Preface

With the emergence of mathematics as a nearly ubiquitous tool for the physical and life sciences, computer science, engineering, medicine, business, finance, the social sciences, and many other aspects of contemporary life, the teaching and learning of mathematical applications and modeling in school and university mathematics have been given heightened attention (Kaiser 2013; National Research Council [NRC] 2013). In the foreword to this volume, Henry Pollak reminds us that mathematics is taught in elementary, middle, and high school because it is useful. Yet, much too often mathematics teachers report students' perception of school mathematics is that it is not relevant or useful, with refrains such as "Where are we going to ever use this (topic)?" or "What is this (topic) good for?" The seminal symposium held at the University of Utrecht in 1968, "Why [and How] to Teach Mathematics So as to Be Useful" provided insights into principles that promised to help improve mathematics education in schools and that could potentially ameliorate student queries such as those above. In his opening address, Hans Freudenthal, a Dutch mathematician, offered three guiding principles in response to the symposium question: focus on mathematics as a human activity, mathematization from contexts, and mathematics for all students (Freudenthal 1968).

The symposium and its published proceedings spawned considerable curriculum work, instructional innovation, and research focusing on the role and nature of applications and especially mathematical modeling in high school and tertiary mathematics in Europe. However, in spite of decades of work by Henry Pollak (a plenary speaker at the Utrecht symposium), the response in the U.S. was more muted and localized. In fact, prior to the publication of the Common Core State Standards for Mathematics (CCSSM) (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA Center and CCSSO] 2010), the curricular context of schooling in the United States provided limited opportunity to make mathematical modeling an explicit topic in the K–12 mathematics curriculum (Zbiek and Conner 2006).

The focus on mathematical modeling in CCSSM is of particular interest because it had been given only minimal attention in past standards documents (e.g., National Council of Teachers of Mathematics [NCTM] 1989, 2000). Now, however, mathematical modeling holds a privileged place in CCSSM in that it is both a Standard for Mathematical Practice and, at the high school level, a conceptual category.

Focusing school mathematics on modeling as described in CCSSM offers both challenges and opportunities. For example, one major challenge to the implementation of modeling is the "conceptual fuzziness" (Lesh and Fennewald 2013) surrounding what counts as a modeling activity. Researchers have been grappling for many years with the distinctions between what constitutes a problem-solving task versus a modeling task (Zawojewski 2013). Another challenge to implementation is a lack of understanding related to modeling mathematics versus mathematical modeling. In CCSSM, "modeling means using mathematics or statistics to describe (i.e., model) a real world situation and deduce additional information about the situation by mathematical or statistical computation and analysis" (Common Core Standards Writing Team 2013, p. 5). Here *modeling* and *model* are used as verbs. However, the word *model* can also be used as a noun. In many instances, CCSSM and progressions documents describe models as physical representations (e.g., multiplication arrays, number lines, geoboards, area models) that can be used to model mathematics. In other instances (e.g., in the Modeling conceptual category), mathematical modeling refers to the activity described above that links classroom mathematics and statistics to situations in everyday life that are not inherently mathematical. Thus, mathematical modeling and modeling mathematics are quite different. Decoupling these two ideas will be but one of many fundamental challenges to implementing this aspect of CCSSM, hence the title of this volume and the focus of the lead chapter.

Zbiek and Conner (2006) argued that engaging students in mathematical modeling activities is appealing for several reasons, including (*a*) to prepare students to work professionally with mathematical modeling, (*b*) to motivate students to study mathematics by showing them the real-world applicability of mathematical ideas, and (*c*) to provide students with opportunities to integrate mathematics with other areas of the curriculum. Our goal in creating this volume was to provide support for the reader in engaging students at all levels in the wonderful endeavor of mathematical modeling.

Focus, Development, and Organization of the Volume

This 2016 volume of *Annual Perspectives in Mathematics Education* (*APME*) addresses these and other challenges and affordances in elevating modeling to a more central position in mathematics education across the K–16 spectrum. The individual chapters reflect a variety of research- and practice-based perspectives on teaching, learning, and assessing modeling from a range of U.S. and international authors. In our Call for Chapter Manuscripts, we asked potential authors to examine and illustrate the benefits and challenges of implementing modeling through one or more of the lenses: modeling constructs; modeling competencies and sub-competencies; task and curriculum design, especially in a digital world; instructional practices that have proven effective, including elaborating equitable and culturally relevant pedagogies; student learning; assessment of modeling experiences; and support of teachers' learning.

We received 110 chapter proposals, each of which was blind-reviewed by at least two members of our Editorial Panel. Authors of 85 of these proposals were invited to prepare full chapter manuscripts that were subsequently again blind-reviewed. Subject to the criteria in the Guidelines for Preparing Manuscripts, a desire for balance, and imposed space limitations, 22 chapters were accepted with minor revisions based on reviewers' and editors' recommendations, and four were provisionally accepted subject to more extensive revisions and resubmission. The latter revised manuscripts were blind-reviewed by three panel members. Ultimately, 25 chapters were selected and organized in seven sections as listed below. The focus of part II is on models as representations of mathematical and statistical ideas. The remaining sections are devoted to mathematical modeling.

Part I	**Understanding Models and Modeling**
Part II	**Using Models to Represent Mathematics**
Part III	**Teaching and Learning about Mathematical Modeling**
Part IV	**Mathematical Modeling as a Vehicle for STEM Learning**
Part V	**Designing Modeling-Oriented Tasks and Curricula**
Part VI	**Assessing Mathematical Modeling**
Part VII	**Supporting Teachers' Learning about Mathematical Modeling**

Each section includes an introduction that provides a context for, and overview of, each chapter in that section. Nine of the chapters in this volume include interactive components that readers can access through the web addresses within the chapters or by viewing the online resources at www.nctm.org/more4u (using the access code that appears on the title page of this book).

In Appreciation

The publication of this volume entailed the contributions of many people. The editors are grateful for the long hours, considered attention, and commitment to quality from all those involved. First, we would like to express our appreciation to both those who submitted chapter proposals and the chapter authors for their evidenced-based and thought-provoking manuscripts as well as their responsiveness to requests for revision(s) under tight timelines.

The Editorial Panel—Michelle Cirillo, Mathew D. Felton-Koestler, John A. Pelesko, Elizabeth Difanis Phillips, Laurie Rubel, Daniel Teague, and Judith S. Zawojewski—were instrumental in shaping this volume. They helped formulate the Call for Chapter Manuscripts; they reviewed chapter proposals and subsequently chapter manuscripts multiple times while providing direction, detailed comments, and suggested edits; and they performed these tasks in a very timely manner. They also contributed to the volume by authoring section introductions or co-authoring significant chapters (that also underwent the blind-review process). Their collective work contributed immensely to the quality of this volume. Our appreciation is gratefully acknowledged.

Additionally, the volume editor would like to acknowledge the superb assistance of Ginger Rohwer and Hope Smith at Western Michigan University. Rohwer, a mathematics education graduate student, created and administered the *APME* 2016 website for managing the electronic flow of manuscripts, reviews, rating summaries, and Panel recommendations. Smith, administrative assistant in the Center for the Study of Mathematics Curriculum, was invaluable in following up my editing with additional copyediting, preparing edited chapters for NCTM, and reviewing page proofs for style and formatting.

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