CHAPTER 3

Investigating the Five Practices in Action

In chapter 1, we presented the five practices for orchestrating a productive discussion and considered what David Crane's class *might* have looked like had he engaged in these practices and how use of the practices in advance of and during the lesson *could* have had an impact on students' opportunities to learn mathematics. In this chapter, we analyze the teaching of Darcy Dunn, an eighth-grade teacher who has spent several years trying to improve the quality of discussions in her classroom.

The Five Practices in the Case of Darcy Dunn

The vignette that follows, Tiling a Patio: The Case of Darcy Dunn, provides an opportunity to consider the extent to which the teacher appears to have engaged in some or all of the five practices before or during the featured lesson and the ways in which her use of the practices may have contributed to students' opportunities to learn. (This case, written by Smith, Hillen, and Catania [2007], is based on observed instruction in the third author's classroom.)

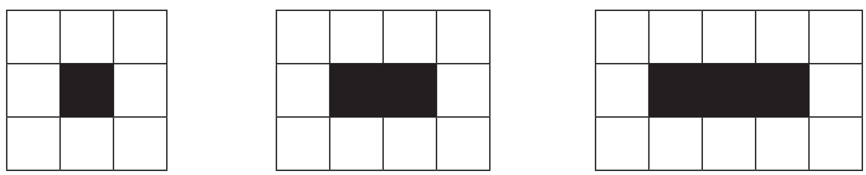
ACTIVE ENGAGEMENT 3.1

Read the vignette Tiling a Patio: The Case of Darcy Dunn and identify places in the lesson where Ms. Dunn appears to be engaging in the five practices. Use the line numbers to help you keep track of the places where you think she used each practice.

Tiling a Patio: The Case of Darcy Dunn

Darcy Dunn was working on a unit on functions with her eighth-grade students early in the school year and decided to engage them in solving the Tiling a Patio task [shown previously as fig. 2.3 but repeated here as fig. 3.1 for the reader's convenience]. As a result of this lesson, she wanted her students to understand three mathematical ideas: (1) that linear functions grow at a constant rate; (2) that there are different but equivalent ways of writing an explicit rule that defines the relationship between two variables; and (3) that the rate of change of a linear function can be highlighted in different representational forms: as the successive difference in a table of (x, y) values in which values for x increase by 1, as the m value in the equation $y = mx + b$, and as the slope of the function when graphed.

Alfredo Gomez is designing patios. Each patio has a rectangular garden area in the center. Alfredo uses black tiles to represent the soil of the garden. Around each garden, he designs a border of white tiles. The pictures shown below show the three smallest patios that he can design with black tiles for the garden and white tiles for the border.



Patio 1 **Patio 2** **Patio 3**

a. Draw patio 4 and patio 5. How many white tiles are in patio 4? Patio 5?

b. Make some observations about the patios that could help you describe larger patios.

c. Describe a method for finding the total number of white tiles needed for patio 50 (without constructing it).

d. Write a rule that could be used to determine the number of white tiles needed for any patio. Explain how your rule relates to the visual representation of the patio.

e. Write a different rule that could be used to determine the number of white tiles needed for any patio. Explain how your rule relates to the visual representation of the patio.

Fig. 3.1. The Tiling a Patio task. (Adapted from Cuevas and Yeatts [2005, pp. 18–22].)

In addition to the fact that the task provided a context for exploring the mathematical ideas that Ms. Dunn had targeted, it had an aspect that she found particularly appealing: all students, regardless of prior knowledge and experiences, would have access to the task. Every student would be able to build or draw the next two patios (part *a*) and make some observations about the patios (part *b*). Although finding the number of white (border) tiles in patio 50 (part *c*) would be more challenging, students could make a table and look for numeric patterns or “see” one of the many relationships between the white and black tiles in the diagram itself.

Ms. Dunn began the lesson by having a student read the task aloud and making sure that all of the students understood what the problem was asking. She told students that they would have five minutes of “private think time” to begin working on the problem individually and reminded them to help themselves to any of the materials (tiles, grid paper, colored pencils, calculators) on their tables. They could then share their ideas about the task with the other members of their table groups and work together to come up with a solution.

As students worked on the task, first on their own and later in collaboration with their peers, Ms. Dunn circulated among the groups, making note of the different approaches that students were using, asking clarifying questions, and pressing the students to think about what the bigger patios would "look like" and how they could figure that out without building or drawing them all. She noted that although all the groups were able to complete parts *a* and *b*, a few students, such as James, were having difficulty describing patio 50, and many were struggling to write symbolic rules for part *d*. Through her questioning during small-group work, these struggling students had made some progress, and she decided that the students could continue working on providing verbal descriptions and converting them to symbolic rules as a whole class.

After about fifteen minutes of small-group work, Ms. Dunn decided that she would ask Beth to present her group's strategy first for part *d*. Several groups had used the same approach, but it had been several days since Beth had contributed to a whole-class discussion in a central way, and Ms. Dunn wanted this quiet student to have a chance to demonstrate her competence. As Beth approached the overhead projector in the front of the room, Ms. Dunn handed her a few overhead pens in different colors and one of the transparencies that she had prepared in advance, showing the first three patios. This way, Beth could easily explain what she did and how it connected to the drawing without having to draw all the patios. The following dialogue ensued between Beth and Ms. Dunn:

Beth: You multiply by two and add six.

Ms. Dunn: You multiply *what* by two?

Beth: The black tiles.

Ms. Dunn: Write it down somewhere. You multiply the black tiles by two, and then add six. Can you show us on the diagram—where do you see it on the picture? Where do you see that, to multiply by two? You can write on the transparency.

Beth: *[Demonstrating her method on the drawing of patio 1]* There's one, then one tile times two equals two, plus six, equals eight, and then, it's eight tiles.

Ms. Dunn: OK, you add six. Where is the constant of six?

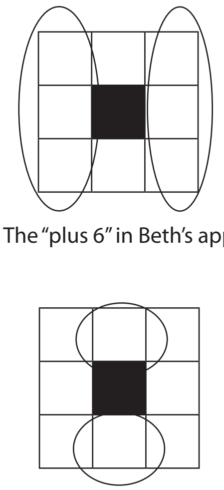
Beth: Because there's three on each side.

Ms. Dunn: Circle them for me.

Beth: *[Makes circles around the tiles on the sides of patio 1, as shown in fig. 3.2a.]*

Ms. Dunn: One, and the two—where's the two? Two ones are where?

Beth: Right there, and right there [*points to the middle tile of the three tiles on the top row and the bottom row of patio 1, as shown in fig. 3.2b.*]



a. The "plus 6" in Beth's approach

b. The "2 times the number of black tiles" in Beth's approach

Fig. 3.2. Beth's approach to patio 1

After Beth's presentation, Ms. Dunn pressed students to express Beth's way of viewing the pattern symbolically as $w = 2b + 6$, where w is the number of white tiles in the patio and b is the number of black tiles. Sherrill commented that the number of black tiles was the same as the patio number, so it didn't matter if they used b (for black tiles) or p (for patio number). Ms. Dunn asked Sherrill to write the generalization for the number of tiles on the newsprint that was hanging on the board so that everyone could keep track of the different ways of finding the total number of white tiles in any patio.

Ms. Dunn then asked for a second method from the class. Several students volunteered to present their work, and after quickly checking the notes that she had made as she had monitored the small-group work, Ms. Dunn selected Faith to go next. On a new transparency, Faith demonstrated her approach to patio 1 [shown in fig. 3.3], explaining, "I did the number of black tiles, and I added two [see step 1 in fig. 3.3]. You do that times two to get the top and bottom [see step 2 in fig. 3.3]. Then I did plus two" [see step 3 in fig. 3.3].

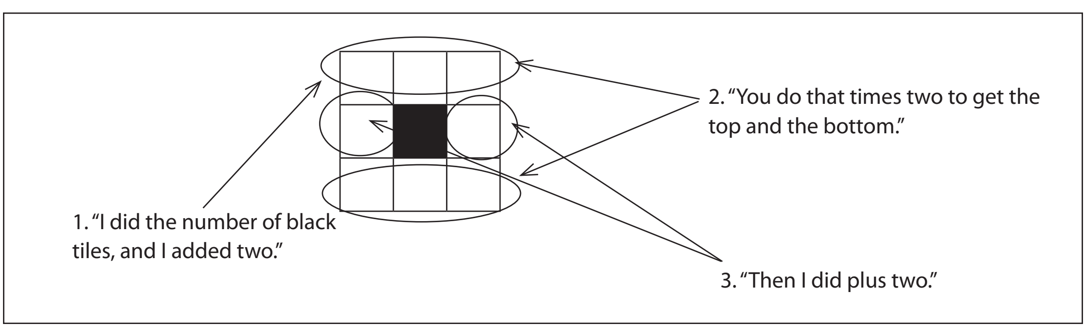


Fig. 3.3. Faith's explanation of her approach on patio 1

When Faith had finished her explanation, Ms. Dunn commented, “OK, I don’t think everyone understood that. Does anyone have a question for Faith?” Pedro was the first to raise his hand, and Ms. Dunn encouraged Faith to call on him. Pedro asked, “Where did your last ‘plus two’ come from?” Faith clarified, “These two right here [pointing to the white tiles to the left and right of the black tile in patio 1], because they’re the two remaining tiles that you haven’t added already.”

Ms. Dunn then asked the class how they could write an equation for Faith’s approach. Damien volunteered that his group had thought about the problem in the same way that Faith did, and they had come up with the equation $w = 2(b + 2) + 2$. At the teacher’s request, Damien went to the front of the room to explain why this equation worked, using the drawing of patio 3. He explained, “The number of black tiles (or the patio number) plus two will always give you the top and bottom rows, and then you always have one on each side which gives you the plus two.” Ms. Dunn asked Damien to add the equation to the newsprint list.

Ms. Dunn then asked Devon if he would be willing to share his approach. Time was running out, and Ms. Dunn wanted to make sure that his approach, which focused on finding the total area of the rectangular region (the patio plus the garden) and then subtracting out the area of the garden, was made public, since it was different from other approaches and had the potential to be a useful strategy for solving problems that students would encounter in the future. Ms. Dunn engaged Devon in the following dialogue:

Devon: OK, like Damien was saying, there’s always going to be two more tiles on the bottom [row].

Ms. Dunn: Draw on it [*hands Damien a transparency*].

Devon: [*Drawing and explaining*] There’s always going to be two more tiles down here [*see step 1 in fig. 3.4*] than there is right here. So, I knew that in patio 50 there was going to be fifty-two on the bottom, ’cause there’s fifty black tiles. And, so I took fifty-two times three, these three [*pointing*], ’cause there’s always three on the side [*see step 2 in fig. 3.4*], no matter what patio it is, and I got a hundred fifty-six. Which gives you the area; then you subtract the black ones [*see step 3 in fig. 3.4*], so you subtract fifty and that gives you a hundred and six.

Ms. Dunn: Oh! That was pretty creative. He took the whole figure, and then subtracted out the area in the middle. Ooh—I like it.

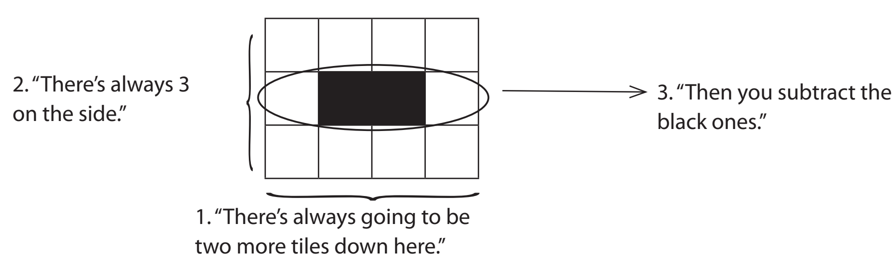


Fig. 3.4. Devon’s explanation of his approach to patio 2

Ms. Dunn then asked the class how they could write Devon's rule, using symbols. Phoebe said that it would be $w = 3(b + 2) - b$. James had a puzzled look on his face, and Ms. Dunn asked him if he had a question for Phoebe. James asked, "Why did you multiply by 3? Everyone else multiplied by 2." Phoebe responded, "Devon is using all three rows of the patio, so he has three rows of $p + 2$, not two rows like Faith had. But then, you have to subtract the black part 'cause it isn't part of the patio." James said, "So you took times three and subtracted, while Faith did times two and added. I get it."

Ms. Dunn very much wanted students to consider the table that Tamika had built when she started the problem. She thought that this representation, which included the first ten patios, would help students see that the number of white tiles increased by 2 as the patio number increased by 1 (that is, that the rate of change is 2)—an idea that had not been salient in any of the presentations so far. Ms. Dunn then planned to ask students to show where this "+2" was in the picture and in the equation. She wanted to make sure that they saw the connection among the picture, the table, and the equation. She then wanted to have students predict what the graph would look like and why, and, ultimately, graph it. But she knew that this work could not be done in the remaining five minutes of class. Instead, she decided that she would begin tomorrow's class with a discussion of the table and the graph.

Ms. Dunn decided to use the limited time she had left to return to the list of equations that the students had produced during the discussion, to which Phoebe had added the last equation. She called the students' attention to the list that was hanging in the front of the room [shown in fig. 3.5] and noted, "We came up with three different ways to find the total number of white tiles in any patio. Can they all be right?" She then asked students to spend the next few minutes discussing this question in their groups. Their homework assignment was to provide a written answer to the question and to justify their conclusion.

$w = 2b + 6$	(Beth and class)
$w = 2(b + 2) + 2$	(Faith and Damien)
$w = 3(b + 2) - b$	(Devon and Phoebe)

In each equation, b is the number of black tiles (or p could be used instead as the patio number) and w is the number of white tiles in the patio.

Fig. 3.5. List of rules for determining the number of white tiles in any patio

Analyzing the Case of Darcy Dunn

Although we could identify many aspects of the instruction in Ms. Dunn's classroom that may have contributed to her students' opportunities to learn mathematics, we will focus our attention specifically on her use of the five practices. In subsequent chapters, we will analyze a broader set of actions that, in combination with the five practices, help account for the success of the lesson. We begin by considering the five practices and whether there is evidence that the teacher engaged in some or all of these practices. Then we consider how Ms. Dunn's use of the practices may have enhanced her students' opportunities to learn.