CHAPTER 1

Before the Unit

Teacher: Know thy impact.

—John Hattie

The ultimate outcome of planning before the unit is for you and your team members to gain a clear understanding of the impact of your expectations for student learning and demonstrations of understanding during the unit.

In conjunction with the scope and sequence your district mathematics curriculum provides, your collaborative team prepares a roadmap that describes the knowledge students will know and be able to demonstrate at the conclusion of the unit. To create this roadmap, your collaborative team prepares and organizes your work around five before-the-unit-begins high-leverage team actions (HTLAs).

HLTA 1. Making sense of the agreed-on essential learning standards (content and practices) and pacing

HLTA 2. Identifying higher-level-cognitive-demand mathematical tasks

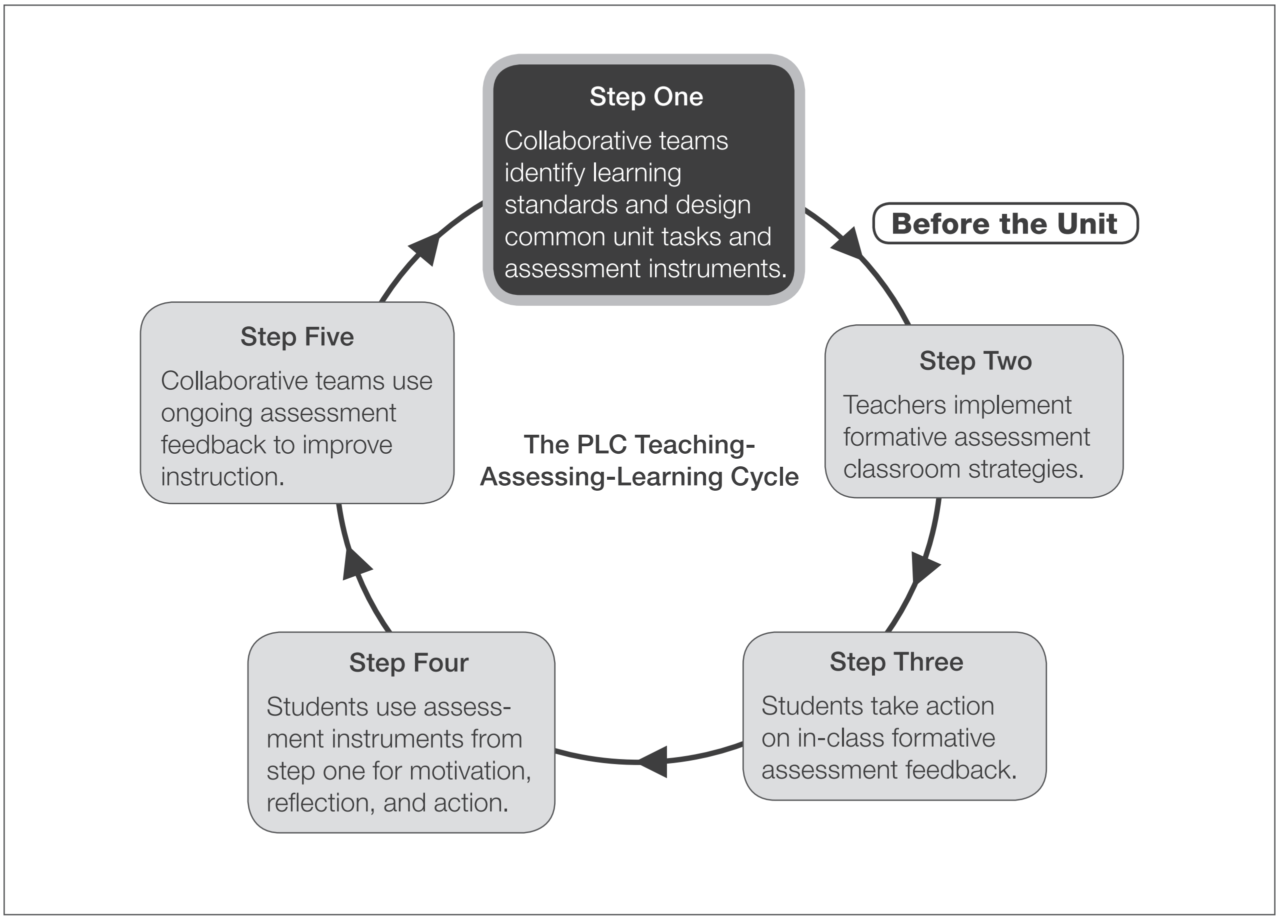
HLTA 3. Developing common assessment instruments

HLTA 4. Developing scoring rubrics and proficiency expectations for the common assessment instruments

HLTA 5. Planning and using common homework assignments

These five team pursuits are based on step one of the PLC teaching-assessing-learning cycle (Kanold, Kanold, & Larson, 2012) shown in figure 1.1 (page 8). This cycle drives your pursuit of a meaningful formative assessment and learning process for your team and for your students throughout the unit and the year.

In this chapter, we describe each of the five before-the-unit-begins high-leverage team actions in more detail (the what) along with suggestions for how to achieve these pursuits (the how). Each HLTA section ends with an opportunity for you to evaluate your current reality (your team's progress). The chapter ends with time for reflection and action (setting your Mathematics at Work priorities for team action).



Source: Kanold, Kanold, & Larson, 2012.

Figure 1.1: Step one of the PLC teaching-assessing-learning cycle.

HLTA 1: Making Sense of the Agreed-On Essential Learning Standards (Content and Practices) and Pacing

An excellent mathematics program includes curriculum that develops important mathematics along coherent learning progressions and develops connections among areas of mathematical study and between mathematics and the real world.

—National Council of Teachers of Mathematics

In most high school mathematics courses, there will be ten to twelve mathematics units (or chapters) during the school year. These units may also consist of several learning modules depending on how your high school curriculum and courses are structured. An ongoing challenge is for you and your team to determine how to best make sense of and develop understanding for each of the agreed-on essential learning standards within the mathematics unit.

Recall there are four critical questions every collaborative team in a PLC asks and answers on an ongoing unit-by-unit basis.

1. What do we want all students to know and be able to do? (The essential learning standards)
2. How will we know if they know it? (The assessment instruments and tasks teams use)
3. How will we respond if they don't know it? (Formative assessment processes for intervention)
4. How will we respond if they do know it? (Formative assessment processes for extension and enrichment)

High-Leverage Team Action	1. What do we want all students to know and be able to do?	2. How will we know if they know it?	3. How will we respond if they don't know it?	4. How will we respond if they do know it?
Before-the-Unit Action				
HLTA 1. Making sense of the agreed-on essential learning standards (content and practices) and pacing	■			

■ = Fully addressed with high-leverage team action

The What

This first high-leverage team action enhances clarity on the first PLC critical question for collaborative team learning: What do we want all students to know and be able to do? In light of the Common Core State Standards for mathematics, the essential learning standards for the unit—the guaranteed and viable mathematics curriculum—include the essential standards students will learn, when they will learn each essential standard (the pacing of the unit), and how they will learn it (via process standards such as the Common Core Standards for Mathematical Practice). The Standards for Mathematical Practice "describe varieties of expertise that mathematic educators at all levels should seek to develop in their students" (NGA & CCSSO, 2010, p. 6). Following are the eight Standards for Mathematical Practice, which we include in full in appendix A (page 163).

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning. (NGA & CCSSO, 2010, pp. 6–8)

While schools and districts use many names for learning standards—*learning goals*, *learning objectives*, and so on—this handbook references the broad mathematical concepts and understandings for the entire unit as *essential learning standards*. For more specific lesson-by-lesson daily outcomes, we use *daily learning objectives* or the daily *essential questions*. We use the terms *learning goals* or *learning targets* to reference the outcome for student proficiency in each standard. The daily learning objectives and the tasks and activities representing those objectives help students understand the four to five essential learning standards for the unit in order to demonstrate proficiency (outcomes) on those standards. The daily learning objectives must also articulate for students what they are to understand and do *that day*.

A unit of instruction connects topics in mathematics that are naturally grouped together—the essential ideas or content standard clusters. Each of those units should consist of about four to six essential learning standards taught to every student in the course. These essential learning standards may consist of several daily learning objectives that require student understanding. The essential learning standards are framed as overarching questions posed to the class during the unit. It might take three to five days of instruction and two to three daily learning objectives to fully answer the essential questions. The *context* of the lesson is the driving force for the entire lesson-design process. Each lesson context centers on clarity of the mathematical content and the processes for student learning.

The crux of any successful mathematics lesson rests on your collaborative team identifying and determining methods of teaching for understanding with the essential learning standards for the unit. Although you might develop daily learning objectives for each lesson as part of curriculum writing or review, your collaborative team should take time during lesson-design discussions to make sense of the essential learning standards for the unit and to consider how the essential learning standards for the unit are connected. This involves unpacking the mathematics content as well as the Mathematical Practices or processes each student will engage in as he or she learns the mathematics of the unit. *Unpacking*, in this case, means making sense of the mathematics listed in the standard, making sense of how the content connects to content learned in other mathematics courses as well as within the current course, and making sense of how students might develop both conceptual understanding and procedural skill with the mathematics listed in the standard.

This first high-leverage team action serves NCTM's (2014) *Principles to Actions* curriculum principle, professionalism principle, and teaching and learning principle as teachers establish goals for student learning. See appendix E (page 175) for how all ten HLTAs support NCTM's guiding practices for school mathematics.

The How

As you and your collaborative team unpack Common Core mathematics content standards or your specific state standards (the essential learning standards) for a unit, it is also important to decide which Standards for Mathematical Practice (or processes) will receive focused development throughout the unit of instruction, and what mathematical tasks you will use during the unit to help students learn both the essential content standards and the Mathematical Practices or process standards. Thus, your team identifies, explores, and discusses:

1. The meaning of the essential *content* learning standards for the unit
2. The intentional Mathematical Practices or processes for student learning and understanding to be developed during the unit
3. The mathematical tasks (higher- and lower-level cognitive demand) to be used during the unit

Unpacking a Learning Standard

How can your team explore the general unpacking of content and linking the content to the Mathematical Practices for any unit? For example, consider the high school content standard cluster *Construct and compare linear, quadratic, and exponential models and solve problems* in the domain Linear, Quadratic, and Exponential Models (F-LE). The essential learning standards for such a unit usually occur during the second semester of either an algebra 1 or integrated mathematics I course (see figure 1.2, page 12).

Content standard cluster: Construct and compare linear, quadratic, and exponential models and solve problems.

F-LE.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

F-LE.1a: Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

F-LE.1b: Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

F-LE.1c: Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F-LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F-LE.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F-LE.4: For exponential models, express as a logarithm the solution to $ab^{ct} = d$, where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Source for standards: NGA & CCSSO, 2010, pp. 70–71.

Figure 1.2: Common Core mathematics essential learning standards for linear, exponential functions unit.

Some of your team members may already have deep knowledge of the relationships between linear and exponential functions (and quadratics, too). Other members may only know about these essential learning standards on a very superficial level. For example, why does F-LE.2 connect arithmetic and geometric sequences to constructing linear and exponential models? Some team members may not yet understand the depth of understanding and the various mathematical representations for students to meet this essential learning standard.

You and your collaborative team can use the discussion questions in figure 1.3 to discuss an appropriate learning progression of the essential standards for this type of unit in your algebra or integrated mathematics I course; identify the depth of the essential learning standards for constructing and comparing linear and exponential models (quadratic, too, if applicable); and discuss explicit connections to previous and future units with functions for your course, or for future high school courses. Additionally, you and your collaborative team should plan and discuss how you want students to demonstrate their understanding of the mathematics content and practices used during the unit. What would you expect to see here?

Your team conversations should focus on student thinking and solution strategies or pathways for the essential learning standards of the unit. This level of unpacking the meaning of the essential learning standards is crucial *before* you can plan effective student engagement within the mathematics instruction.

For example, once your team identifies the depth of the unit's essential learning standards, you can then discuss specific solution strategies and learning processes (problem solving, reasoning, precision, and modeling) that you want students to explore during the unit. You and your collaborative team will need to decide which tools (for example, graphing calculators, statistics programs, graph paper, paper and pencil, or physical models) students will use to construct, model, or build the functions. You will need to specify which essential standards students need to explore first to create a deeper meaning for them of the various types of representations: tables, graphs, and function rules.

Directions: Within your collaborative team, answer the following questions that address linear and exponential functions.

1. What does it mean to construct and compare function models?
2. How might students engage in the construction and comparison of function models? What types of mathematical tasks should they do?
3. What Mathematical Practices should we highlight during a unit of instruction for linear and exponential comparison content?
4. How might students connect numerical representations (arithmetic and geometric) to the visual (graphical) and analytical (function) representations of linear and exponential models?
5. What is the role of quadratics in this content standard cluster, and how could we connect quadratics to the content from this unit? (See F-LE.3.)
6. Why is it important for students to be able to compare these three types of functions as they progress through the course?
7. What is the natural progression of these topics in your current high school course sequence? (See F-LE.4.) How can you help your students to understand this progression?

Figure 1.3: Sample essential learning standard discussion tool for linear and exponential functions.

Visit **go.solution-tree.com/mathematicsatwork** to download a reproducible version of this figure.

For example, once you make sense of how to compare linear and exponential functions and connect them to their various representations, you might decide that students will need to engage in Mathematical Practice 4, "Model with mathematics," in order to build both linear and exponential functions. You might also determine that students will engage in Mathematical Practice 5, "Use appropriate tools strategically," as you plan opportunities for them to use technology as a tool for creating linear or exponential functions. (Selecting good tasks will be discussed further in the following section on the second high-leverage team action). Students engaged in these Mathematical Practices will be asked to check for reasonableness of computations to determine if the solution is appropriate given the original problem context.

For example, students can use a graphing calculator or a software tool to examine a set of data and determine the best-fitting model from a list of familiar functions. Consider the world population task in figure 1.4 (page 14). Your students could create a scatterplot of the data and then use the regression capabilities of the graphing calculator or computer software to determine the best-fitting model for the data. In this mathematical task, students compare linear, exponential, and quadratic models too.

Directions: The world population (in millions) from 1960 through 2010 is shown in the following data table. Answer each question.

Year	**1960**	**1965**	**1970**	**1975**	**1980**	**1985**	**1990**	**1995**	**2000**	**2010**
Population (millions)	3,039	3,345	3,707	4,707	4,086	4,454	4,851	5,688	6,083	6,902

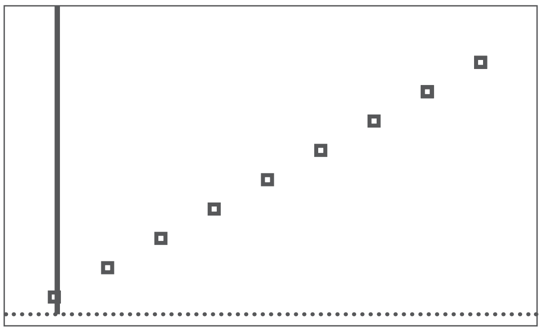
1. Use a graphing and statistics tool to make a scatter plot of the data. Examine the data tables and use the regression capabilities of the graphing tool to determine the best-fitting models for the data.

   Linear function:

   Exponential function:

   Quadratic function:

Visually, all three models appear to be reasonable fits over the given domain. To decide which model is best to use, consider the context of the problem, especially if the model is going to be used to extrapolate for years outside the given domain.



2. Use the three models from question one to determine a value for the world population in 1950. The actual world population in 1950 was 2,556,000,053. Which of the three model's 1950 population value comes closest to the actual value?

3. The world population reached 7 billion in 2011, and is expected to reach 8 billion by 2027. If this prediction is true, which model—linear, exponential, or quadratic—would be the best predictor?

4. Is the world population rate of change staying the same, increasing, or slowing down over the next fifty years?

Source: Zimmerman, Carter, Kanold, & Toncheff, 2012.

Figure 1.4: Predicting world population: Creating and comparing a linear, quadratic, and exponential function.

Visit **go.solution-tree.com/mathematicsatwork** to download a reproducible version of this figure.

The key element of this first high-leverage team action is your ability to make sense of the essential learning standards and to also *plan* for student engagement in the Mathematical Practices that support the standard. See Appendix A (page 163) for a complete listing of the Mathematical Practices and Appendix B (page 167) to review evidence for each Mathematical Practice. In general, to begin any discussion of the essential standards with your team, you can use the discussion tool in figure 1.5 for each unit of the course.

Directions: Discuss the following prompts or questions with your collaborative teams to unpack essential learning standards, prerequisite skills for the unit, associated Mathematical Practices or processes relevant to the current unit of study for your grade level, and pacing decisions for the unit.
1. List the agreed-on essential learning standards for this unit. 2. What is the prerequisite knowledge needed to engage students with the essential learning standards? 3. What is the time frame available to teach this unit, and how will that time be distributed for each essential learning standard? 4. What are the mathematics vocabulary and literacy skills necessary for student success in this unit? 5. What are specific teaching strategies we can use to most effectively teach each essential learning standard for the unit? (See the questions in figure 1.3, page 13.) 6. Which Mathematical Practices or processes should we highlight during the unit in order to better engage students in the process of understanding each essential learning standard?

Figure 1.5: Discussion tool for making sense of the agreed-on essential learning standards for the unit.

Visit **go.solution-tree.com/mathematicsatwork** to download a reproducible version of this figure.

After using the discussion tool in figure 1.5, you and your collaborative team can use the result of your conversations to create a transparent map of the unit and to articulate the intent of the unit to all team members.

Consider the sample unit plan for an algebra I or an integrated mathematics I or II course presented in figure 1.6 (pages 16–17) and designed to support the first high-leverage team action—Making sense of the agreed-on essential learning standards (content and practices) and pacing—for a unit supporting quadratic functions.

<table>
<tr><td>Unit Name: Quadratic Functions</td><td>Unit Number: 7</td></tr>
<tr><td>Time Frame
Eighteen fifty-minute class periods (including review and test)</td><td>Purpose
This unit examines the parameters of a quadratic function, graphing with and without a table, identifying key features of a quadratic function, and then comparing all functions learned throughout the year.</td></tr>
<tr><td colspan="2">Common Core Essential Learning Standards
1. Interpret functions that arise in applications in terms of the context.
F-IF.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
2. Analyze functions using different representations.
F-IF.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
F-IF.7a: Graph linear and quadratic functions and show intercepts, maxima, and minima.
F-IF.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
F-IF.8a: Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
F-IF.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
3. Build a function that models a relationship between two quantities.
F-BF.1: Write a function that describes a relationship between two quantities.
F-BF.1a: Determine an explicit expression, a recursive process, or steps for calculation from a context.
4. Build new functions from existing functions.
F-BF.3: Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
(Limit to quadratic functions for this unit.)</td></tr>
<tr><td>Essential Learning Standards
Student will be able to:
1. Interpret functions that arise in applications in terms of the context.
2. Analyze functions using different representations.
3. Build a function that models a relationship between two quantities.</td><td>Enduring Understanding
Students can model and interpret real-world situations with quadratic functions.
Essential Questions
1. How are different representations of quadratic functions used to identify key features of the graph?
2. What are the distinctions of a quadratic function compared to linear and exponential?
3. What is the minimum or maximum of the graph, and how does it behave?
4. How do the a, b, and c values change the function and the domain and range of the function?</td></tr>
</table>

Key Mathematics Vocabulary
• *zeros* • *extreme values* • *standard form* • *factored form* • *perfect square trinomial* • *discriminant* • *quadratic equation* • *radical* • *maximum and minimum* • *parabola* • *vertex* • *leading coefficient* • *axis of symmetry* • *x-intercept* • *roots (solutions)*
Prior Knowledge (What Knowledge and Skills Need to Be Spiraled?) **Prerequisite understanding for this unit includes:** • Domain and range • How to graph a point and make a graph from a table • Rate of change • GCF and like terms • Linear functions, factoring, and solving quadratics

Source for standards: NGA & CCSSO, 2010, pp. 69–70.

Figure 1.6: Sample unit plan progression of content for a quadratic functions unit of study.

Visit **go.solution-tree.com/mathematicsatwork** to download a reproducible version of this figure.

When unpacking the essential learning standards, your team will also develop a lesson progression that makes sense for student understanding of the essential learning standards for the unit.

Making Sense of the Unit Content Progression

Once your team identifies the essential learning standards for the unit, you decide how much time to dedicate to each essential standard as well as a natural progression of lessons during the unit. As with any planning, this is a starting point to understanding the depth of student understanding required for the unit and how to organize the development of mathematical concepts within the unit. Your team should share how each concept is connected to previous standards and upcoming standards in order to make explicit and logical connections for the unit content for students. Figure 1.7 (pages 18–19) provides a sample unit progression for exponential functions across standards for students. An additional sample unit plan is provided at **go.solution-tree.com/mathematicsatwork**.

Unit Seven Plan: Twenty Instructional Days

Day 1	Day 2	Day 3	Day 4	Day 5
F-LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*	**F-LE.1:** Distinguish between situations that can be modeled with linear functions and with exponential functions.*	**F-LE.1:** Distinguish between situations that can be modeled with linear functions and with exponential functions.*	**F-LE.1:** Distinguish between situations that can be modeled with linear functions and with exponential functions.*	**F-LE.3:** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
F-LE.5: Interpret the parameters in a linear or exponential function in terms of a context.*	**F-BF.1:** Write a function that describes a relationship between two quantities.*	**F-BF.1:** Write a function that describes a relationship between two quantities.*	**F-BF.3:** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	**F-IF.7:** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases, and using technology for more complicated (exponential only).
Day 11	**Day 12**	**Day 13**	**Day 14**	**Day 15**
F-IF.8a: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of a function.	**F-IF.9:** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.	**F-IF.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative.	**F-IF.6:** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. (Percent rate of change)	**A-SSE.3:** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*

Day 16	Day 17	Day 18	Day 19	Day 20
S-ID.6: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.	**S-ID.6:** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.	Review for unit seven	Review for unit seven	Assessment for unit seven

Notes for Unit Seven

This unit connects to the students' prior knowledge on the domain Expressions and Equations (8.EE) under the *content standard cluster* Work with radicals and integer exponents.

8.EE.1: *Know and apply the properties of integer exponents to generate equivalent numerical expressions.* This unit is packed with content, and thus it needs twenty days to unfold the clusters of comparing, building, and interpreting exponential functions.

The unit provides a deep examination of exponential functions and the laws of exponents, including definition of exponent notation, sum law for exponents, product law for exponents, definition of negative exponent notation, and basic characteristics of exponential functions.

The unit also extends ideas and understandings from unit 3 on linear functions to the parallel ideas for exponential functions with special emphasis on the notion of constant percent rate of change (as compared to constant rate of change in chapter 3), repeated multiplication as the big idea, geometric sequences and recursive definitions, the meaning of the dependent variable, the meaning of the independent variable, parameters and their meanings, ways of measuring amount of growth via the constant difference (linear), and constant ratio (exponential).

The unit also compares linear and exponential functions in a context and from various representations of those types of functions.

Statistics are also brought back into this unit through fitting an exponential function to data.

*Specific modeling standards appear throughout the high school standards marked with an asterisk. In addition, "model with mathematics" is an expected mathematical practice and process for students.

Source: Adapted from Grossmont Union High School District, El Cajon, California (July 2013).
Source for standards: NGA & CCSSO, 2010, pp. 70–71.

Figure 1.7: Sample unit progression for exponential functions—across standards.

Visit **go.solution-tree.com/mathematicsatwork** to download a reproducible version of this figure and an additional sample.

This first high-leverage team action is both a district and a teacher team responsibility. The district office needs to provide guidance about the proper scope and sequence of the essential learning standards of the unit. At the same time, your course-based team members need to be clear on the intent of the essential standards, the rationale for teaching the standards in a specific order, and the nuances of the meaning and intent of each essential learning standard. At a minimum, your team should take the time to write a set of notes similar to the ones shown at the end of figure 1.7 as you work to better understand the intent of the mathematics content and progressions for the unit.

Your Team's Progress

It is helpful to diagnose your team's current reality and action prior to launching the unit or chapter. Ask each team member to individually assess your team on the first high-leverage team action using the status check tool in table 1.1. Discuss your perception of your team's progress on making sense of the agreed-on essential learning standards (content and practices) and pacing. It matters less which stage your team is at and more that you and your team members are committed to working together to focus on understanding the learning standards and the best mathematical tasks, activities, and strategies for increasing student understanding and achievement as your team seeks stage IV—sustaining.

Your responses to table 1.1 will help you determine what you and your team are doing well with respect to your focus on essential learning standards and where you might need to place more attention before the unit begins.

Once your team unpacks and understands the essential learning standards you are ready to identify and prepare for higher-level-cognitive-demand mathematical tasks related to those essential learning standards. It is necessary to include tasks at varying levels of demand during instruction. The idea is to match the tasks and their cognitive demand to the essential learning standard expectations for the unit. Selecting mathematical tasks together is the topic of the second high-leverage team action, HLTA 2.