Preface

> Recommendation 1: Problem solving must be the focus of school mathematics in the 1980s.
>
> *—An Agenda for Action: Recommendations for School Mathematics of the 1980s*

> Problem solving . . . can serve as a vehicle for learning new mathematical ideas and skills. . . . A problem-centered approach to teaching mathematics uses interesting and well-selected problems to launch mathematical lessons and engage students. In this way, new ideas, techniques, and mathematical relationships emerge and become the focus of discussion. Good problems can inspire the exploration of important mathematical ideas, nurture persistence, and reinforce the need to understand and use various strategies, mathematical properties, and relationships.
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> *—Principles and Standards for School Mathematics*

THE TWO statements above, made twenty years apart by the National Council of Teachers of Mathematics (1980, p. 2; 2000, p. 182), serve as evidence of a long-term commitment of the Council to making problem solving a central theme of school mathematics instruction. The first statement was made at a time when the NCTM was just beginning to assert itself as a leader in efforts to change the nature of mathematics teaching in our schools. The second statement demonstrates that after two decades of curriculum development, research, and considerable reflection, the Council has developed a mature position about the role that problem solving should play in mathematics instruction.

The second statement also captures the essence of what this volume and its companion for grades 6–12 are about, namely, that

the role of problem solving in mathematics instruction should change from being an activity that children engage in after they have studied various concepts and skills to being a means for acquiring new mathematical knowledge. But to suggest, as do the authors of *Principles and Standards,* that problem solving "can serve as a vehicle for learning new mathematical ideas and skills" (NCTM 2000, p. 182) is one thing; to provide the sort of coherence and clear direction that teachers need is another matter. These volumes represent a serious attempt to provide this coherence and direction.

In conceptualizing these volumes, the Editorial Panel was guided by what it saw as a central message of all four NCTM *Standards* documents (1989, 1991, 1995, 2000), namely, their emphasis on the importance of viewing classroom mathematics teaching as a system. According to Hiebert and his colleagues (1997), the five dimensions of this system are (1) the nature of classroom tasks, (2) the role of the teacher, (3) the social culture of the classroom, (4) mathematical tools as learning supports, and (5) equity and accessibility. Changing any of the elements of this system requires parallel changes in each of the other dimensions.

The system of mathematics classroom instruction that has characterized U.S. schools for at least the entire past century can be characterized in terms of the foregoing dimensions roughly as follows. Classroom tasks come mainly from the worked examples and homework exercises in the textbook. These are predominantly short, out of context, and symbolic, with emphasis on mastering and maintaining procedural skills. The teacher's role is to work examples for the students using direct teaching with the expectation that students will listen and learn to apply the same procedures that the teacher demonstrates. Students then practice those procedures through individual classwork and homework in which they try many more exercises that are very similar to those the teacher just demonstrated. If any applications of these procedures to real-world problems are included, they are briefly stated and straightforward "word problems" presented immediately after the procedures that students are expected to use to solve the problems.

The social culture of the traditional classroom includes the agreement that the teacher and the answer key in the textbook are the sole mathematical authorities. Students who develop proficiency in using the procedural strategies given in the textbook and demonstrated by the teacher are rewarded with praise and high grades. The nature of the students' thinking and the strategies,

both mathematically valid and invalid, that they may have tried for solving problems are generally of much less interest than getting the right answer using the method shown in the textbook.

The most unfortunate consequence of instruction of the sort just described is that too often students leave school with at best a command of a set of facts, procedures, and formulas that are understood in a superficial or disconnected way. Even worse perhaps, they have little or no notion of how they might use what they have learned as they pursue their lives outside of school.

The chapters of this book together describe in some detail the characteristics of a classroom system called "teaching mathematics through problem solving" in which the main goal is for students to develop a deep understanding of mathematical concepts and methods. The key to fostering understanding is engaging students in trying to make sense of problematic tasks in which the mathematics to be learned is embedded. In addition to the mathematics that is the residue of work on the tasks, the kind of sense making and problem solving in which students engage involves doing mathematics. As students attempt to solve rich problem tasks, they come to understand the mathematical concepts and methods, become more adept at mathematical problem solving, and develop mathematical habits of mind that are useful ways to think about any mathematical situation.

This approach to classroom instruction involves much more than finding and using a collection of "fun" problems. First and foremost, the problematic tasks that are chosen must have embedded in them the mathematics that is to be learned. Second, the tasks must be accessible and engaging to the students, building on what they know and can do. Third, the teacher's role is very important in ensuring that the classroom norms are supportive of students' learning in this way and in pressing students to think deeply about their solution methods and those of their classmates and, most important, about the mathematics they are learning. Teachers also have a role in ensuring that students have access to appropriate technological and intellectual tools for learning, including facility with important paper-and-pencil procedures. A final challenge for teachers and curriculum developers is to find ways to ensure that the understanding that comes from learning mathematics through problem solving is accessible to all students.

This volume focuses on mathematics in prekindergarten through grade 6, and its companion volume deals with the secondary grades. The issues and the organization of the two volumes are similar, a reflection of the overlap of teaching issues across all grade levels.

This volume consists of three main sections—"Issues and Perspectives," "In the Classroom," and "The Role of Technology"—and a final chapter that presents a research perspective on teaching mathematics through problem solving. No single section addresses the entire set of issues concerning teaching mathematics through problem solving, but the volume as a whole presents much of what we mathematics educators as a profession know and have experienced about the topic.

The chapters in section 1 raise issues and points of view about teaching mathematics through problem solving that anyone who teaches mathematics to young children should carefully consider. In chapter 1, Diana Lambdin suggests several benefits of teaching through problem solving. In a nutshell, she argues that a close relationship exists between problem solving and understanding and that tremendous benefits result from learning mathematics with understanding. She brings her chapter to a close by showing how learning through problem solving promotes deep, rich understanding of mathematical concepts and processes.

Chapter 2, written by E. Paul Goldenberg, Nina Shteingold, and Nannette Feurzeig, focuses "not only on how mathematics reflects important ways of thinking that we believe *all* subjects should support and *all* children can and should acquire but also on the special role that mathematics plays in honing, refining, and extending these ways of thinking" (p. 16). The authors discuss five "habits of mind" that are especially relevant in prekindergarten through sixth grade and provide some concrete suggestions about how these habits of mind fit into teaching that emphasizes problem solving.

In the next chapter, Beatriz D'Ambrosio takes us on a trip from ancient to modern times to look at how conceptions of problem solving and the role it plays in the mathematics curriculum have changed over time. She notes that problem solving has been an important component of the school mathematics curriculum for at least 150 years and argues that teaching mathematics through problem solving emerged rather slowly and has recently begun to appear in some school mathematics textbooks.

Section 2, comprising chapters 4–12, focuses on how teaching mathematics through problem solving might play out in the classroom. Taken as a whole, these chapters serve to describe how the five dimensions of the classroom teaching system discussed previously might be thought of when problem solving becomes the means through which understanding of important mathematics is attained. A chapter by James Hiebert opens this section, and in it

he identifies several "signposts" that can guide teachers in giving students opportunities to develop deep understanding of important mathematics. He notes, "Just as signposts along the road can highlight for travelers important information for reaching their destination, so signposts for the classroom can highlight for teachers essential features for helping students achieve the intended learning goals" (p. 54).

In chapter 5, John Van de Walle points out that adopting a problem-solving stance toward instruction requires the teacher to pay special attention to those tasks or problems that are at the heart of this approach. He addresses three fundamental questions: How do we choose good tasks? What do they look like? and How can we make them accessible to every child so that all children can learn?

Susan Jo Russell, Rebeka Eston, Jan Rook, Malia Scott, and Liz Sweeney, the authors of chapter 6, pose the question "What does it mean to have a mathematics curriculum that focuses on problem solving?" (p. 85). They point out that they have moved from viewing problem solving as separate from the rest of the mathematics curriculum to regarding the use of problems as a mechanism for focusing on the "coherent development of important mathematical ideas that are core to the curriculum" (p. 85).

An essential ingredient of teaching mathematics through problem solving is "listening" to students as they do mathematics. For Erna Yackel, the author of chapter 7, listening includes paying attention to what children do as well as to what they say. In general, listening to children in the mathematics classroom involves attempting to figure out how children make sense of and solve the problems they are given in mathematics class. She challenges teachers to think about how to use listening to move children forward in their thinking and enhance learning for all.

In chapter 8, Frances Curcio and Alice Artzt "explore the parallels between the process of teaching a mathematics lesson that leads to deep understanding and the process of solving a mathematics problem" (p. 137). Specifically, they discuss teaching mathematical problem solving within a teaching-as-problem-solving framework, and they show how teachers' knowledge, beliefs, goals, and problem-solving behaviors can be transferred to their students in ways that develop the students' understanding of important mathematics.

Because teaching mathematics through problem solving involves substantive changes in the nature of classroom activity

and discussion, as well as changes in what is expected of both students and teachers, teachers should establish and sustain a risk-free classroom environment in which students' reasoning, not just answers, is valued. In chapter 9, Michelle Stephan and Joy Whitenack use examples from a first-grade classroom to illustrate how the teacher and the students can create a classroom environment conducive to rich problem solving as they establish both social and sociomathematical norms in the classroom.

Carmel Diezmann, Carol Thornton, and James Watters assert that teachers should pay particular attention to the needs of exceptional students if they intend to provide worthwhile problem-solving experiences for all the students in their classrooms. In chapter 10, the authors suggest ways to tailor mathematics instruction to meet the needs of students with learning difficulties as well as those with special talents in mathematics. They note that for both groups of students, their potential for learning is extended when they are challenged to engage in real problem solving.

In chapter 11, Lyn English points out that "[p]roblem posing—like its companion, problem solving—should be an integral component of the mathematics curriculum across all content domains" (p. 197). She suggests that problem posing tends not to be given the attention it deserves in mathematics class, even though it occurs naturally in everyday life. She discusses the fundamental importance of building inquiry-oriented classroom environments that foster problem posing and engage students in constructive dialogue and debate about their mathematical conjectures and constructions.

In the final chapter of this section, Yoshinori Shimizu gives an overview of the Japanese approach to teaching mathematics through problem solving and describes a typical organization of mathematics lessons in Japanese elementary schools. He also presents a specific problem that is typically found in Japanese textbooks and the related anticipated students' solutions to it to show how students share their solutions during the whole-class discussion. He ends his chapter by offering teachers some practical ideas that he has picked up from his work with Japanese teachers.

Section 3 contains chapters by Warren Crown (chapter 13) and Michael Battista (chapter 14). Crown discusses the role that technology can and cannot play in helping teachers use a problem-based approach in their mathematics teaching. He identifies various kinds of technology available for use in elementary classrooms and points out their strengths and weaknesses in support-

ing teaching through problem solving. Battista illustrates how special computer software can be used to support problem-based learning of geometry. He gives examples of sequences of problems that guide students' construction of geometric concepts and illustrates the nature of students' reasoning and learning as they work on these problems.

In chapter 15, Jinfa Cai summarizes what research tells us regarding four questions frequently asked about teaching through problem solving: (1) Are young children really able to explore problems on their own and arrive at sensible solutions? (2) How can teachers learn to teach through problem solving? (3) What are students' beliefs about teaching through problem solving? (4) Will students sacrifice basic skills if they are taught mathematics through problem solving? Cai concludes that some aspects of teaching mathematics through problem solving have considerable support from empirical research, but some important issues need additional research.

A special feature of this volume is the inclusion of a collection of Teacher Stories that amplify the perspectives and suggestions offered by the chapter authors. These stories, written by teachers involved in professional development seminars, serve to illustrate many of the ideas about teaching mathematics through problem solving discussed in those chapters.[1] The teachers were asked to choose classroom episodes that interested, intrigued, puzzled, or surprised them. They wrote to reflect on their own practice and to articulate questions or concerns that they wanted to share with the other teachers in the group. These stories, therefore, were not created to illustrate exemplary practice, to explain what works, or to tell other teachers how to carry out a specific activity or implement a teaching technique. Rather, they are attempts to capture episodes of mathematical activity in real classrooms, accompanied by the authors' own thoughts. In a sense, the stories bring to life many of the ideas about teaching mathematics through problem solving presented in the other chapters. The stories represent a wide range of classroom settings—large cities, small urban centers, and suburban towns. We are grateful to these teachers for sharing their practice with us. We hope their stories will serve to spark other

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teachers' thinking about how to begin to teach mathematics through problem solving. Furthermore, we also hope that this collection of teachers' stories and perspectives offered by the other authors will provide both the coherence and the clear direction concerning teaching through problem solving that teachers have been seeking.

Finally, the conceptualization and preparation of this volume was undertaken by a small team of mathematics educators who thought long and hard about what it might mean to use problem solving "as a vehicle for learning new mathematical ideas and skills" (NCTM, 2000, p. 182). Without their very able assistance, this volume would have never been completed. Not only did each of them write a chapter and review drafts of chapters, but they each also gave us invaluable feedback whenever we asked for it. We wish to extend our sincerest thanks to these dedicated individuals, the members of the Editorial Panel:

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