remainder.
    - Add one to the quotient and drop the remainder.
    - Express the remainder as a fraction.
- Provide students with a variety of division problems in which the divisor is greater than the dividend, so that the quotient will be a fraction. Give students time to explore using various representations including concrete and pictorial models.
- Facilitate classroom discussions in which students explain their thinking and the meaning of their solutions given the constraints of the problem.

What the STUDENTS do:

- Model and solve division problems in which they interpret the remainder as a fraction and explain their thinking.
- Explain their reasoning for interpreting the remainder as a fraction.
- Solve a variety of division problems determining what to do with the remainder. Explain their thinking.
- Model problems in which the divisor is greater than the dividend and share their thinking about the quotient being a fraction.
- Explain their reasoning, connecting to a generalization that a fraction is a type of division problem.

Addressing Student Misconceptions and Common Errors

Students may initially think that you cannot divide a "smaller number by a bigger number" since this will be a new situation for them to consider. Provide them with good problems to solve and give them many opportunities to explore with models so that they are developing conceptual understanding. It is important that they understand this concept in a way that makes sense to them rather than be shown how to do it. The role of the teacher is to provide sensible problem situations, ask supporting questions, and facilitate conversations in which the students are making sense of the situation and why their answers make sense.

Students who struggle with interpreting the remainder of division examples need more experience solving problems using concrete models so they understand that the remainder tells what part of a group is left over. Asking questions such as "How many are left?" and "How many would it take to make another full group?" and modeling what part of a full group is left over will help them to understand the meaning of the remainder when it is expressed as a fraction.

Notes

STANDARD 4 (5.NF.B.4)

Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. *Interpret the product $\frac{a}{b} \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.* For example, use a visual fraction model to show $\frac{2}{3} \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$. (In general, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.)

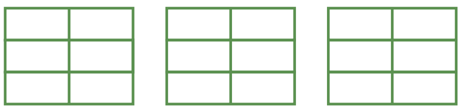
In Grade 4, students used models to multiply a fraction by a whole number (for example, $4 \times \frac{3}{5}$), connecting to the meaning of whole number multiplication. Fifth grade students extend this concept by using models to represent situations in which they need to multiply a whole number by a fraction ($\frac{3}{5} \times 4$) or a fraction by a fraction ($\frac{1}{4} \times \frac{3}{5}$). Provide students with real-life contexts and situations to model in order to give them experiences they need to develop understanding of what is happening when they multiply a fraction by a fraction.

What the TEACHER does:

- Explicitly connect multiplication of whole numbers to multiplication with fractions by giving students connected situations that they can model.
- Scaffold problems beginning with unit fraction factors and build to multiplying with other fractions and mixed numbers.

Example 1:

Frank baked 3 pans of brownies. He cut 6 brownies in each pan. How many brownies did Frank bake? (3 groups of 6)



$3 \times 6 = 18$

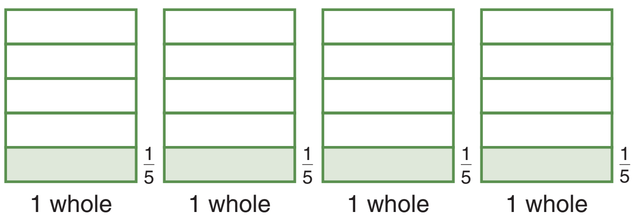
3 groups of 6 = 18

Example 2:

Marcella made 4 gallons of punch. One-fifth of the punch was orange juice. How much orange juice did she use in the punch?

A student might think of this as $\frac{1}{5}$ of each gallon being orange juice. Because there are 4 gallons that would show that $\frac{4}{5}$ of a gallon is orange juice.

$\frac{1}{5} \times 4$



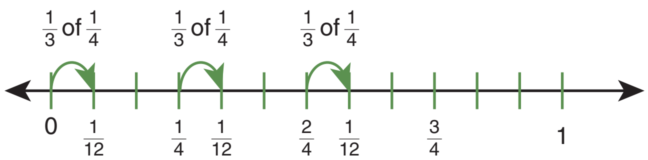
$\frac{1}{5} \times 4$ means $\frac{1}{5}$ of each group of 4 $= \frac{4}{5}$

Example 3:

The distance from Elsa's house to her grandmothers is $\frac{3}{4}$ of a mile. She biked $\frac{1}{3}$ of the way there and stopped to rest. How far did Elsa travel before her rest stop?

A student might say, because I wanted one-third of the distance, I divided each fourth into 3 sections since I thought $\frac{1}{3}$ of the way would be the same as $\frac{1}{3}$ of each fourth. When I put them together, the total distance she biked before resting would be $\frac{3}{12}$ of a mile.

$\frac{1}{3} \times \frac{3}{4}$



$\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$

$\frac{1}{3}$ of $\frac{3}{4} = \frac{3}{12}$

- Give students time to work in groups to explore solutions using models, including area models, fraction bars, and number lines.
- Monitor group work by noting what students are doing and asking supporting questions.
- Facilitate class discussions in which students model and explain their thinking.

(continued)

GRADE 5

What the TEACHER does (continued):

- Use formative assessment tasks to determine whether students understand what is happening when they multiply two fractions and why the product is smaller than the factors. Students need to understand that because each factor represents a part of the whole, when they multiply a fraction times a fraction, they are taking part of a part. For example, given the example $\frac{1}{2} \times \frac{1}{4}$, students should understand that they are taking a part ($\frac{1}{2}$) of a part ($\frac{1}{4}$) and the result ($\frac{1}{8}$) will be smaller than the part they had at the beginning.
- As students demonstrate conceptual understanding of what happens when they multiply fractions using models, pictures, words, and numbers, encourage students to look for patterns so they can generalize a procedure for multiplying fractions and justify why that procedure works. Why can you multiply numerators and multiply denominators to get the product of two fractions?

What the STUDENTS do:

- Explore what happens when multiplying a whole number by a fraction by solving a variety of word problem contexts using models, pictures, words, and numbers.
- Explain their reasoning to partners, groups, and to the class. Compare different strategies focusing on how strategies are similar and how they are different.
- Explore multiplication of a fraction by a fraction by solving a variety of word problem contexts using models, pictures, words, and numbers.
- Explain their reasoning to partners, groups, and to the class. Compare different strategies focusing on how strategies are similar and how they are different.
- Look for patterns when multiplying fractions. Explain why those patterns work using models, pictures, words, and numbers.
- Apply the patterns to determine a procedure for multiplying fractions.

Addressing Student Misconceptions and Common Errors

Students may see the pattern and see that to multiply fractions you "simply" multiply the numerators and multiply the denominators. This is the correct algorithm or procedure. However, only references to real-life situations and using models and visual representations will help students develop a conceptual understanding of what is actually happening when they multiply fractions.

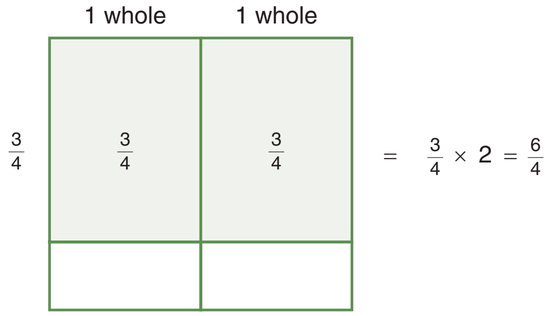
Notes

b. *Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.*

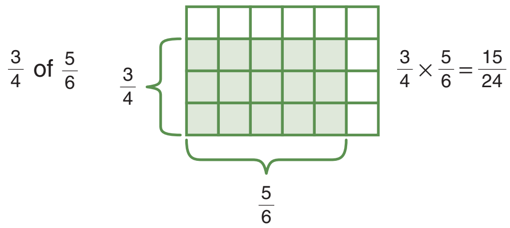
Using area models was a focus of work with multiplication of whole numbers in Grades 3 and 4. Fifth graders extend this work to examples with area models that have fractional side lengths. Students should have a variety of problems to solve using area models. This Standard can be taught in conjunction with earlier exploration of multiplication of whole numbers by a fraction, fractions by fractions, and mixed numbers.

What the TEACHER does:

- Provide students with problem contexts in which they find the area of a rectangle with one side that is a fraction and the other side a whole number.
- Facilitate a discussion in which students determine the part of the unit square used to tile the rectangle based on the dimensions of the side.
- Have students work in pairs or groups using grid paper to model the problem and discuss how to find the area of the rectangle.



- Ask students to explain their thinking using pictures, words, and numbers.
- Extend the problem to situations finding the area of a rectangle with both sides as fractions or mixed numbers.
- Have students work in pairs or groups using grid paper to model the problem and discuss how to find the area of the rectangle when one or both sides include a fraction.



- Facilitate classroom discussions in which students explain their reasoning and strategies to solve the problems using pictures, words, and numbers.
- Ask students to compare previous work and generalizations with multiplying fractions and mixed numbers to the solutions of these problems. What is similar? What is different?

What the STUDENTS do:

- Work in groups to solve the area problems using pictures and models.
- Explain the measures of the pieces they use to tile the rectangle, determining what part of a unit square each piece represents.
- Share the strategies and reasoning they use to solve each problem using models, pictures, words, and numbers.
- Compare the process and results of solving these problems with the previous work involving multiplication of fractions.

Addressing Student Misconceptions and Common Errors

Watch for students who have difficulty determining the part of the unit square. Thinking in terms of the whole rectangle will help them define the number of parts when the dimensions are fractional parts of the whole. Reinforcing when they multiply a fraction by a fraction they are taking part of a part will help students to see that the “overlap” is the number of pieces (or numerator), and the total number of pieces in the whole is the denominator.

STANDARD 5 (5.NF.B.5)

Interpret multiplication as scaling (resizing), by:

a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

Students explore a variety of multiplication situations in which they resize one of the factors and consider what happens to the size of the product. This Standard gives students the opportunity to reason about the size of the product when one of the factors is scaled or resized. Ample class time should be spent on giving students the opportunity to justify their thinking by creating mathematical arguments, comparing their thinking and strategies with those of their classmates, and thinking about similarities and differences in the various strategies.

Charlie's room is 15 feet by 12 feet. He has an awesome closet that is 3 feet by 12 feet. He wants to carpet the bedroom but tile the closet. How does the amount of carpet he needs to buy compare with the amount of tile he needs?

Molly's garden is 10 feet by 8 feet. Anna's garden is twice as long as half as wide. Without multiplying, compare the areas of their gardens and explain your reasoning.

Mark's garden is twice as long as Molly's and twice as wide. Without multiplying, compare the areas of their gardens and explain your thinking.

What the TEACHER does:

- Provide students with a variety of multiplication examples and problems in which one or both of the factors is scaled (resized)
- Give students time to discuss the impact of scaling factor(s) on the size of the product and justify their reasoning.

What the STUDENTS do:

- Reason about the impact of scaling one or both factors on the size of a product before multiplying.
- Justify their thinking using pictures and models.
- Compare their strategies with that of classmates to find similarities and differences in reasoning.

Addressing Student Misconceptions and Common Errors

Students will likely have many misconceptions about what happens to the product when one or both factors are scaled. For example, if both the length and width of a rectangle are doubled, some students will assume the product (area) is doubled. When they test their conjecture by drawing a picture, they will see that the product is actually four times greater. Allow students to explore a variety of multiplication scaling situations by drawing pictures and making models that will help them to make conjectures as to *why* the results are true, which is less likely to happen if they simply multiply.

Notes

b. *Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence* $\frac{a}{b} = \frac{(n \times a)}{(n \times b)}$ *to the effect of multiplying* $\frac{a}{b}$ *by 1.*

As students work with various models for multiplication of whole numbers, fractions, and mixed numbers, visual representations will help them to understand the size of the product when they multiply a fraction by a whole number, a whole number times a fraction, or a fraction by a fraction. This Standard also revisits building sets of equivalent fractions to extend previous work with equivalent fractions to generalizing that as they build sets of equivalent fractions they are multiplying the original fraction by names for 1.This Standard should not be taught as a standalone topic; these understandings should be incorporated into all student discussions about multiplication of fractions when appropriate.

What the TEACHER does:

- Give students explicit opportunities to examine and predict the size of a product when they multiply
  - a given whole number by a fraction;
  - a given fraction by a whole number;
  - a fraction by a fraction;
  - multiplication with mixed numbers by whole numbers, fractions, and mixed numbers.
- Ask questions that provide students with opportunities to share ideas, clarify their understanding, develop mathematical arguments, and make generalizations about the products of multiplication with fractions.

What the STUDENTS do:

- Explain that when multiplying a whole number greater than 1 by a fraction, the size of the product will be less that the whole number (because they are taking a part of the whole number) and greater than the fraction (because they have more than 1 group of the fraction).
- Use visual representations including area models, fraction strips, and number lines to support their thinking.

Addressing Student Misconceptions and Common Errors

Students are often puzzled when they find that the product is less than one or both of the factors. In previous work with multiplication of whole numbers, the product was always greater than both factors. Give students many opportunities to use visual models to "see" what is happening when they multiply with fractions. Discussions in which students explain their thinking will also help to identify and address misconceptions.

Notes

STANDARD 6 (5.NF.B.6)

Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

This Standard should be incorporated throughout this cluster. Problems provide a context for thinking about what is happening when students multiply fractions and mixed numbers.

What the TEACHER does:

- Provide a wide variety of multiplication problems with fractions and mixed numbers using the situations described in Table 2, page 256.

  Example 1:

  Louise ate $\frac{1}{2}$ of the bunch of grapes that were in the fruit bowl. Her brother ate $\frac{1}{4}$ of the grapes that were left. What part of the bunch of grapes did Louise and her brother eat?

  Example 2:

  Marty is training for the swim team. On Monday he swam 16 laps. On Tuesday he swam $1\frac{1}{2}$ times as many laps. On Wednesday he swam $1\frac{1}{2}$ as many laps as Tuesday. How many laps did Marty swim on those 3 days?

- Give students opportunities to work with partners, in small groups, and individually to solve problems using visual models and determining whether their answers make sense.

What the STUDENTS do:

- Use a variety of strategies, including make a model, draw a picture, make a table, look for a pattern, and guess and check, to solve problems that provide a context for multiplying fractions and mixed numbers.
- Think about the reasonableness of their solutions in terms of the context and the numbers.
- Explain their solution process using models, pictures, words, and numbers.

Addressing Student Misconceptions and Common Errors

Watch for misconceptions from previous multiplication standards. Students who struggle understanding why they should multiply in these problems need more experience using visual representations. It is helpful to have them break the problem into smaller parts and explain their thinking as they complete each part of the problem.

Notes

STANDARD 7 (5.NF.B.7)

Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]

[1] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

This is students' first experience with division of fractions. They use their understanding of whole number division to visualize what happens when they are dividing whole numbers by fractions and fractions by whole numbers. Beginning with a review of division of whole number situations will help students to apply their understanding to fraction situations. (See Grade 3, Operations and Algebraic Thinking, Clusters A and B.)

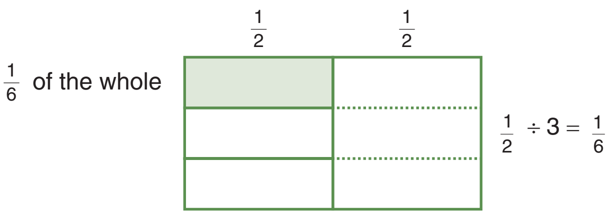
a. *Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.* For example, create a story context for $\frac{1}{3} \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $\frac{1}{3} \div 4 = \frac{1}{12}$ because $\frac{1}{12} \times 4 = \frac{1}{3}$.

Start with problem situations and visual representations for what is happening when dividing a fraction by a whole number. This may be confusing to some students, and they will need many concrete experiences to develop understanding. After students can visualize situations and connect those experiences to writing division equations, they can talk about the equations in terms of a missing factor and make generalizations. It is important that students be given opportunities to make sense of this and not be given a rule ("invert and multiply") that makes no sense to them and can cause misconceptions and errors.

Example (Partitive division: Knowing how many groups and finding the number of items in a group):

I have $\frac{1}{2}$ lb. of chocolate raisins and I want to divide it up to put the same amount of chocolate in each of 3 small bags. How much should each small bag of chocolate raisins weigh?

(Students ask themselves, "If I divide $\frac{1}{2}$ into 3 groups, how much will be in each group?")



What the TEACHER does:

- Review partitive model of division of whole numbers. Give students problems to solve and have them explain their thinking.

  Whole number example:

  My mom gave my four sisters and me 100 quarters to spend at the arcade. If we each get the same number of quarters, how many do we each have to spend?

- Relate this work to division of a fraction by a whole number. Ask how it is similar and how it is different.

  Fraction example:

  The snow plow has $\frac{1}{4}$ of a ton salt that must be spread on 3 streets. If the driver wants to use the same amount of salt on each street, how much salt will he spread on each?

- Pose a variety of problems in which students model dividing a unit fraction by a whole number.
- Facilitate conversations in which students explain their work.
- Following multiple experiences modeling, connect that work to written equations.
- Discuss patterns students see in written equations and connect those equations to the relationship between multiplication and division.

  $\frac{1}{2} \div 3 = \_\_\_\_ \qquad 3 \times \_\_\_\_ = \frac{1}{2}$

What the STUDENTS do:

- Solve problems involving division of a unit fraction by a whole number using models and pictures.
- Explain their work using pictures, words, and numbers.
- Connect visual representations to writing division equations.
- Look for connections between division with fractions and multiplication with fractions using previous experiences with the relationship between multiplication and division.

Addressing Student Misconceptions and Common Errors

Dividing a fraction by a whole number is likely to cause students initial confusion around understanding how you can possibly divide a fraction (part of a whole) by a whole number. One misconception is that you always have to "divide the bigger number by the smaller number." Connect to work with earlier standards in this domain in which students interpreted a fraction such as $\frac{3}{4}$ to also mean 3 divided by 4. It is important to give students many opportunities to solve problems with visual representations to develop understanding that this is the same as the sharing situations they used when dividing whole numbers. Do not rush students into writing equations. Allow students to write their own problems modeled after those you have given. This will help them to think about the situation and when it makes sense to divide a fraction by a whole number. Do not give them the traditional rule for division of fractions. Rather, take time for classroom discussions in which students explain their thinking and work to make sense out of the solution process and to determine the reasonableness of their answers. The role of the teacher is to clarify student thinking by posing good questions.

Notes

b. *Interpret division of a whole number by a unit fraction, and compute such quotients.* For example, create a story context for $4 \div \frac{1}{5}$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div \frac{1}{5} = 20$ because $20 \times \frac{1}{5} = 4$.

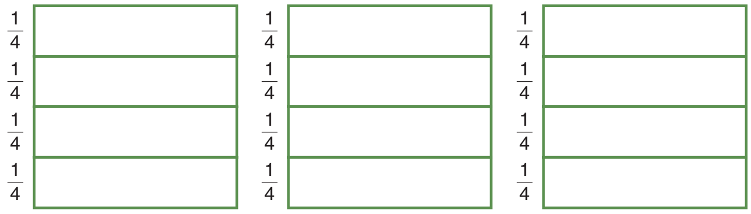
This Standard extends the ideas in the previous Standard to situations that call for dividing a whole number by a fraction. Using the measurement meaning of division (looking for the number of groups that can be made when the total and the number of items in a group are known) will help students to visualize and model what is happening when dividing a whole number by a fraction.

Example:

I have 3 quarts of lemonade. Each cup holds $\frac{1}{4}$ of a quart. How many cups can I fill?

(Students ask themselves, "How many groups of $\frac{1}{4}$ can I make out of 3?")

$3 \div \frac{1}{4}$ How many $\frac{1}{4}$s are in 3 wholes?



I can fill 12 cups.

What the TEACHER does:

- Review the measurement model of division of whole numbers. Give students problems with whole numbers to solve and have them explain their thinking.
- Relate this work to division of a whole number by a fraction. Ask how it is similar and how it is different.
- Pose a variety of problems in which students model dividing a whole number by a unit fraction.
- Facilitate conversations in which students explain their work.
- Model questions that students can ask themselves to help interpret the problem.
  - How many groups of $\frac{1}{4}$ can I make out of 3?
  - Relate this question to the visual representations students use to solve the problem.
- Following multiple experiences modeling, connect that work to written equations.
- Discuss patterns students see in written equations and connect those equations to the relationship between multiplication and division.

$3 \div \frac{1}{4} = \_\_\_\_\_ \qquad \frac{1}{4} \times \_\_\_\_\_ = 3$

What the STUDENTS do:

- Solve problems involving division of a whole number by a unit fraction using models and pictures.
- Explain their work using pictures, words, and numbers.
- Ask themselves appropriate questions to relate the meaning of the problem to the action needed to solve the problem. ("How many groups of ____ can I make out of ____?")
- Connect visual representations to writing division equations.
- Look for connections between division with fractions and multiplication with fractions using previous experiences with the relationship between multiplication and division.

Addressing Student Misconceptions and Common Errors

Watch for students who are having difficulty identifying what operation to use in solving problems with fractions. Using key words is not helpful and removes making sense from the process. Rather, have students model the problem using pictures and ask supporting questions, such as "What do you know? What do you want to find out? How can you show that in your picture?" As students solve mixed problems, adapt your questions to help students think about the meaning of the operations and how it can help them determine which operation to use.

Give students a variety of problems and ask them to model and write an expression they would use to solve the problem. Ask them to explain their model and expression.

c. *Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.* For example, how much chocolate will each person get if 3 people share $\frac{1}{2}$ lb of chocolate equally? How many $\frac{1}{3}$-cup servings are in 2 cups of raisins?

Solving real-world problems should be included throughout this Standard. Once students have had experiences with dividing a fraction by a whole number and a whole number by a fraction, they can solve mixed problems that involve both situations. At some point students should solve problems with fractions and mixed numbers that include all four operations.

What the TEACHER does:

- Pose a variety of problems in which students divide a whole number by a unit fraction or a unit fraction divided by a whole number.
- Facilitate conversations in which students explain their work.
- Following multiple experiences modeling expect students to connect their models to written equations.
- Discuss patterns students see in written equations and link patterns and problems to the relationship between multiplication and division (Table 2, page 256).

What the STUDENTS do:

- Solve problems involving division of a whole number by a unit fraction or a unit fraction divided by a whole number.
- Explain their work using pictures, words, and numbers.
- Ask themselves appropriate questions to relate the meaning of the problem to the action needed to solve the problem.
- Connect visual representations to writing division equations.
- Look for connections between division with fractions and multiplication with fractions using previous experiences with the relationship between multiplication and division.

Addressing Student Misconceptions and Common Errors

Students may struggle determining which number goes where in the division problem. “Am I dividing the fraction by the whole number or the whole number by the fraction?” Drawing a picture using the information in the problem and focusing on what they want to find out will help. Model asking questions and encourage them to ask themselves similar questions, such as

- What is being divided or broken up?
- Am I trying to determine how much in a group or how many groups?
- What visual representations can I use to show the actions of the problem?

Notes

Sample PLANNING PAGE

Standard: 5.NF.A.2. *Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.* For example, recognize an incorrect result $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$, by observing that $\frac{3}{7} < \frac{1}{2}$.

Mathematical Practice or Process Standards:

SFMP 2. Reason abstractly and quantitatively.

Students will use previous understanding of fractions to determine whether a fraction is closest to 0, $\frac{1}{2}$, or 1 whole.

SFMP 3. Construct viable arguments and critique the reasoning of others.

Students justify their reasoning with clearly stated arguments.

SFMP 4. Model with mathematics.

Students use fraction models to help them determine benchmark fractions.

SFMP 8. Look for and express regularity in repeated reasoning.

Students look for patterns and explore how to determine a benchmark fraction without the use of models.

Goal:

Students use models, patterns, and reasoning to determine if a fraction is closest to 0, $\frac{1}{2}$, or 1 whole in preparation for estimating with benchmark fractions.

Planning:

Materials: Fraction models including area models, fraction strips, and number lines. A variety of fractions written on index cards gathered into a deck.

Sample Activity:

Make a table with 3 columns on the blackboard. Label the columns "0 $\frac{1}{2}$ 1."

Choose an index card from the deck. Ask students to determine whether that fraction is closest to 0, $\frac{1}{2}$, or 1 whole, justifying their response using any of their fraction models. If the class agrees, the fraction is written in the correct column on the table. Continue with a variety of fractions monitoring when students seem less reliant on the models. Discuss patterns they see on the table. Continue with additional fractions and ask students to determine the closest benchmark fractions, reasoning with the patterns they found rather than using the models.

Notes

Sample PLANNING PAGE (Continued)

Questions/Prompts:

- How did you know $\frac{7}{8}$ is closest to 1 whole?
- What do you notice about the fractions that are closest to zero? Why do you think that is true?
- What do you notice about the fractions that are closest to one-half? Why do you think that is true?
- What do you notice about the fractions that are closest to one whole? Why do you think that is true?
- Are there any fractions that do not fit the pattern?
- Add some more fractions to each of the columns.
- How can benchmark fractions help you to determine if your answer to an addition or subtraction problem is reasonable?

Differentiating Instruction:

Struggling Students: Struggling students may need more experience with models. They should compare each fraction to each of the benchmarks and determine which is the closest.

Use simpler fractions to begin with and ask explicit questions that will help students make generalizations, such as fractions that are close to 0 have numerators and denominators that are far apart. Fractions close to $\frac{1}{2}$ have a numerator that is close to half of the denominator. Fractions close to 1 have a numerator that is very close to the denominator.

Extension: Extend this work to identifying benchmarks with mixed numbers.

Notes

PLANNING PAGE

Standard:

Mathematical Practice or Process Standards:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

PLANNING PAGE

Standard:

Mathematical Practice or Process Standards:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

Reflection Questions: Number and Operations—Fractions

1. How do unit fractions form the foundations part for developing the concept of equivalence in Grade 3 and with fractional number operations in 4 and 5? Give an example of how unit fractions build conceptual understanding at your grade level.

2. How does understanding the meaning of the denominator and the meaning of the numerator affect conceptual understanding of addition and subtraction of fractions? How are addition and subtraction of whole numbers the same as addition and subtraction of fractions? How are they different?

3. How does understanding of the meaning of multiplication and division of whole numbers relate to understanding the meaning of multiplication and division of fractions? How can this relationship help to build conceptual understanding when multiplying and dividing fractions and mixed numbers?

4. Look at the Related Content Standards listed in each cluster overview for this domain. Make a chart with a sequence of Standards across Grades 3–5. From this work describe how fraction concepts grow from one grade to the next as well as within a specific grade level.