CHAPTER

7

Selecting and Designing Groupworthy Tasks

WE HAVE talked at length about strategies for making small groups places where children will all participate and learn mathematics. As important as these strategies are, children will not learn mathematics unless these groups have challenging, mathematically rich, groupworthy tasks to work on. In chapter 4, we looked carefully at one task, showing how the task's features support teachers' efforts to involve all children with the big ideas that form the foundation of third-grade math. We now turn our attention to finding and creating tasks that will support your efforts to move children toward a deeper understanding of the math they need to learn in your class.

You can get some help in finding good tasks in a number of sources that may be familiar to you. The National Council of Teachers of Mathematics (NCTM) *Standards* documents (1989, 1991, 1995, 2000) have mathematically meaty tasks that you can adapt easily to complex instruction groupwork. Marilyn Burns's *About Teaching Mathematics: A K–8 Resource* (2000) also has some excellent tasks for groups: you can adapt many of these for different grade levels without compromising their educational potential. We think, for example, the "$1.00 word" problem (Burns 2000, p. 16), displayed below, is both groupworthy and mathematically challenging. You could use it as is in grades 3, 4, or 5.

1. If a = $.01, b = $.02, c = $.03 and so on, what is the value of your first name?
2. Using this alphabetic system, one of the days of the week is worth exactly $1.00. Which one is it?
3. Find other words that are worth exactly $1.00.

We also highly recommend *Elementary and Middle School Mathematics* (Van de Walle, Karp, and Bay-Williams 2010), not only because it offers mathematically meaty problems, but also because it explains clearly what is difficult in each curriculum area. Prospective teachers often find it so useful that they do not sell it back to bookstores, so used copies can be hard to find. The book has been through many editions, however. We believe that *all* editions are useful, and the earlier ones are somewhat cheaper. Figure 7.1 gives an example of the tasks the book offers.

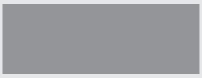
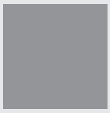
	If this rectangle is *one-third* of the whole, what could the whole look like?
	If this rectangle is *three-fourths* of the whole, what could the whole look like?
	If this rectangle is *four-thirds* of the whole, what could the whole look like?

Fig. 7.1. (VAN DE WALLE, JOHN A.; KARP, KAREN S.; BAY-WILLIAMS, JENNIFER M., ELEMENTARY AND MIDDLE SCHOOL MATHEMATICS: TEACHING DEVELOPMENTALLY, 7th Edition, © 2010, p. 298. Reprinted by permission of Pearson Education, Inc., Upper Saddle River, NJ)

Appendix A will say more about resources you can use to find good tasks.

We know, however, that you almost certainly have a mathematics curriculum to follow, as well as particular mathematical skills and knowledge that you need to teach. Building on what we have said in earlier chapters, especially chapter 4, about characteristics of groupworthy tasks, this chapter offers you guidance in adapting and designing tasks that will engage students, challenge them, and support their learning of significant mathematics, specifically the mathematical ideas and skills that your curriculum emphasizes.

In either looking for a task in the math materials you already have or designing a task from scratch, you will start with the mathematics you want your students to learn, the big ideas that you want them to engage with. Figuring out the big ideas may take a bit of work. Textbooks tend to organize around operations and skills rather than ideas. Conversations with one or two other teachers will probably help. Even if your colleagues teach different grades and do not use complex instruction, if they have in the past helped you think about your students' mathematical confusions, they can probably help you identify the big ideas that undergird the math you teach. Van de Walle, Karp, and Bay-Williams (2010) is another good resource for this. Identifying the big ideas is only the first step, but it is a crucial one.

We begin our discussion of task construction with an example from a fifth-grade teacher we admire. The task requires students to examine what they are doing mathematically when they use the standard multiplication algorithm. It engages them with place value as it manifests itself in multiplication, with the meaning of partial products in this computation, and with relationships of the numbers with one another. This initial section of the chapter presents the task itself, mathematical goals and expectations that the teacher had when he designed it, and some students' work. We then describe several ways that we have adapted the task for use with other learners. Finally, we explain why we like the task and what we have learned from working with it.

The chapter's second section lays out criteria that guide us in creating and revising groupworthy tasks and offers guidance for those wishing to create their own tasks. We know that most teachers need to ground their math curriculum in state and district standards, which may connect to the Common Core Standards for Mathematics that many states have adopted, and in the textbook that their district has chosen. Section three offers suggestions for locating good tasks in your textbook and adapting ones that have some good features but are not groupworthy as written. Most, although certainly not all, textbooks *do* contain at least a few tasks that will work well for groups with only minor alterations. In section three, we present three problems that we found in textbooks and recommend almost as is, explain why we like them, and offer task cards that a teacher might use when assigning them to groups.

As promised, we start with a fifth-grade task and explore how small adjustments to the task can significantly increase students' interactions with important mathematical ideas.

A Fifth-Grade Task: 15 × 49

Joe Cleary, a fifth-grade teacher in Holt, Michigan, wanted to deepen his students' understanding of the meaning of multiplication. During the first weeks of his unit on multiplication, he used a number of tasks that involved his students with different representations and solution methods for a variety of multiplication problems.

For the end of the unit, Joe developed a groupworthy task that he used before students took the unit test. He had the following goals.

- Help students bring together various ideas about multiplication
- Help students summarize and synthesize what they learned
- Help students "show off" their new understandings and realize how much they had learned

Concerning this task's origin, Joe writes (Cleary, personal communication, August 30, 2010):

> As for the 15 × 49 task, it evolved from an Investigations in Number, Data, and Space (TERC 2006) lesson on multiplication. The lesson in Investigations asked the students to think about multiplication as clusters of problems and to work on solving equations as a series of "clusters" or partial products. I had worked with my class on these kinds of clusters and on other ways of solving multiplication equations. It was this work that led me to create a complex task that asked students to bring together their strategies.

Figure 7.2 gives the original version of Joe's task. Before you read any further, try this task yourself.

Joe chose the numbers 15 and 49 because of the many partitioning and compensation strategies students could use to solve this problem. For example, he predicted that his fifth-graders might multiply 15 × 50 to get 750 and then subtract 15, because one fewer 15 would give them 49 × 15 rather than 50 × 15, to get a final solution of 735, as shown in figure 7.3.

As a group, construct a poster that shows four ways of solving and explaining the following number sentence:

$$15 \times 49 = ?$$

Your explanation might include grids, pictures, charts, algorithms, written explanations, or any other way your group can think of to explain the equation.

Each member in the group should be prepared to show and explain your group's answers.

Fig. 7.2

$$49 \times 15 = ?$$
$$(49 + 1) \times 15 = 50 \times 15 = 750$$

But this is 50 fifteens and we only need 49 fifteens, so we have to subtract a fifteen:

$$750 - 15 = 735$$

Fig. 7.3

Or, they might multiply 49 × 10, get 490, then multiply 49 × 5; or take half of 490, since 5 × 49 would be half of 10 × 49; get 245; and add those products to obtain 735, as in figure 7.4.

$49 \times 10 = 490$

Half of 10 is 5, so 49×5 is half of 490, which is 245.

$$\begin{array}{r} 490 \\ \underline{+245} \\ 735 \end{array}$$

Fig. 7.4

The Students' Solutions

The fifth graders came up with a number of different ways to solve the problem Joe had given them. Every group included the traditional algorithm shown in figure 7.5.

$$
\begin{array}{r}
{}^{4}49 \\
\underline{\times\ 15} \\
245 \\
\underline{\ \ 490} \\
735
\end{array}
$$

Fig. 7.5

As Joe had anticipated (see fig. 7.3), two groups, noticing that 49 is one less than 50, computed 50 × 15 and explained, "We rounded 49 to 50 and subtracted 15 from the product."

Several groups created a grid that enabled them to compute each of the partial products separately and then to add them up, as shown in figure 7.6.

×	40	9
10	400	90
5	200	45
	600	135 = 735

Fig. 7.6

Two groups partitioned 49 into 40 + 9, computed the products of 40 × 15 and 9 × 15 using the standard algorithm, and added the two products. Several groups also added the number 49 fifteen times in a column. All the groups also had responses that were representations rather than solution methods. For example, four groups wrote a story problem that illustrated the *meaning* of the number sentence 15 × 49 = 735. Examples included the following:

> Clara baked some batches of cookies. There were 15 cookies in a batch …
>
> Tom went to the store and bought 49 packages of cups. Each package has 15 cups …

Another group included prime factorization (see fig. 7.7) as a representation of the problem.

(5 x 3) x (7 x 7)

V V

15 x 49

Fig. 7.7

Two groups represented the multiplication with an area model, constructing rectangles that were 49 units by 15 units, as shown in figure 7.8.

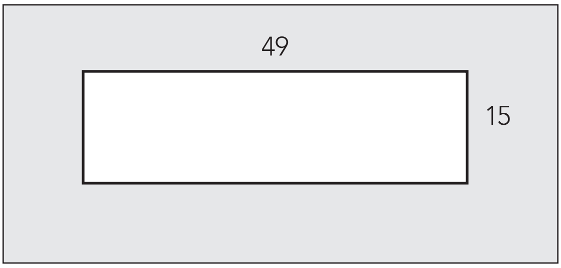


Fig. 7.8

Having asked for solution methods, Joe was surprised to see area-model representations (fig. 7.8) that identified one possible meaning for the number sentence but did not show a way to compute an answer to the problem. However, he encouraged the students to show how they might use their rectangles to solve the problem. To do so, one group divided the rectangle into 100-unit and 10-unit sections, as shown below:

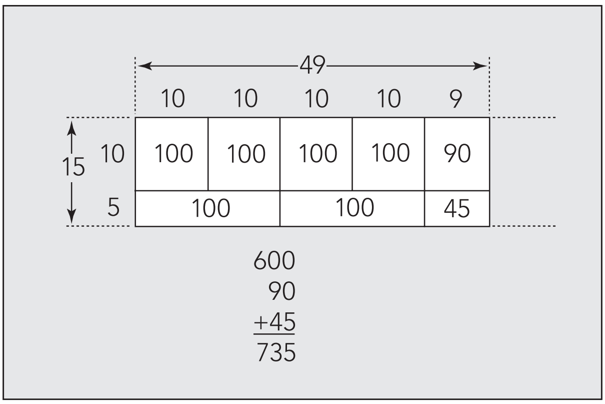


Fig. 7.9

When Joe showed us the fifth-graders' work, we saw that the students' responses, although not what their teacher had originally intended, demonstrated an understanding of multiplication's meaning in many ways. Joe reported that he was able to conclude the work on the task with a discussion in which students looked across the different groups' responses to make some comparisons and contrasts among them. The success of this last pedagogical move made us wonder whether the effort of trying to make explicit connections among various solution strategies might not cause students to dig deeper into the meaning of the numbers in the familiar multiplication algorithm. Together, we decided to alter the task by adding a directive to look for connections among their solution strategies. Figure 7.10 shows our new task.

> As a group, use four different strategies to solve 24 × 15.
>
> Your final product should do the following.
>
> 1. Show each strategy, and how you used the strategy to solve 24 × 15. You can use grids, pictures, charts, written explanations, or any other tools you need to make your explanation clear and easy to understand.
> 2. Show connections across the strategies.

Fig. 7.10

Again, we suggest you try this new version of the task before reading further. This time, focus on trying to identify and show the connections among your methods: for example, you might notice the numbers that occur in partial products in several of them. If you have trouble doing this, you are in good company: read on.

Prospective Teachers Work on 24 × 15

Several of us gave this new version of the task to prospective teachers in our methods classes. We describe their work here, both because it illustrates some of the strategies we expect younger students to try and because it enables us to delve deeper into the mathematical possibilities arising from our modification. Like Joe's students, the prospective teachers gave the traditional algorithm a prominent place on their posters. One class had just read a chapter that laid out many strategies for solving multiplication problems, and, as we had expected, they turned to the book to find examples of alternative algorithms. They also included a number of other algorithms that used the distributive property or compensation strategies. Several groups used one or more of the approaches shown in figure 7.11. The directive to "show connections among the strategies" puzzled the prospective teachers, however.

Solution 1	**Solution 2**	**Solution 3**	**Solution 4**
Traditional Algorithm $\begin{array}{r} {}^{2}24 \\ \times\ 15 \\ \hline 120 \\ \underline{24} \\ 360 \end{array}$	20 × 10 = 200 20 × 5 = 100 4 × 10 = 40 $4 \times 5 = \underline{+\ 20}$ 360	20 × 15 = 300 $4 \times 15 = \underline{+\ 60}$ 300 + 60 = 360	25 × 15 = 375 375 – 15 = 360

Fig. 7.11

To help them see connections, we asked the groups to choose a number that was visible in the process of one solution and see if they could find it in another solution. For example, we pointed out the 200 in solution 2 (see fig. 7.11 above) and asked, "Can you find 200 in any of the other solution methods? Does it mean the same thing? What else can you find in more than one method?" We hope that the prospective teachers will someday ask *their* students these kinds of questions, and we encourage you to try these questions when using this task.

Several groups looked for the 200 in the traditional algorithm and realized that it was there in the 24, which is really 240 or 200 + 40, as shown in figure 7.11, solution 2. All groups found some numbers that showed up across solution methods. We were pleased that, by the end of the lesson, many students had concluded that all the various algorithms for multiplication involved "taking numbers apart and putting them back together." This insight is central to understanding any approach to multidigit multiplication or, for that matter, any other operation that involves place value, one of the big ideas that students work on throughout elementary school.

When we took this second version of the task—the one requiring students to look for connections among multiplication strategies—to other fifth-grade classrooms, we found that the children, like the prospective teachers, found multiple ways to solve the multiplication problem. But the suggestion that they locate "connections" among solution strategies, however, puzzled them just as much as it had puzzled the prospective teachers. Like the prospective teachers, the fifth graders needed some help thinking about what this would mean. We suspect that students might find connections more easily if we engaged them in some preliminary work finding connections across solution strategies with smaller numbers such as 12 × 6.

We like this task for a couple of reasons. First, it is straightforward: We ask students to think about a variety of ways to solve a multiplication problem, explain each of their approaches, and locate connections among the various solution methods. The task and the task cards that we used remind us that once students are used to using complex instruction guidelines, a task card need not be elaborate to serve its purpose adequately.

Second, nothing in this problem distracts the children from the mathematics. Often the most attractive math problems given in textbooks or other curricula involve students in making decisions that are not really mathematical. Recall, for example, the ubiquitous problems involving calculating how many ice-cream-flavor-and-cone combinations one can make with chocolate, vanilla, and chocolate-chip ice cream and three different kinds of cones. Or, take a look at the problem that we analyze in appendix B, in which students must decide where they can paint a mural and what it should look like, given particular amounts of different colors of paint. Joe's problem asks students to think first about different ways to solve and represent a multidigit multiplication problem. Our extension has them think about the connections among their different approaches. This investigation should take students directly into the mathematics.

A third reason we like this task is that it offers the students a chance to pull together what they know about place value and multiplication and showcase their understandings.

This opportunity for reflection helps students move beyond just doing mathematics to actually thinking about what they are doing as they do mathematics. As they learn how to think about their mathematical work rather than just about the mathematics, they will gain tools that will support deeper engagement in more challenging mathematical work.

Perhaps this task's most significant strength, however, is a feature that we originally saw as a weakness: none of the groups who originally undertook the task were clear on what making connections among the various strategies would mean. Even when we explained it, both children and adults needed support in identifying connections. We hesitated to recommend the task's second version to others, because we were afraid that teachers would feel discouraged if their students failed to find connections. We then realized, however, that the very fact that students of all ages had to struggle to see mathematical connections among strategies indicated that the task would challenge students who had always succeeded in solving school math problems before as well as those with a more mixed history. The second version would create an opportunity for *everyone* to learn important math with and from one another.

By asking for four different strategies, the task provides entry for children with widely different skills: even those who can't yet multiply two-digit numbers can add a column of fifteen 49s. By asking for connections, the task challenges all students, including those who often move through math assignments at warp speed. Moreover, the groups' struggles point them toward a deeper understanding of the mathematics. We want a complex instruction task to do this: it is worth the time it takes precisely because it challenges all students and pushes them intellectually, moving them toward clearer understanding of big mathematical ideas. Watching prospective teachers struggling to see connections among different ways of solving a multiplication problem alerted us to a weakness in our teaching as well as in the prospective teachers' earlier schooling: when we introduce students to new ways to represent and solve multiplication problems, we usually hope to help them grasp the mathematics behind the algorithm they have learned to use competently. If they cannot see connections between that algorithm and other approaches to a multiplication problem, we know that we have not achieved this goal. We think that the version of this task that requires students to look for connections among approaches serves the same diagnostic function in fifth grade that it does in the college classroom.

Struggling to make connections, as the prospective teachers found, helps children and adults get deeper into the mathematics. As we saw above, just about everyone will probably use the traditional algorithm to solve a multidigit multiplication problem. Making connections helps them make sense of what they are doing when they use that algorithm.

This task reminded us that it is okay if a task overshoots what students can currently do unaided. Teachers need to ask themselves whether their students, in groups, can work on it productively and whether it will engage them in supportive conversations with each other about rigorous math. Lev Vygotsky (1978) and his zone of proximal development (ZPD) remind us of the importance of presenting students with tasks beyond what they can solve independently. Vygotsky's ZPD is the space in which a learner can succeed only with others' aid. Learning occurs as students work in this space and become able to perform tasks independently for which they earlier needed help. Ideally, groupworthy tasks fall into this zone: no one in the group can do these tasks alone, but students can make progress on them if they work together and help one another. A good task affords multiple paths of entry, based on children's current skills and knowledge; pushes students beyond what they already know and can do alone; and supports them in doing together what they cannot yet do independently. When we ask our students to make connections among different strategies for solving multiplication problems, we provide that push.

Before leaving this task behind, we want to mention that teachers we know have adapted it to meet different mathematical goals, using different numbers and, occasionally, different operations. Substituting decimal numbers for 24 and 15, and leaving the rest of the task the same, will get students thinking about place value in a very different, and less familiar context. A teacher of younger children asked her students to show three ways of *adding* 49 and 15. And one of us asked students to show four different ways to compare $^6/_9$ and $^5/_8$. A group of teachers or prospective teachers could undoubtedly find many other ways to rewrite this task to serve the goals of their curricula.

What to Think about When Adapting a Task to Make It Groupworthy

Although group tasks in textbooks often have considerable potential as prompts for mathematical conversation, they can usually benefit from some editing that aims to center the children's attention on the mathematics, as Joe's task does. We ask ourselves the questions below when adapting a textbook task. Only rarely do we solve all the problems that these questions suggest. However, we do find that considering each question at least briefly is helpful. After a little practice, you will find that you can do this quickly. We have found, too, that when we work with a colleague, we can answer more of the questions to our satisfaction. Our questions focus on three different aspects of task creation: the mathematical ideas, the context the problem creates, and its groupworthiness.

1. Questions that help you to figure out what mathematical reasoning children will need to do to complete the task

A. Think first about the task's mathematical demands. Figure out what important mathematical ideas you want your students to be working on. Ask yourself first what ideas the task *could* get your students thinking about, and then which ones you want to focus on.

B. Ask yourself whether this task will challenge *all* your students mathematically. If not, figure out how you can adapt it so that it will.

C. How can you adapt the task so that it affords more than one starting point, requires more than one solution path, and thus makes some mathematical connections visible? What strategies will your students use to succeed with this task? What might they try? What prior knowledge do they bring to this task that will help them succeed?