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Shifting Expectations: Classrooms as Learning Environments

Vignette 1: Students Learning Together

The students in Ms. Shaw's ninth-grade class are working on the following probability problem:

> *Imagine that you have two pockets and that each pocket contains a penny, a nickel, and a dime. You reach in and remove one coin from each pocket. Assume that for each pocket, the penny, the nickel, and the dime are equally likely to be removed. What is the probability that your two coins will total exactly two cents?*

Steven, Antonio, Olivia, and Fasik sit together around a table. To determine the denominator of the probability, they have been counting all the possible coin combinations on their individual papers. They have been debating the number of possible outcomes.

Steven wrote the following:

(P, P) (P, N) (P, D) (N, D) (N, N) (D, D) (N, P) (D, P) (N, N)
9 possible

Antonio wrote this:

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D ___P___ P      P ___ D___ D            D ___N___N
     |                 |                      |
     N                 N                      P
                     9 ways
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Steven and Antonio seem to agree that nine combinations of coins are possible. Then Antonio pauses and says, "But there are *two* pockets, so it's 18."

"Why do you want to multiply?" asks Steven. Olivia laughs. Quietly, Fasik says he got six ways, because he counted (N, D) and (D, N) as the same outcome. Hearing the other students, though, he changes his response to nine.

What just happened would be a point of breakdown in many classrooms. Fasik and Antonio have miscounted the outcomes for different reasons. Their responses use some logic, but at this point making sense of each other's thinking is hard.

Key ideas of probability underlie this confusion:

- How do you count the coins systematically to find all the outcomes?
- How do you account for the two pockets in your counting?
- What counts as a unique outcome?

These are questions that need to be raised and then answered for this activity to be mathematically meaningful for the students. Fortunately, Ms. Shaw can draw on resources to move this conversation from the point of breakdown to a mathematically meaningful interaction.

Ms. Shaw, along with her colleagues, has worked on an approach to group work in her classroom designed to support learning by engaging students in thinking about such vital mathematical questions. Having students work together on rich problems allows them to bring up their mathematical understandings to serve as a foundation for their learning. Ms. Shaw and her colleagues use Elizabeth Cohen's (1994) seminal work on collaborative learning to support students in airing their thinking and guide them toward deeper understandings of mathematics.

By structuring an activity that raises these fundamental mathematical issues, Ms. Shaw has already done crucial work in engaging the students' sense making. As educators, we want to ensure that students will build on these preliminary understandings to come to mathematically sound ones.

Vignette 2: Facilitating Deeper Sense Making

Ms. Shaw is circulating around the classroom. This group is at an impasse. Kneeling on the floor to be at eye level with the students, she smiles and says, "You're arguing. I love arguing! Okay, Fasik. Tell me what the argument is about." The other students all start talking over Fasik. Ms. Shaw holds up her hand, signaling them to be quiet.

Fasik, a recent immigrant who is learning English, points to the (N, D) and (D, N) on his paper and says hesitatingly, "These two are the same." He also points to (P, N) and (N, P) along with (P, D) and (D, P), saying, "And these and these."

"Fancy set notation!" says Ms. Shaw. "Does everybody see what Fasik is thinking?"

Steven then speaks up, "No, they are different because they are in a different order."

Antonio loses his patience and says, "But I get 18 'cause you have two pockets, so it's all of this"—he circles his finger around his tree diagram—"times two."

Ms. Shaw turns to Olivia. "What do you think?" Olivia says she agrees with Steven "that it's just nine."

"Because?" Ms. Shaw asks, her eyebrows raised expectantly.

"Because those"—Olivia points to the (N, D) and (D, N) on Fasik's paper—"are different and the two pockets doesn't matter."

"Okay. Let's see," Ms. Shaw says. "How are we going to straighten this out? I like your diagram, Antonio. Let's see whether we can use it to answer this question." She puts Antonio's paper in the center of the table and then acts out the scenario. "Okay, you get a penny out of one pocket, right?" She pretends to pull a penny out of her pocket. "And out of the other pocket, you can get another penny. Where is that outcome on Antonio's diagram?"

Ms. Shaw continues to act out the outcomes, asking the students to find each event on Antonio's diagram.

Complicated pedagogical work happens in this second vignette. To support her students in airing their thinking, Ms. Shaw acknowledges the importance of mathematical disagreements ("I love arguing!"). At the same time, she evaluates the quality of their arguments, making it clear that students need to justify their positions (“Because?” she prompts Olivia later in the conversation). Second, she manages students’ attention in important ways. She helps Fasik, hesitant to speak in front of his peers, to get the floor by signaling his groupmates to be quiet. She puts his paper in a place that his peers can see, allowing him to use simple language (“These two are the same”) alongside his solution to explain his thinking about complex ideas. All the while, Ms. Shaw talks with this group and scans the rest of the classroom to make sure the other students are staying focused. She prioritizes quality conversations with her students and knows ahead of time what to monitor for and some key questions that will help her quickly assess any group’s progress.

Although Antonio and Fasik’s answers are wrong, she praises the systematic representations they have made in trying to solve this problem. These ways of counting are an important start toward an accurate answer, and Ms. Shaw realizes that the boys need a better idea of exactly what they are counting. Because Antonio is the only one to use the tree notation, she puts his work at the center during her enactment of the coin pulling. Doing so not only values the intellectual contribution of his work but also helps the other students understand an important representation for counting a sample space.

In this book, I will illuminate some of the concepts and tools that Ms. Shaw and her colleagues have used to shift their classrooms into places that support the development of students’ mathematical thinking.

Developing Understanding in Different Learning Environments

How do we teach more students harder mathematics? American mathematics teachers are under pressure to increase the rigor of their courses while simultaneously making them accessible to more students. Unlike a mere twenty years ago, the upper-level mathematics curriculum is not solely the province of the college-bound. State and national standards, increased graduation requirements, and the expansion of standardized testing have raised the bar for all students, so teachers must bring challenging mathematics to a broader range of learners.

During that same period, we have discovered much more about how people learn mathematics. Studies of expert performance show that deep conceptual understanding of mathematics is needed to use that knowledge in different contexts. Consider that, in Ms. Shaw’s classroom, the students have an opportunity to make sense of the idea of a *sample space*. In their daily lives, they will not have to worry about the two-pocket coin problem. But they will need a flexible conceptual understanding of probabilistic reasoning, because many financial and medical decisions require an understanding of these concepts. If students develop a good grasp of core ideas in Ms. Shaw’s classroom, they are in a much better position to take charge of those consequential choices in their futures.

Teachers need to create *learning environments* that support student participation (Bransford, Brown, and Cocking 2000). Learning environments include the activities, context, roles, and relationships that participants have in a given setting that shape meanings and, consequentially, understandings. Thinking about the classroom as a learning environment means thinking not only about what the teacher is doing but also about the activities, the students, and the students’ roles and relationships with one another, the teacher, and the content.

Ms. Shaw and her colleagues have chosen a rich problem that students can engage with. They use deliberate structures to involve students with the problem and monitor their progress. They no longer simply plan what they, as teachers, will do in the classroom. They have to develop a set of principles, structures, and strategies for interacting with students around mathematics. In this way, students have opportunities to engage their own ideas in various ways: drawing their own representations, discussing ideas with each other, justifying their ideas, and airing their confusions.

That last point is essential. When teachers provide a place for students’ prior knowledge, children develop a more robust understanding of ideas. Too often in school, students learn what they need to know for a test and then revert to their misconceptions in their daily lives, failing to make critical connections between what they learned and how they live. Students lose learning between courses, leading teachers to complain of poor retention. The more varied ways of knowing, representing, and talking about math that are accepted, the greater the chances that more students will feel that their prior knowledge is valuable and that they will be able to make the meaningful connections that lead to understanding. Teachers need to respond to thinking that they may not always anticipate, increasing the complexity of the work.

For all this to be possible, students need to feel comfortable in their mathematics classrooms. Mathematics is notoriously disliked as a subject, the butt of jokes on TV shows and in comic strips. Particularly by adolescence, students have firm ideas about *who* is good at math—ideas often based on stereotypes of class, race, gender, or reputations of prior academic achievement. By valuing different mathematical contributions along with correct responses, such as Antonio's tree representation, Ms. Shaw and her colleagues redefine what being smart in math class means, giving more students an intellectually legitimate way into the subject. At the same time, by valuing the intellectually diverse mathematical contributions of their students, these teachers open the door for their own professional learning. The focus on student thinking requires a genuine curiosity about young people and their ideas.

> **"Kids walk into math class thinking that school is about producing something on paper. I need to change their mind and get them to see it's about learning: starting with what they know, getting confused, and then developing tools to get through that confusion. The most important tools come from working with each other: listening, communicating, working through, and persisting. In the end I want it to be about 'Do I get this? Do you get that?' Writing it down is the last step."**
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> ***—Laura Evans, Complex Instruction Educator, Mathematics Teacher, and Coach***

How Might Collaborative Learning Address Problems with Typical Math Instruction?

For several reasons, typical mathematics instruction has not succeeded for most learners. According to the Organization for Economic Cooperation and Development's PISA 2009 study, not only do American fifteen-year-olds underperform in mathematics compared with their age-mates in other countries, but also the negative impact of socioeconomic status on mathematical performance is greater here. Although enrollment in advanced mathematics classes has increased overall with changing graduation requirements, many poor students and students from historically disenfranchised ethnic and racial groups do not enroll in advanced mathematics (Planty, Provasnik, and Daniel 2007).

In secondary mathematics classrooms, the focus on teacher talk during instruction leaves students in the role of moving lessons forward by supplying answers to relatively simple questions:

Teacher: See the vertical angle here. It measures what?

Student: 70?

Teacher: Right, 70 degrees.

This pattern of instructional dialogue is called *IRE*, which stands for initiation–response–evaluation (Cazden 2001). The teacher *initiates* an interaction by posing a question. Students then *respond* to the question. Then the teacher *evaluates* their answers.

According to the Third International Mathematics and Science Study (TIMSS), the dominant pattern of classroom instruction in the United States is *learning terms and practicing procedures*

(Stigler and Hiebert 1999). IRE discourse supports this form of mathematics instruction. Teachers give students definitions or demonstrate procedures. Then teachers question to see whether the students can recall the definition or apply the procedure to similar problems during instruction. Although the TIMSS study is more than a decade old, more recent studies of classroom teaching confirm that few mathematics lessons include opportunities for student sense making or questions that move understanding forward (Banilower et al. 2006). That is, the emphasis on deep conceptual understanding of mathematics needed for flexible thinking is missing.

IRE discourse is not inherently problematic. It has a role in many kinds of instructional conversations. Even in Ms. Shaw's classroom, we see moments of IRE structure in vignette 2, when she asks, "Where is that outcome on Antonio's diagram?"

However, student learning opportunities in her classroom are qualitatively different because of how the overall environment is organized. For instance, the students had a chance to make sense of the problem before she questioned them. In this way, the issues of uniqueness that come out in Fasik and Antonio's solutions are more authentic for the students than if she simply told the students how to ensure the correct sample space. Also, many questions she posed require more challenging mathematical thinking than they would in an IRE-dominated learning environment. For example, the students all produced different representations of the sample space. When Ms. Shaw organized the conversation around Antonio's tree diagram, the students had to make connections between their representation of the possible outcomes and his. They are also pressed to explain their thinking, not simply to produce a correct answer. In fact, if Ms. Shaw were looking only at their written work, she would not see Antonio's confusion about the two pockets or Fasik's about the distinction between (N, D) and (D, N) as outcomes.

Traditional IRE-dominated classrooms that focus on learning terms and practicing procedures constitute a kind of learning environment, even though they are not envisioned in this way. In these learning environments, quick, accurate recall and calculation are often the most important ways of being smart. If we look at the learning environment, this makes sense. Correct answers keep the lessons moving forward in an IRE dialogue, so they are valued, often at the expense of valuing students' mathematical thinking. In contrast, in our vignette, Antonio's incorrect answer of 18 outcomes in the sample space did not preclude Ms. Shaw from valuing his sophisticated tree representation.

Traditional learning environments give the impression that mathematics questions only ever require a short time to solve. Mathematician and educational researcher Alan Schoenfeld (1988) has documented this common belief among students and linked it to traditional instruction. Unlike the worksheets that include similar problems of increasing difficulty—a hallmark of traditional classrooms—the probability task presented here was one of three that Ms. Shaw's students were thinking about that day. By giving fewer problems that demand a greater depth of thinking, Ms. Shaw allowed her students opportunities to make important connections by giving sustained attention to their work.

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In a traditional setting, learners receive knowledge passively. Their individual understandings and identities do not influence what goes on in the classroom. In contrast, the students in our vignette are actively involved in making sense of the mathematics. Their confusions and disagreements become the basis for the instructional dialogue. With instruction focused on students' ideas, students not only feel intellectually valued but also have the potential to find a place for their out-of-school selves. Educational researchers Jo Boaler and James Greeno (2002) compared high school calculus classes that engaged students in mathematical ideas with more traditionally didactic classes. Students reported feeling like they "fit" more in the classrooms that incorporated dialogic classrooms than in the classrooms that relied heavily on the teacher's monologue. Students learn not just about mathematics in school but also about who they are as mathematics learners.

In addition to keeping students' sense of self outside the classroom doors, traditional mathematics classrooms make participation optional. Students know if they hang back and wait, another "smarter" student will supply the missing answer. Because classroom participation and achievement are related (Cohen 1994), a classroom where participation is optional disadvantages students who are reluctant to participate, whether due to their temperaments or their prior achievement. A collaborative learning environment strives to increase student learning by increasing participation. Mindful of this connection, Ms. Shaw drew Olivia and Fasik, who were both relatively quiet, into the conversation by soliciting their thinking.

Traditional mathematics instruction has not reached all students equitably. Although all students can be taught mathematics at a greater level of depth, historically marginalized groups of students are often severely underserved in mathematics classrooms. As we witnessed in Ms. Shaw's classroom, teachers can encourage recent immigrants like Fasik to speak in small-group settings. Group work also allows nonverbal representations of mathematics to play a larger role in the classroom. When students create and discuss their own representations, such as Antonio's tree diagram of the sample space, it not only highlights their sophisticated thinking but also might give English language learners, or others who might struggle with academic language, greater access to the mathematical content. In fact, small-group settings furnish important settings for English language learners to develop their facility with academic language (Gibbons 2003).

Summary

In this chapter, I outlined the motivation for finding new ways of organizing the learning environment of the classroom by comparing traditional instruction to the ideal implementation of collaborative classrooms. I held up an *ideal* collaborative classroom, because many attempts to incorporate collaborative learning often fall short of this. In this book, I outline a set of concepts and tools that can help teachers adjust their instruction to consider the whole learning environment of the classroom and to move their collaborative learning toward this ideal (see table 1.1).

In chapter 2, I will discuss more fully equitable classroom learning environments and how collaborative learning can contribute—or sometimes work against—student participation. In the chapters that follow, I introduce some concepts, strategies, and tools for supporting successful group work in the mathematics classroom.

The goal of this book is to support teachers in developing tools for effective group work in their secondary mathematics classrooms. Effective group work can leverage the learning potential of student-to-student interaction. It can also address some problems in typical mathematics instruction by supplying a framework for teachers to create engaging learning environments. Like all complex tools, group work needs to be used carefully to be effective. Thus, I will outline ways to choose tasks, help students adjust to new ways of approaching schooling, and address the kinds of status problems that can threaten the most earnest attempts at collaborative learning.

> **"One of the big shifts is moving away from being an 'I-teacher' to being a 'them-teacher.' Instead of asking yourself, 'What do *I* say now? What do *I* do next?' you ask, 'What do *they* need next? What are *they* doing?'"**
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> ***—Laura Evans, Complex Instruction Educator, Mathematics Teacher, and Coach***

Table 1.1. Shifts in major components of the teaching system moving from a traditional to a collaborative learning environment

Component	Learning Environment	
	Traditional	**Collaborative**
Teacher role	• Effective presentation of ideas • Attending to correct answers	• Designing effective learning environments • Instruction centered on student thinking
Mathematics	• Ready-made: nothing to figure out, just to receive • Hierarchical view: sequential topics organized so that one cannot progress without mastering prerequisites	• In the making: students can make sense of mathematics • Connected view: a network of ideas whose connections students can explore through different forms of mathematical thinking
How to be smart in mathematics	• Quick, accurate recall and calculation • Complete worksheets with many problems of increasing difficulty	• Create sophisticated visual and symbolic representations of mathematical thinking • With peers, analyze fewer problems that require a greater depth of thinking
Pace of mathematics classes	• Mathematics problems take a short time to solve	• Mathematics problems must be considered in depth to be understood
Student role	• Students are passive recipients of knowledge • Individual understandings and identities do not influence the classroom • Work completion most highly valued form of participation	• Students' confusion and disagreements are expected and become the basis for instructional dialogue • Students have the potential to find a place for their broader identities • Careful thinking most highly valued form of participation
Participation demands	• Participation is optional • Wait for the "smart" student to answer questions • Status strongly influences participation	• Participation is expected of every student, regardless of prior achievement or temperament • Increase participation by fostering equal-status interactions
Assessment tools	• Contribute to comparison and competition among students • Make definitive judgments about achievement and smartness	• Focus on individuals' learning • Encourage persistence by offering opportunities to revise
Multiyear learning	• Prioritizes teacher autonomy • Little to no coordination across mathematics classrooms	• Prioritizes student learning • Careful sequencing of mathematics and student skills to assist in the development of both