

Counting and Cardinality

Counting and Cardinality

Domain Overview

KINDERGARTEN

Students enter kindergarten with a broad range of experiences with numbers. Some will be able to count by rote from 1 to 100 (or a subset of that range). Others may have limited experience with counting to 10. Keep in mind that the content standards identify what students should know and be able to do by the end of kindergarten.

Therefore, you will need to scaffold individual standards to meet the needs of students. For example, it is likely that you will begin the school year focusing on rote counting (sequencing number names) to 20 and at the same time work only on counting physical objects to 5. By the end of the year, students should be able to successfully complete all of these standards.

SUGGESTED MATERIALS FOR THIS DOMAIN

K

✓	Objects for counting such as beans, linking cubes, counter chips, coins
✓	Five frames (Reproducible 1)
✓	Ten frames (Reproducible 2)
✓	Double ten frames (Reproducible 3)
✓	Hundreds chart (Reproducible 4)
✓	Dot cards (Reproducible 5)
✓	Numeral cards (Reproducible 6)

KEY VOCABULARY

Vocabulary should be explored with kindergarten students using pictures and visual representations.

K

✓	add to combine or join together <i>related words: add, and, plus, join, put together, (+)</i>
✓	compare to look for similarities or differences among numbers or their size
✓	count to say numbers in order; to assign a value to a group of items based on one-to-one correspondence
✓	difference the amount by which one number is greater or less than another number. The difference can be found by subtracting, comparing, or finding a missing addend.
✓	equal (=) same as in value or size
✓	fewer than less than
✓	five frame a graphic representation that is useful to help students to count, see number relationships, and learn basic facts (Reproducible 1)
✓	greater than more than
✓	hundreds chart a 10-by-10 grid with the counting numbers from 1 to 100 listed; used to develop and demonstrate patterns and strategies for counting, addition, subtraction, and place value
✓	* numeral a symbol that represents a number; 3 is the numeral that represents a count of 3 objects
✓	number a count or measurement
✓	subtract to take one number away from another; to find the difference between two numbers <i>related words: subtract, minus, take from, take apart (–)</i>
✓	ten frame a graphic representation that is useful to help students to count, see number relationships, and learn basic facts (Reproducible 2)
✓	total (sum) the result when two or more numbers are added together

*Students are not responsible for these vocabulary words; however, they should understand the mathematical concept.

Counting and Cardinality

K.CC.A*

Cluster A

Know number names and the count sequence.

STANDARD 1 **K.CC.A.1:** Count to 100 by ones and by tens.

STANDARD 2 **K.CC.A.2:** Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

STANDARD 3 **K.CC.A.3:** Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

*Major cluster

Counting and Cardinality K.CC.A

Cluster A: Know number names and the count sequence.

Kindergarten Overview

This cluster is about rote counting. Students do not need to have an understanding of what the numbers mean or of place value within this cluster. They will work with those concepts in the Order and Algebraic Thinking (OA) and Number and Operations in Base Ten (NBT) domains. Once students can count, they begin to connect number words with counting quantities. This should be accomplished in small increments.

Standards for Mathematical Practice

SFMP 6. Attend to precision

Students are learning numbers by rote counting. Vocabulary development, especially for students who have not had previous experience, includes counting as well as learning number names as they count by ones and by tens.

Related Content Standards

1.NBT.A.1 2.NBT.A.2

Notes

STANDARD 1 (K.CC.A.1)*Count to 100 by ones and by tens.*

Students work over the course of the year to count from 1 to 100. They begin counting by ones, and as the range of numbers grows, they also count by tens. Although this standard includes recognizing numerals, it does not include reading or writing numerals.

What the TEACHER does:

- Provide many opportunities for students to count. Begin with a small range of numbers, such as 1 to 10, and increase the range depending on student needs. As students begin to rote count fluently, introduce the numeral representations for each number name. Give students a variety of opportunities to match the numeral with the number name.
- Use a variety of nursery rhymes, children's books, and songs to help students associate number sequence with situations that are already familiar to them.
- Once students can count to a given number, use a section of the hundreds chart to help them recognize the numerals that represent these numbers. Use matching games and activities to help students connect the number name with the numeral. The goal for this standard is for kindergarten students to count to 100 by the end of the year. This should develop over time depending on the readiness of students.
- As students become fluent in rote counting by ones, introduce counting by tens (10, 20, 30). Using a hundreds chart, ask them to identify any visual patterns they see in the numerals, such as they all end in 0 or the first digit goes in order.

What the STUDENTS do:

- Students begin by sequentially counting by ones. They start with a limited range of numbers and increase the range until they can count to 100.
- They begin to match the number name with the numeral. They play games and complete activities using numeral cards and portions of the hundreds chart to connect numerals with number names.
- Students count to 100 by tens. As they learn the numerals for the digits 0 to 9, they begin to recognize patterns in the written numerals.
- Motion with fingers on both hands to count by tens.
- Use dimes to count by tens; this is a good way to introduce coins to students when they are ready to understand that one dime can represent ten cents.

Addressing Student Misconceptions and Common Errors

Students who confuse the sequence of numbers (ex. 1, 4, 7, 3, 9, 2), skip numbers (ex. 1, 2, 3, 5, 6, 7, 9 . . .), or repeat numbers (1, 2, 3, 4, 2, 3, 4) need more experience counting within a smaller range of numbers. Students should be fluent within a range before increasing the range.

Words for the teen numbers may be confusing since they do not follow the pattern of other decade numbers (ex. fourteen vs. twenty-four). Provide more practice with reciting teen numbers and connecting the number name with the written numeral.

Focus on oral patterns such as the sequence of the ones place digits in the twenties is the same as the sequence of the ones place digits in the thirties.

20, 21, 22, 23, 24, 25, 26, 27, 28, 29

30, 31, 32, 33, 34, 35, 36, 37, 38, 39

Notes

STANDARD 2 (K.CC.A.2)

Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

Once students are fluent at counting beginning with 1, they begin to work on counting forward from a number other than 1 within a given range. This is a prerequisite skill for counting on as students begin to work with addition. This standard does not include writing numerals, which will follow at a later time.

What the TEACHER does:

- Give the students a start number greater than 1 and ask the students to count on to that number within a range of numbers. For example, give a starting number of 5 and ask the students to count to 10. As in the previous standard, this is rote counting using number names. Understanding *number* as a count of objects is not necessary within this standard.
- While students are increasing the range of numbers to which they are counting, they are beginning to work on the standards within K.OA and K.NBT domains using lesser numbers.

What the STUDENTS do:

- Students begin to rote count from a number other than 1 (counting on) without having to go back and start at 1.
- Given the number 3, the student should be able to continue the count (4, 5, 6) without starting from 1. Complete similar examples within a given range of numbers. Although recognition of written numerals may help students to count on, it is not essential. The expectation for this standard is rote counting.

Addressing Student Misconceptions and Common Errors

Students who struggle with developing this standard, particularly with numbers greater than 10, should master counting within a sequence before counting forward from a number in that sequence. For example, students should be able to rote count to 20 before they are expected to count on from 8. Begin with smaller numbers and progress to greater numbers. Limit how far you want students to count and then increase the range.

Notes

STANDARD 3 (K.CC.A.3)

Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

Students recognize and write the numerals 0 to 20. Begin with the single-digit numerals and represent the number of items in a set with the appropriate numeral. Additionally, given a numeral, students can represent that number of items in a set.

What the TEACHER does:

- Provide students with a variety of opportunities to recognize written numerals from 0 to 9. This can be accomplished with numeral cards, a deck of cards with both the numeral and a picture of the count of that many objects (Reproducible 6).
- Begin with a small range of numbers (0, 1, 2, 3). As students recognize those numerals with ease, add more cards to the range.



- Show students a collection of items and have them match the appropriate numeral card with the set.
- As students begin to recognize numerals, provide practice writing the numerals, using various modes including writing the numerals in the air as you model, writing numerals on large chart paper with a paint brush, writing in sand or shaving cream, or tracing the numerals on paper. Student readiness will vary with the development of eye-hand coordination and small motor skills.
- Teach this standard together with K.CC.B.2 so that as students count objects, they match the number of objects in a set with the numeral. Focus on activities that connect the concept of a specific quantity of objects and how they are represented by a number. The numeral is the written representation for that number.
- Start with a small range of numbers and continue with counting to 20, using a variety of objects to count including five frames, ten frames, and double ten frames. Include opportunities to count throughout the day such as counting steps, counting the number of students buying lunch, or counting the number of students who walk to school. Ask students to recognize and write numerals for the numbers they use throughout the day.

What the STUDENTS do:

- Recognize numerals from 1 to 20.
- Match a collection of items with the appropriate numeral.
- Over time, write numerals from 1 to 20. Readiness to write the numerals will vary with the development of eye-hand coordination and small motor skills.
- Represent sets of objects with numerals after they have had experience with CC. Cluster B: Count to tell the number of objects.
- Use a variety of modalities to trace numerals in the air, in sand, on paper.

Addressing Student Misconceptions and Common Errors

It is common for kindergarten students to invert or reverse numerals. With additional experience, most children will self-correct. Give children opportunities to have a variety of kinesthetic experiences to form numerals (write numerals in sand, rice, etc.) before they use paper and pencil.

Counting and Cardinality

K.CC.B*

Cluster B

Count to tell the number of objects.

STANDARD 4

K.CC.B.4: Understand the relationship between numbers and quantities; connect counting to cardinality.

- When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
- Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
- Understand that each successive number name refers to a quantity that is one larger.

STANDARD 5

K.CC.B.5: Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1 to 20, count out that many objects.

*Major cluster

Counting and Cardinality K.CC.B

Cluster B: Count to tell the number of objects.

Kindergarten Overview

Students move from rote counting to finding the number of objects in a set. Cardinality refers to the actual count or number of items in a set. This cluster connects to the previous cluster. As students show proficiency rote counting within a range of numbers, for example, 1 to 10, they can begin to find the number of objects in a set within that range. While working within clusters A and B, it is important for students to connect the physical objects (3 counters) with the oral number word (*three*) and the numeral (3). Students should begin with counting physical objects, progress to pictures, and then connect the physical representations to the numeral.

Standards for Mathematical Practice

SFMP 4. Model with mathematics.

SFMP 6. Attend to precision.

Students continue to develop counting skills extending rote counting to actually counting concrete objects. They begin to develop the idea of one-to-one correspondence as they realize one number name goes with each item. Counting sequentially, starting with 1 and later counting by adding one to the number of items in a collection, helps students to know number names and the correct order of numbers as they match each count with one item.

Related Content Standards

K.CC.C.6 K.CC.C.7 1.NBT.A.1 2.NBT.A.3

Notes

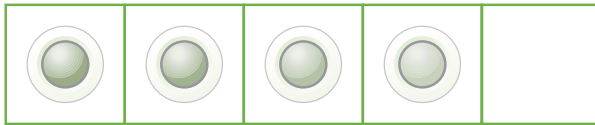
STANDARD 4 (K.CC.B.4)

Understand the relationship between numbers and quantities; connect counting to cardinality.

- When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.*
- Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.*
- Understand that each successive number name refers to a quantity that is one larger.*

What the TEACHER does:

- Provide opportunities for students to count using a variety of objects such as buttons, counters, shells, coins, and dot cards.
- Ask student to count objects, beginning with a smaller range of items and increasing as students count accurately.
- After students have counted items placed in organized arrangements (straight line, circle), arrange objects randomly.
- Use five frames to model linear representations of object and to help students begin to see patterns that make 5 (Reproducible 1).

Show representations of 4 items

- Ask, “How many are there?” to reinforce that the last number name tells the count of items rather than the counting process itself.
- Use formative assessment protocols to be sure that students understand the last number said tells the number of items in a collection.
- Have students count 5 cubes. Add one more cube to the set and ask how many now? (6). Progress to a similar setup but do not add the cube . . . ask “How many will there be if I add one more cube?” This helps students to visualize the process.
- After many experiences, ask, “If you had 5 cubes and added one more, how many would there be?” without using materials. Begin with numbers 1–5 and then increase the range of numbers to 10. It will take time for students to develop this conceptual understanding, so this standard should be developed over several months.
- As students are ready, extend this work to 10 using ten frames (Reproducible 2).

What the STUDENTS do:

- Say the number name in consecutive order as they point to each object. Some students may find it helpful to move the objects as they count.
- Start by counting objects that are in a straight line and then move to organized representations (ex. arrays, circles) and finally randomly arranged objects.
- Indicate by counting that the last number said tells the number of items.
- Count on to the original number of items in a set, first by adding one item and later by mentally counting up one.

Addressing Student Misconceptions and Common Errors

Watch for students who find it confusing to say one number name with one object as they count (one-to-one correspondence). Begin with a smaller number of objects and model saying the number name as you physically move the object. Have students do the same.

Watch for students who double count an object. Physically moving the object and saying one number name for each object will help to reinforce one-to-one correspondence; that is, one object goes with one number name. Students may see 5 items spread out as different from 5 items close together. Students should physically move the objects matching one item from one set with one item from the other set to understand that the count of 5 remains the same no matter how the objects are organized.

STANDARD 5 (K.CC.B.5)

Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

This standard builds on the previous standards in this cluster. Students continue to count items in a set, using physical and pictorial representations. In addition, given a number, students count out that quantity of items. Numeral recognition is developed throughout this cluster, so students should also recognize a written numeral and count a number of counters given the number orally or given the written numeral. Provide a variety of concrete experiences before students draw pictures.

What the TEACHER does:

- Provide students with a bag, box, or bucket of objects and ask them to count out a certain number of objects. For example, say, “Show me 5 buttons.” Begin with numbers to 5 and extend the range to 10, 15, and 20 as students show skill counting out objects.
- Ask students to match numerals with the number of items in the set they have counted.
- Give students a numeral card and ask them to read the number. Students then count out that many items to represent the number.
- Give a drawing of countable items, such as flowers, teddy bears, or cars, then ask students to circle a number of items and write the numeral.

What the STUDENTS do:

- Count out a number of items using a variety of concrete objects.
- Match a numeral card with the number of items in a set.
- Given a written numeral, count that number of items from a collection of items.
- Given a drawing of items such as flowers, teddy bears, or cars, circle a number of items and write the numeral.
- Draw a given number of items.

Addressing Student Misconceptions and Common Errors

Some students may be able to match a quantity with a number (or numeral) but cannot produce that number of objects when given materials or asked to draw a picture. Looking for a specific quantity when given a choice of collections has a lower level cognitive demand (is easier) than having to produce a set of objects given a number. This standard will take time to develop.

How many peanuts?



Versus

Show me 5 peanuts

Notes

Counting and Cardinality

K.CC.C*

Cluster C

Compare numbers.

STANDARD 6

K.CC.C.6: Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.¹

¹Include groups with up to 10 objects.

STANDARD 7

K.CC.C.7: Compare two numbers between 1 and 10 presented as written numerals.

*Major cluster

Counting and Cardinality K.CC.C

Cluster C: Compare numbers. Kindergarten Overview

Students build on the work of the previous clusters to develop strategies to compare two concrete quantities and later connect that idea to comparing two number words and two numerals. The language of more than (greater than) and less than (fewer than) can extend to “how many more?” . . . and “how many less?” which begins the concept of additive thinking (one more than, two more than, one less than, etc.). Developing this language and giving students a variety of experiences will lay a solid foundation for future work with addition and subtraction.

Standards for Mathematical Practice

SFMP 4. Model with mathematics.

SFMP 6. Attend to precision.

This will be students’ first experience with comparing quantities. Precision with language is critical in this cluster. Scaffolding experiences that start by using concrete materials with obvious comparisons and honing in on quantities that get closer in size will provide students with the time needed to understand the concepts.

Related Content Standards

K.MD.A.2 1.NBT.B.3 2.NBT.A.4

Notes

STANDARD 6 (K.CC.C.6)

Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.¹

¹Include groups with up to 10 objects.

Students begin to work with two sets of objects to compare the number in each set. Note the scaffolding of tasks in the descriptions that follow.

What the TEACHER does:

- Provide activities for students to compare quantities to determine which has more and which has less. Make the two groups very obvious so counting isn't needed to determine which has more. The focus is on developing appropriate vocabulary including greater than and less than. For example, show students a large bag of popcorn and a small bag of popcorn and develop the concept and language of more and less. "The big bag has *more than* the little bag." "The little bag has *less than* the big bag."
- Show two bags that are the same size and begin to work with the idea of equal using the language of *same as*.
- Once comparison vocabulary has been developed, students compare the number of items in two sets of objects and determine which has more and which has less. It is helpful to compare two different types of items (blue chips and red chips, circles and triangles), so there is no confusion when students begin to compare. There is a hierarchy of strategies involved when comparing. Students should use the strategy that makes the most sense to them.



- *Matching:* Line up the items in each set using one-to-one correspondence. Which set has more? (triangles). Which set has less? (circles). Asking questions like "how do you know?" starts to develop reasoning and mathematical arguments called for in the Standards for Mathematical Practice.



- *Observation:* I see there are more triangles than circles. When students use this strategy, it is important for them to explain how they "see" more triangles than circles. While an acceptable strategy, it is often difficult for them to explain how they know.
- *Take away or fair share:* Each time I take a circle, you take a triangle. When all of the circles are gone, there will still be some triangles. Follow up with questions such as "Are there more triangles or circles? How do you know? Are there fewer triangles or circles? How do you know? Which shape has more? Which shape has fewer?"
- *Compare counts:* Students count the number in each group and compare the counts. For example, there are 3 circles and 5 triangles, so there are fewer circles than triangles because 3 is less than 5.

- Transition to situations in which students are comparing the number of like items. For example, compare 5 peanuts with 7 peanuts.
- Once students recognize sets that are greater than or less than, give them situations in which they identify how much more than or how much less than one set is compared to another set (with differences of 1, 2, and 3).
- Provide situations where students identify equivalent sets. This may be their first experience with equality. Using the term *same as* may be more meaningful than *equal to* in describing equivalent sets.

What the STUDENTS do:

- Develop vocabulary of greater than (more than) and less than (fewer than) to compare the number of items in two sets.
- Use various strategies that make sense to compare items in two sets, including matching, observation, take away/fair share, counting.
- Explain their reasoning to show that one set has more or less than another.
- Identify how many more or how many fewer items one set has than another.
- Recognize two sets that have the same number of items using the description *same as*.

Addressing Student Misconceptions and Common Errors

Students who have trouble with the vocabulary of comparison need more opportunities to compare obvious amounts and practice the different ways to describe the comparison. For example, there are more teddy bear counters than chips. There are fewer chips than teddy bear counters.

The strategies above are listed in order of development. Students who are struggling to compare the size of two sets of items should line them up with one-to-one correspondence. If they are unable to keep the objects lined up, provide a sheet of one-inch graph paper and keep the items small enough so one item can fit in each square. Ask questions such as, Which row has more? How do you know? Which row has fewer (less)? How do you know?

Keep the number of objects in each set within the range of student success and then build to using greater numbers of items. Continue giving students opportunities to describe their thinking and to use comparison vocabulary.

Notes

STANDARD 7 (K.CC.C.7)

Compare two numbers between 1 and 10 presented as written numerals.

This is the culminating standard of the Counting and Cardinality domain because it requires students to synthesize all of the previous standards. Students must be able to count items in a group, recognize number words and numeral representations, compare two groups of objects to identify which is greater or less, and associate numbers with each set to begin understanding the abstract nature of comparing numbers given only the numerals.

What the TEACHER does:

- Once students show proficiency with comparing sets of objects (up to 10), repeat the same activities with different materials or pictorial representations. Have students place the numeral card for each set next to the items.
- Ask students to describe the comparison. There are more triangles than circles, so 5 is more than 3. As with the previous standards in this domain, students should have a variety of experiences with concrete and pictorial representations and then make explicit connections to the number names and numerals. Include situations so all of the comparison vocabulary is developed.

3 ○○○
5 ▼▼▼▼▼

What the STUDENTS do:

- Given two sets of concrete materials, students label each set with the appropriate numeral.
- Students compare the number of items in each set using comparison vocabulary and then connect the comparison of physical objects to the number names in describing the comparison.

Addressing Student Misconceptions and Common Errors

Since this standard requires facility with all of the previous standards in this cluster, students who cannot accurately compare the number of physical objects are likely to struggle with comparing the numbers written as numerals. These students need additional practice comparing sets of objects and describing their reasoning before working with the numerals. Modeling the transition between the vocabulary of comparing the count of physical objects and using the same vocabulary with the number of items will help students to practice the vocabulary of greater than (more than), less than (fewer than), and same as.

Notes

Sample PLANNING PAGE

Standard: K.CC.B.5. Count to answer “how many” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1 to 20, count out that many objects.

Mathematical Practice or Process Standards:

SFMP 4. Model with mathematics. Student uses a variety of concrete objects to show a given count.

SFMP 6. Attend to precision. Students will use correct number names and written numerals to accurately sequence numbers as they count out the number of items in the set.

Goal:

As students show proficiency with rote counting, for example, from 1 to 10, they can begin to find the number of objects in a set within that range (cardinality). It is important for students to connect the physical objects, with the number word and the numeral. Students should begin with counting physical objects, progress to pictures, and then connect the numeral to the physical representations.

Planning:

Materials: Counters including chips, buttons, shells, and the like; five and ten frames; numeral cards.

Sample Activity:

- Model counting the number of objects in a set of 3 chips. Count orally as you move the items.
- Provide each table with a collection of items to count (less than 6 to start). Let students take turns counting items.
- As students show accuracy with counting, increase the number of objects.
- Have students match the numeral card with the correct count.
- Students who can write numerals can label each collection by writing the numeral.

Questions/Prompts:

Show me 10 counters.

How can you prove there are 10?

Match the numeral for 10 with the items you counted.

If a student double counts an item, prompt him to say one number with each item he moves.

Write the number to show how many you have counted.

Differentiating Instruction:

Struggling Students: Begin with numbers less than 6. Have students move objects as they count. Be sure they are moving one item with each number. Next, have students point to each item as they count. Do not have students write numerals until they can count accurately.

Extension: Give the students the number and ask them to show that many items and, later, draw that many items. Let students model numbers with ten frames and double ten frames explaining different strategies to know the number. For example, I know there are 15 because 5 spots are empty.

PLANNING PAGE

Standard:

Mathematical Practice or Process Standards:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

PLANNING PAGE

Standard:

Mathematical Practice or Process Standards:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

PLANNING PAGE

Standard:

Mathematical Practice or Process Standards:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

Reflection Questions: Counting and Cardinality

The order of these standards is not meant to be linear—in other words, you do not teach one standard and then move to the next. Rather, they connect within a cluster, across clusters, and across domains.

1. Look at the standards in Cluster A: Know number names and the count sequence. How are these standards related? Discuss how to organize these standards to help meet the individual needs of your students.
2. Cluster B: Counting to tell the number of objects addresses students' ability to count objects. How does this cluster build from Cluster A? How can you assess student needs to determine the range of numbers to work with, beginning with the standards in Cluster A and then moving to Cluster B? For example, some students may need to work on counting to 5; others may be ready to count to 10 or even 20.
3. The standards in Cluster C talk about early comparison of two quantities. What are some grade-level-appropriate activities that kindergarten students can do to begin to understand the meaning of greater than (more than), less than (fewer than), and same as (equal)?