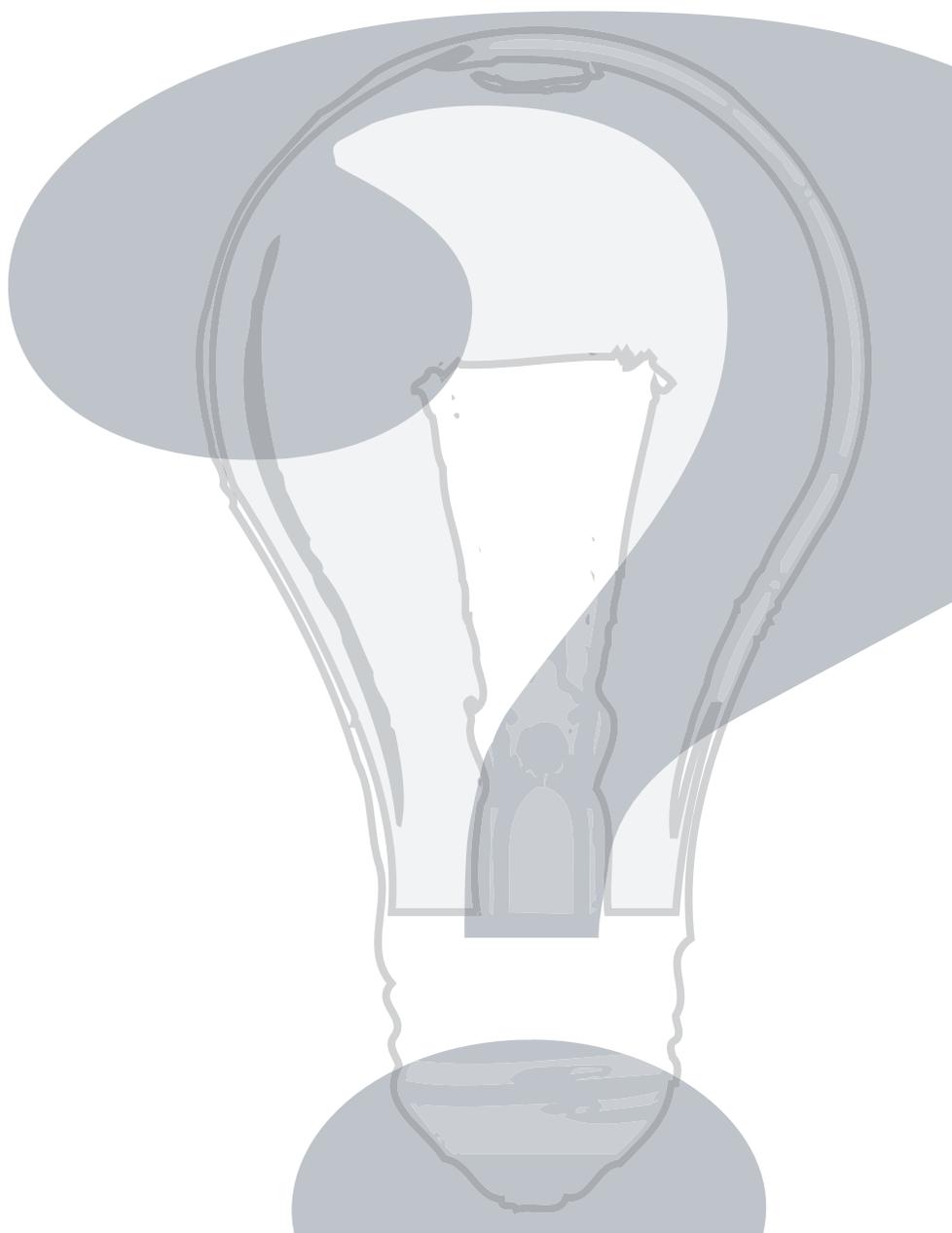


# Teaching Mathematics through Problem Solving

Grades 6 – 12



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