

The Tree of Mathematics

Mathematics occupies a peculiar position in cultural life today. “Everybody knows” that it is one of the most basic, and also ancient, types of knowledge; yet it is not part of normal cultural discourse, and few people know much about its historical development, or even that it has a history.

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Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences (2003, vol. 1, p. 3)

Elementary mathematics is simple but challenging. It is beautiful but baffling to many. Although it is supposed to spark creativity, it tends to instead intimidate with its multitude of rules to memorize. It causes anxiety when it should cause excitement. And although it is deeply rooted in our collective history, it at times feels detached from the real world. Does the history of mathematics matter for a mathematics teacher? Does knowledge of the history behind mathematical concepts facilitate better teaching? Is it really true “that no subject loses more than mathematics by any attempt to dissociate it from its history” (Whitrow 1932, p. 225)?

1.1. HOW OLD IS MATHEMATICS?

Mathematics is a unique school subject. It leads students through human history by way of mathematical concepts and ideas. Around the world, mathematics curricula roughly follow the historical development of central mathematical concepts. Think, for example, of the number systems: natural numbers, fractions, decimals, integers, irrational numbers, and finally, complex numbers. This is the order in which we introduce students to the number systems, and in a broad sense it slides along the time line of the discovery and development of different numbers in mathematics (Ifrah and Bellos 2000).

“The entire body of mathematical knowledge is very much like a tree” (Kennedy 1995, p. 460). The branches of the Tree of Mathematics represent the mathematical concepts, and no matter how abstract or complex a concept is, its origins can be traced back to the roots (figure 1). Every single mathematical formula or rule can be derived from earlier mathematical ideas. Every single mathematical concept makes sense in the light of its historical development.

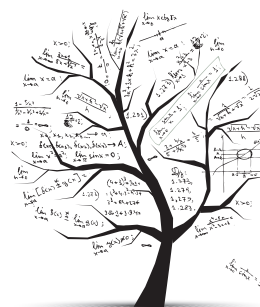


Fig. 1. The roots of the Tree of Mathematics are grounded in counting and measuring.

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The roots of the Tree of Mathematics are grounded in two basic human activities: (1) counting and (2) measuring. Every mathematical concept, no matter how complicated, can be traced back to basic mathematical concepts such as addition of natural numbers or a straight line. Human needs for counting and measuring gave birth to the oldest mathematical subjects, arithmetic and geometry.

Curious Facts The constantly growing, branching, and blooming Tree of Mathematics is very old. The roots of arithmetic, for instance, go unimaginably deep in the soil of time. One of the earliest, if not *the* earliest, mathematical artifacts in existence is the Ishango bone (see figure 2). It was unearthed in 1950 in the then-Belgian colony of the Congo (now the Democratic Republic of Congo). The bone is a fibula of a large mammal, probably a baboon or large cat, and is approximately 25,000 years old. It is 10 cm long and bears an articulated, organized series of notches (Brooks et al. 1995). It looks like a tally stick, doesn't it?



Fig. 2. The Ishango bone is on exhibition at the Royal Belgian Institute of Natural Sciences.

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Although its original purpose remains a subject of debate, the grouping of etchings on the bone suggests more advanced mathematics than just tallying (Marshack 1972). For instance, the number of notches in row a and the number of notches in row b add up to the same number, 60 (figure 3).



Fig. 3. The diagram of notches on the Ishango bone suggest that it was used for more than just tallying.

Williams, Scott W. 2008. *The Notches on the Ishango Bone* <http://www.math.buffalo.edu/mad/Ancient-Africa/ishango_bone.jpg> (accessed August 31, 2011)

The first known number system was developed in Babylon (modern-day southern Iraq) about 4,000 to 5,000 years ago. In some respects, this system is considered to be superior to our present one—and far superior to later Greek and Roman systems (figure 4).



Fig. 4. The Plimpton 322 clay tablet is in the G.A. Plimpton Collection at Columbia University. Its main content is a table of numbers in sexagesimal notation.

<http://commons.wikimedia.org/wiki/File:Plimpton_322.jpg>, via Wikimedia Commons

Measuring land around the fertile Nile River in Egypt gave a boost to the development of another mathematical branch called geometry. The earliest known standard units used by people to measure length are the 6,000-year-old Egyptian cubits. The common cubit was the length of the

forearm—from the elbow to the tip of the middle finger—and measured approximately 50 cm (figure 5; Whitrow 1932).



Fig. 5. Ancient Egyptian cubits are the earliest known standard units used by people to measure length.

Coudée-turin.jpg: Bakhaderivative work: JMCC1, CC BY-SA 3.0 <<https://creativecommons.org/licenses/by-sa/3.0/>>, via Wikimedia Commons

As humanity progressed, the Tree of Mathematics grew further from its humble roots. Practical challenges and the inner logic of mathematics provided constant nutrients for the tree. The growth of the Math Tree is not unlike the growth of a real tree.

In humans and other animals, growth can occur in most parts of the body. As we mature, our bones, skin, and muscles all increase in size. Trees do not grow like this. Trees grow by producing new cells in a very limited number of places. These places of cell division are called *meristems*. Meristems are zones of intense activity. They are where all new cells are formed and where they expand. (Virginia Tech Forestry Outreach Site 2020, p. 1)

The Tree of Mathematics has its own “meristems.” These are the crucial moments in mathematical history when new concepts were created and started developing. At these mathematical meristems, one can find the essence and the most important meaning of mathematical concepts and ideas. Starting from these points in the history of mathematics, one can follow a branching twig to master the concepts, expand on the ideas, and ultimately get to the meristem of a new mathematical concept.

1.2. SCHOOL MATH CURRICULUM AND THE TREE OF MATHEMATICS

In school mathematics, however, some connecting branches and twigs of the Math Tree are missing. New mathematical concepts pop up like rabbits from a magic hat. Visually, the Tree of Mathematics presented to students can be depicted as in figure 6. The disjointed branches and twigs make school mathematics look like a large collection of rules, formulas, and procedures. It looks more like a pile of twigs than a tree, and the deep meaning, inner logic, and accessible, natural source of understanding of mathematical concepts is somewhat obscured.

Let’s look at an example. Graph interpretation competence is essential to understanding today’s world. Despite years of schooling, though, many high school students do not have the necessary ability to interpret graphical information (Tairab and Khalfan 2004). Data representation appears in the North American curricula as early as second grade: “By the end of Grade 2 students will read (also collect and organize) primary data presented in concrete graphs, pictographs, line plots, simple bar graphs, and other graphic organizers” (OME 2005, p. 51).

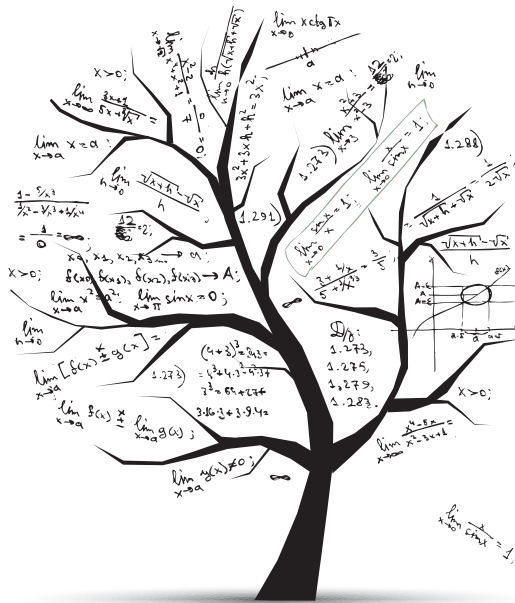


Fig. 6. This representation of the Tree of Mathematics is missing some twigs.

This expectation bundles together thousands of years of development of presenting data graphically. Some scientists argue that it is possible that Neanderthal humans were creating pictographs known as *petroglyphs* as far back as 700,000 BCE (EAE 2020). The first *bar graph* appeared as late as the 14th century, and *line graphs* are an even later invention, thought to be from the 17th century (Friendly 2008). Skipping the historical and logical steps of development of more abstract ways to represent data might be one of the reasons for the aforementioned students' difficulties.

Also, if the Tree of Mathematics is missing branches, students do not see the picture of mathematics as a “global” affair, as an enterprise in which many, if not all, cultures are active participants. The perception of mathematics as the result of the work of Europeans (and mostly Western Europeans for that matter) keeps permeating our mathematics teaching. We know that “how we see ourselves and others is shaped by the history we absorb” (Joseph 2011, p. 2). The subject of mathematics provides a unique opportunity to expose our diverse student body to a global, interconnected, and all-inclusive worldview. Let's look, for instance, at one of the basic mathematical concepts, the all-important, both in arithmetic and algebra, mathematical tool—fractions. Fractions have been around in mathematics and the everyday life of people in Asia, Africa, and the Americas for more than 4,000 years. The apparatus of fractions accepted as standard in the modern European mathematical tradition and the one that we primarily teach in school is just one of the branches of the “fraction tree.” Other methods, techniques, and approaches (see chapter 5) to fractions exist from different non-European cultures that might, for one thing, prove themselves more coherent and efficacious for our students. And for another, students who have been exposed to them would see mathematics as a work of the world where no culture is excluded, where all people are active participants. The same goes for every concept and area of school mathematics—be it negative numbers or variables, measurement, or arithmetic operations. The history of mathematics is a veritable portal through which culturally responsive pedagogy enters math classrooms.

A famous mathematician once said through his character, a young girl named Alice, that you must “begin at the beginning . . . and go on till you come to the end” (Carroll 1998, p. 182). That very much applies to the task of learning and teaching mathematics. Climbing the Tree of Mathematics is a great metaphor for the process of mastering mathematics. To get to your destination somewhere in the crown of the tree, you begin at the roots and follow the branches that lead to your desired twig. You cannot jump from branch to branch like a squirrel. If you hop over parts of the tree, how can you be sure that you will land on the right twig? And would you feel safe sitting on this twig if you were not certain that its connections to the tree were robust and sturdy?

In this book, the basis for our explorations of the basic middle school mathematical concepts will be the history of their development.

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