

## Math Is *Supposed* to Make Sense!

Not long ago, I was called on to help my 14-year-old granddaughter, Maisey, who was struggling to pass algebra. Her motivation to *pass* algebra was strong—if she didn't, she wouldn't be eligible to play basketball. Her motivation to *learn* algebra in any meaningful way, however, was far less strong. She was convinced that there was nothing to be gained by trying to make sense of math, since there was little in math that made sense. Rather, she believed she just needed to find a way to remember which rule to use for which kind of problem. Maisey developed these beliefs over many years; getting past them is proving to be quite a challenge for her, as well as for the adults trying to help her.

Maisey's story is not unique. Unfortunately, many students, like many adults, have come to believe that math has more to do with abstract rules, memory tips, and magic tricks than it has to do with making sense. They have become used to being told what to do and have come to believe that mathematics may or may not make sense—often, the latter. The structure of our mathematics classrooms and the prevalence of a lecture approach to teaching can contribute to a student's belief that math just doesn't make sense. Changing that idea so that students come to not only believe, but demand, that math should make sense to them can be the difference between a program that simply covers material and one that turns students into mathematical thinkers.

## What It Means to Make Sense of Mathematics

We should distinguish between *making sense* and the closely related ideas of *logic*, *reasoning*, and checking that an answer is *reasonable*. Making sense involves all of these things and more. When a student makes sense of what she's doing in mathematics, it's like a light bulb going on in her head. She sees how an answer or a process follows from what came before, she mentally verifies that what she sees is reasonable for the context or situation, she can see a reason why certain things were done, and what she sees fits with what she has previously learned, knows, or believes to be true. She feels comfortable that what she has done, or what she sees that someone else has done or said, makes sense. It becomes part of a bigger understanding that mathematics makes sense as a discipline—that the subject of mathematics has an underlying structure we can rely on. And the experience of making sense in that particular situation or for that particular problem contributes to a growing conviction that math is supposed to make sense.

## Mathematical Habits of Mind

Arguably, the most important outcome for students in an effective mathematics program is the development of mathematical habits of mind—the ability to think mathematically, analyze situations, understand relationships, and adapt what they know to solve a wide range of problems, including problems that may not look like any they've encountered before. These habits of mind have been concisely characterized by Cuoco, Goldenberg, and Mark (2010) to include the following:

- ♦ **Performing thought experiments**—connecting experiences and previous understanding to mentally model a problem, situation, process, or operation.
- ♦ **Finding, articulating, and explaining patterns**—noticing regularity in situations, procedures, and so on and being able to accurately describe and explain the pattern.
- ♦ **Creating and using representations**—using different representations (words, pictures, objects, tables, graphs, symbols, equations, etc.) to solve problems and to develop and demonstrate understanding.
- ♦ **Generalizing from examples**—trying one or more specific examples as an entry point to a complex problem.
- ♦ **Articulating generality in precise language**—developing the nontrivial ability to clearly describe a mathematical pattern or relationship using correct mathematical vocabulary.
- ♦ **Expecting mathematics to make sense**—coming to believe, and even insist, that what we do in mathematics should make sense.

The last item on this list—expecting math to make sense—is without question the most important goal we should have for every student we see or for every student of every teacher we work with.

## When Students Don't Make Sense of What They Do

I often tell the story of Marisa, a 5th grader highlighted in a video that is part of the Math Reasoning Inventory project (Burns, 2012). In a one-on-one situation, a teacher gives Marisa the following problem:

There are 295 students in the school. School buses hold 25 students. How many school buses are needed to fit all the students?

Marisa almost immediately dives into a calculation, lining up the number 25 under the number 295, writing an addition sign, and carefully (and accurately) adding 295 plus 25. She tells the teacher that her answer is 320 school buses. When the teacher asks her how she did it, she explains, "Since I heard the word 'all,' I figured it was plussing, so I plussed 295 and 25 and I got 320." When she first announces her answer before the explanation, she hesitates just an instant, perhaps considering that there probably would not need to be more buses than the number of students they are going to carry. But apparently Marisa decided that, since she had done what she had learned to do (paying attention to which clue words or key words might appear in a problem, and accurately computing), it didn't matter whether the answer made sense or not. After all, *this was math*.

## Expecting Math to Make Sense

We often see included on problem solving lists or stated in learning standards that students should confirm that their answer is reasonable. Yet all of us have seen instances where a student will give an answer that obviously doesn't make any sense, as Marisa did. Unfortunately, too many students see "checking for reasonableness" as simply one more requirement to mindlessly check off, rather than thinking about whether their answer truly makes any sense or is reasonable for the problem, for a real situation, or in any other way. With an emphasis on rules, procedures, and tricks, we seem to have created a society in which many people (not just students) don't actually expect that math will make sense, so they aren't particularly concerned if it doesn't.

What an unfortunate situation—one of the most note-worthy characteristics of mathematics is that, by its very nature, it makes more sense than probably any other school subject. Mathematics has many dimensions and can be used in different ways, but its inherent structure, order, and logic unify mathematical topics and applications, and mathematics generally operates consistently and predictably. I'm afraid that the way we teach math sometimes belies that quality. In our zeal to cover a certain amount of material in a limited time, we can inadvertently short-circuit any effort toward understanding. We may convince ourselves that if students will just memorize the procedure, they'll do OK on the test. And sometimes, it just may not be prac-

tical to try to help every student conceptually understand the steps in a complex procedure like the long-division algorithm or the quadratic formula. But we can still help students come to see the inherent consistency within mathematics and to understand numbers, operations, and relationships well enough that what they see and do in mathematics makes sense to them.

We want students to believe deeply that mathematics makes sense—in generating answers to problems, discussing their thinking and other students' thinking, and learning new material. If something *doesn't* make sense, alarms should go off in their heads. It should be disturbing—or at least cause a minor upset. That moment of cognitive dissonance should lead the student to take some kind of action until whatever it is does make sense.

- If a student is solving a problem and an answer doesn't make sense (like Marisa's response that 320 buses would be needed to hold 295 students), that upset should cause the student to go back and reconsider the approach or calculation that led to that answer.
- If another student is explaining his thinking, and it doesn't make sense, the upset should lead the listener to ask a clarifying question.
- If the teacher is explaining a new mathematical concept, or the student otherwise encounters a new mathematical idea, and that concept or idea doesn't make sense, the upset should lead the student to ask a question or dig deeper until it does.

Consider what it would mean if every student—and every teacher—absolutely and positively believed that math is supposed to make sense. And whenever something mathematical didn't make sense to a student, that student would say, "Wait a minute. Wait a minute. That doesn't make sense to me (yet). And math is supposed to make sense!" If we were to accomplish that core goal, perhaps all of our other efforts to improve mathematics teaching and learning would have a real chance of success.



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