■ PART I

Assessment in Action

Introduction

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In this section on Assessment in Action in the classroom, each chapter frames formative assessment as an essential classroom practice and presents specific classroom-based strategies that can be embedded into teachers' everyday instructional practices. Formative assessment involves activities undertaken by both teachers and students to modify teaching and learning activities, with a focus on learning rather than on evaluation, ranking, or judgment (Black and Wiliam 1998; Gipps 1994; Sadler 1998). A key aspect of formative assessment is that the information it generates is used by both teachers and students to improve learning.

Formative assessment strategies make students' mathematical thinking and understanding visible, thus serving as methods for "eliciting and using evidence of student thinking," as called for by the National Council of Teachers of Mathematics' (NCTM) *Principles to Actions: Ensuring Mathematical Success for All* (NCTM 2014). As an example, the chapter by Kim and Lehrer describes how "formative assessment talk" generates information to support students' progressions along learning trajectories in ways that are not possible using traditional evaluative (e.g., IRE) modes of discussion. In order to elicit and assess active thinking rather than passive recall, more diverse and complex assessment tasks and strategies are required (NCTM 1995; Shepard 2001). This point is illustrated well in the strategies for assessing and supporting children's understanding (rather than rote memorization) of number facts presented in the chapter by Bay-Williams and Kling. Similarly, the chapter by Silver and Smith makes salient the connection between cognitively challenging tasks and formative assessment. As Wiliam (2007) points out, "the task of the teacher is not necessarily to teach, but to create situations in which students learn" (p. 1087).

The chapters in this section make explicit connections between planning, instruction, and assessment. Their authors describe ways in which teachers can use formative assessment to determine next instructional steps "in-the-moment" during a lesson and in planning subsequent lessons. For example, Slavit and Nelson's chapter describes how problem-based instruction creates the need for formative assessment strategies and provides a context in which assessment and instruction become increasingly interconnected in teachers' daily practice. The chapter by Fennell, Kobett, and Wray provides a variety of formative assessment strategies and describes how such strategies can be used to "in*form*" instruction and planning. Each chapter provides a scenario or context to ground and illustrate the highlighted strategies, across grade levels (including elementary and middle grades; see chapter 15 by Marynowski in part III of this book for formative assessment in secondary classrooms), mathematical content (e.g., number facts, measures of center, and linear relationships), and in multiple contexts (e.g., problem-based learning, learning trajectories, implementing cognitively challenging tasks, and general classroom practice). The following paragraphs provide summaries of each chapter in this section, specifically highlighting how the chapter presents formative assessment in action in the mathematics classroom.

In chapter 1, **Integrating Powerful Practices: Formative Assessment and Cognitively Demanding Mathematics Tasks,** Silver and Smith describe how instructional moves that maintain students' engagement in cognitively challenging mathematical work and thinking simultaneously serve the purpose of formative assessment. The lines of research indicating the power of cognitively challenging tasks and of formative assessment techniques in supporting students' learning have previously been disconnected; here, the authors bring together these ideas and illustrate their interconnectedness in classroom practice: "engineering effective classroom discussions, questions, and learning tasks" are instructional moves essential for providing opportunities for formative assessment (e.g., Wiliam 2011) and for engaging students in cognitively challenging mathematical work and thinking (e.g., Henningsen and Stein 1997). To illustrate, the authors present the case of an eighth-grade mathematics lesson on linear relationships, set in a problem-solving context where students determine the cost of pizzas with different numbers of toppings. Prior to the lesson, the teacher selected a cognitively demanding task, anticipated students' strategies, created a monitor chart to identify these strategies during the lesson, and planned questions to ask during small-group work and the whole-group discussion. During the lesson, the teacher monitored students' work, asked questions to assess and advance students' thinking, and engaged students in a whole-group discussion. In presenting the case, Silver and Smith provide explicit connections between the five practices for orchestrating whole-group discussions (Smith and Stein 2011) and formative assessment (Suurtamm 2012).

In the chapter **Developing Fact Fluency: Turn Off Timers, Turn Up Formative Assessments,** Bay-Williams and Kling call attention to the perils of timed, frequent tests that can have potential and long-term impacts on children's mathematical confidence and view of mathematics. They contrast teaching patterns of "Memorize-Test-Continue" (M-T-C) that can neglect reasoning strategies with "Reasoning strategies, Practice, and Monitoring" (R-P-M) that can support students as they work toward mastery. The authors distinguish the R-P-M approach as shifting the learning focus from memorization to strategy development and meaningful practice, as well as shifting the assessment focus from timed tests to observations and interviews. They argue that this approach can accomplish the mastery and retention of facts that traditional approaches have failed to produce. Bay-Williams and Kling suggest five strategies to more appropriately assess fact fluency: no longer use time tests, make tests shorter to create time for students to reflect on their strategies, have students describe their strategy for solving a problem, include self-assessment, and provide teacher feedback that is more detailed than a mere score.

In chapter 3, **Using Learning Progressions to Design Instructional Trajectories,** Kim and Lehrer describe how a fifth-grade math teacher used evidence of students' learning from a Learning Progression Oriented Assessment System (LPOAS) to support students' statistical reasoning. This LPOAS involved four elements: construct maps, assessment items, scoring exemplars, and lessons. The authors argue this LPOAS supports teachers in designing an instructional path that aligns students' current understandings with a conjectured learning progression. The teacher's knowledge of mathematics is key in this approach where instructional decisions are based on the mathematical substance of students' thinking. Assessment tasks were used to determine students' current understandings along a progression of three conceptual building blocks. The teacher then asked "leveraging" questions and "engineered" formative assessment talk (FAT) based on the conjectured learning progression. The authors suggest teachers use a construct map when scoring students' responses to assessment items, identify leverage points that bridge current levels of understanding with learning performances, and design questions and supporting representations to help students move from their current understandings to those with greater disciplinary scope and precision.

In **How Changes in Instruction Support Changes in Assessment: The Case of an Inclusive STEM-Focused School,** Slavit and Nelson describe how curriculum and instructional choices influence assessment in a grades 6–12 STEM-focused school. In a problem-based learning approach, teachers at this school prioritized formative assessment in instructional design. A key component of assessment at this school was presentations to authentic audiences, such as local business professionals and professors. This required the development of rubrics where teachers had important conversations about clarifying the learning goals and criteria for success. At the end of the year, students reported positive experiences based on teacher feedback and flexibility in the way teachers viewed their learning. The project-focused learning environment changed the way teachers viewed assessment, and they felt their new formative assessment strategies made students' learning more visible.

In the final chapter of this section, **Classroom-Based Formative Assessments: Guiding Teaching and Learning,** Fennell, Kobett, and Wray argue for the value of using formative assessment as an everyday practice in mathematics classrooms. They present five classroom-based formative assessment (CBFA) techniques, validated through classroom use, that teachers can implement on a regular basis to guide and inform planning and teaching: observations, interviews, "show me," hinge questions, and exit tasks. The chapter describes each CBFA technique and provides concrete suggestions for when and how to use the technique in the mathematics classroom. The authors describe observations, interviews, and "show me" as informal techniques that could be used within any lesson to monitor students' progress and to help teachers determine the pace of the lesson, identify misconceptions, and consider potential next steps. Hinge questions and exit tasks are more formal in the sense that the question or item requires careful construction to elicit evidence of students' understandings of the main mathematical idea(s) of the lesson. Hinge questions are open-ended or specially crafted multiple-choice items that can serve as a deciding element for determining next instructional steps in planning and teaching. Exit tasks provide written documentation about each student's understandings and proficiency. In the chapter overall, Fennell and colleagues frame formative assessment techniques as in-the-moment opportunities for teachers (and students) to garner, and immediately use, evidence of learning to adapt instruction and meet students' learning needs.

Together, the five chapters in part I on Assessment in Action provide concrete suggestions for using formative assessment as an everyday classroom practice. As you read these chapters, consider the following questions:

- How do teachers connect planning, teaching, and assessment into a seamless cycle of instructional moves?
- In what ways can teachers elicit and support students' thinking?

- How are students included in the assessment process?
- In what ways does classroom assessment inform both teachers' and students' next moves?

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Integrating Powerful Practices: Formative Assessment and Cognitively Demanding Mathematics Tasks

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As students shuffled out of her classroom, Ms. Dyson sat at her desk and reflected on what had just occurred in her eighth-grade mathematics class. Here is a portion of her reflection:

> The Building a Pizza task worked really well today! There were a few bumps at the start, but I was able to get the confused students on track. The class was excited when they went to the Domino's website and saw that the problem was real—the price of a topping was not given! Students worked hard and generated lots of mathematics—plotting points by treating the number of toppings and corresponding cost as ordered pairs, looking for patterns in prices for medium pizzas that varied by number of toppings, and forming generalizations to express the cost for a pizza with respect to the number of toppings.
>
> Groups 1, 2, and 5 stated a generalization in words for the total price of a medium pizza, but they had trouble expressing it algebraically. Group 4 got the price per topping but had trouble writing the equation, initially confusing what was constant and what varied. Several groups had difficulty interpreting the graph produced by Group 4. Based on what I saw today, Group 3 seems ready to move on, so tomorrow I will ask them to explore how the generalization they found in part *c* would be affected if we changed the conditions by including two toppings in the base price and only charging for extra toppings. Most of the groups need more experience with the concepts in context and more practice with writing and graphing equations to express generalizations, so I will ask them to extend this investigation to small and large pizzas.

The first part of Ms. Dyson's reflection on the lesson is fairly typical of what teachers might glean from lessons that "go well," but the latter part is not at all typical. Even without knowing the details of the task, which we will present later, we can see that Ms. Dyson's comments suggest both that she learned quite a lot about her students during the lesson and that she was using that information in planning tomorrow's lesson.

How did Ms. Dyson uncover so much about her students' mathematical understandings during one lesson? How might her students benefit from the detailed insights she developed about what they know and can do, and about areas where they might need further conceptual development or skill practice? In this chapter we consider these questions in relation to some recent research on mathematics instruction.

Some Relevant Recent Research on Mathematics Instruction

As we elaborate below, research on effective mathematics instruction has established two distinct, robust findings. One is that students learn mathematics well in classrooms where they have regular opportunities to work on cognitively challenging tasks that promote mathematical problem solving, reasoning, and understanding, as long as their teachers support their work on the tasks in a manner that does not lower the cognitive demand as the lesson unfolds. A second robust research finding is that students learn mathematics well in classrooms where teachers employ formative assessment techniques to elicit, interpret, and use evidence about what students have learned to inform instructional decisions. These evidence-based characterizations of effective mathematics teaching have been disconnected in both the research literature and in practitioner-oriented outlets in large part because they derive from different perspectives on classroom instruction and from distinct lines of empirical inquiry. In this chapter we interweave these distinct characterizations to produce an integrated perspective that we believe can inform and support efforts to improve mathematics teaching.

Cognitively Demanding Mathematical Tasks

Mathematics classroom instruction is organized around and delivered through the mathematical tasks, activities, and problems found in curriculum materials. For example, the students in all seven countries analyzed in the TIMSS video study (National Center for Educational Statistics [NCES] 2003) spent more than 80 percent of their time in mathematics lessons working on tasks. Thus, students' opportunities to learn mathematics are determined to a great extent by the mathematical tasks they encounter in the classroom. Though mathematical tasks are a constant presence in mathematics classrooms, they also exhibit considerable variation.

Tasks vary not only with respect to mathematics content but also with respect to the cognitive processes they entail. Tasks that offer opportunities for students to sharpen their mathematical thinking and reasoning by requiring them to analyze mathematics concepts or to solve complex problems can be considered cognitively demanding or high-level tasks. In the Building a Pizza task shown in figure 1.1, for example, no solution path is explicitly suggested or implied, and students could use a variety of approaches (e.g., plot the number of toppings and cost as points on a graph to find the rate of change, build a table with the given values and interpolate, or find the difference in the number of toppings and the difference in the cost and then divide). In addition, students must determine and enact a reasonable course of action and justify the plausibility and accuracy of their solutions.

In contrast, cognitively undemanding tasks—low-level tasks that require little more than memorization and repetition—offer little or no opportunity to develop proficiency with complex, high-level cognitive processes. For example, it is likely that students would expect to solve the

Writing Equations task shown in figure 1.1 using a specific, memorized procedure (e.g., the point-slope form of a line, or a combination of the slope formula and the slope-intercept form of a line). Low-level tasks typically require neither decision-making nor justification.

Building a Pizza	Writing Equations
You and your friends are going to buy pizza from Domino's. From previous orders you know that a medium pizza with 2 toppings costs \$14.00 and a medium pizza with 5 toppings costs \$20.00. *a.* Assuming Domino's charges the same amount for each topping added to a plain cheese pizza, determine the cost per topping. *b.* If you wanted to order a medium cheese pizza, with no additional toppings, how much would you expect to pay? *c.* Write a general rule you could use to determine the price of any medium Domino's pizza. For each part of the task, be sure to explain how you got your answer and why it makes sense. *Adapted from Mathalicious (http://www.mathalicious.com/lessons/domino-effect)*	For each pair of points, find the rate of change, the *y*-intercept, and the equation of the line that passes through the points. *a.* (3,2) and (7,-4) *b.* (2,3) and (6,4) *c.* (1,6) and (3,2) *d.* (0,-2) and (3,4) *e.* 1,-4) and (-4,7)

Fig. 1.1. Mathematics tasks with different cognitive demands

Deciding to use a high-level mathematics task in a lesson is an important step, but the payoff from this decision depends on how the task is enacted in the lesson. Selecting high-level tasks for use in mathematics classrooms does not guarantee that the tasks will be used in ways that maintain the demand characteristics essential to opportunities for students to learn mathematical thinking and reasoning. Research has shown that the cognitive demands of mathematical tasks can change as tasks are introduced to students and/or as tasks are enacted during instruction (Stein, Grover, and Henningsen 1996). The mathematical tasks framework (MTF) shown in figure 1.2 models the progression of mathematical tasks from their original form, as they appear in the pages of textbooks or other curriculum materials, to the tasks that teachers actually provide to students, and then to the tasks as they are enacted by the teacher and students in classroom lessons (Stein et al. 2009).

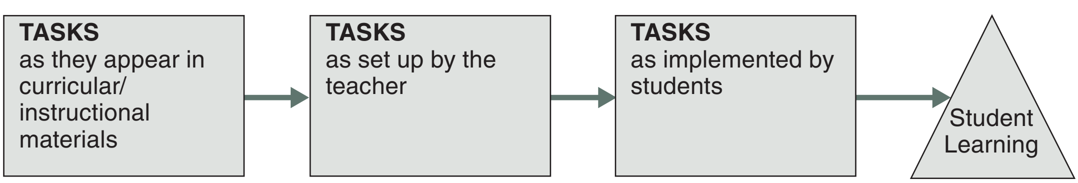

Fig. 1.2. The mathematical tasks framework

The first two arrows in the figure identify critical phases in the instructional life of tasks at which cognitive demands are susceptible to being altered. The tasks, especially as enacted, have consequences for student learning of mathematics, as is shown by the third arrow in the figure and the “Student Learning” triangle that follows it. The features of an instructional task, especially its cognitive demands, may be altered as a task passes through these phases (Stein, Grover, and Henningsen 1996; Stigler and Hiebert 2004).

Researchers who have used the MTF, and related conceptualizations, as a lens for studying mathematics classroom teaching have noted that implementing cognitively challenging tasks in ways that maintain students’ opportunities to engage in high-level cognitive processes is not a trivial endeavor, especially for teachers of mathematics in the United States (e.g., Henningsen and Stein 1997; NCES 2003). Nevertheless, evidence from research conducted in a variety of U.S. classroom contexts has found that it is possible for American teachers to do this well, with clear benefits for their students.

Research has found that greater student learning occurs in classrooms where cognitively demanding mathematical tasks are used frequently and where high-level cognitive demands are maintained throughout an instructional session (Boaler and Staples 2008; Hiebert and Wearne 1993; Stein and Lane 1996; Stigler and Hiebert 2004; Tarr et al. 2008). For example, in a longitudinal comparison of three high schools over a five-year period, Boaler and Staples (2008) determined that the highest student achievement occurred in the school in which students were supported to engage in high-level thinking and reasoning. Boaler and Staples attribute students’ success to the ability of the teachers to maintain high-level cognitive demands during instruction. Studies by Tarr and colleagues (2008) and Stein and Lane (1996) found that classrooms in which teachers consistently encourage students to use multiple strategies to solve problems and support students to make conjectures and explain their reasoning were associated with higher student performance on measures of thinking, reasoning, and problem solving.

Formative Assessment

Another body of research suggests that student achievement is amplified when teachers employ formative assessment techniques in classroom instruction. Black and Wiliam (1998) synthesized the results of dozens of studies of formative assessment, and they found strong evidence of greater student achievement in classrooms where teachers used such techniques. Ehrenberg and colleagues (2001) reported that the impact on student achievement of teachers using formative assessment as part of instruction was far greater than that obtained by reducing class size. Other empirical studies have demonstrated that teachers can learn to use formative assessment in the mathematics classroom with positive effects on students’ learning (e.g., Wiliam et al. 2004). Although some have pointed to weaknesses and gaps in the evidence base (e.g., Bennett 2011), the preponderance of research evidence appears to support the positive influence on student learning of formative assessment in classroom instruction.

Formative assessment refers to a process of eliciting and interpreting evidence about what students have learned and then using this information to make instructional decisions (Wiliam 2011, p. 50). In contrast to summative assessment, which involves the evaluation *of* student learning, progress, or achievement to assign grades or appraise programs, formative assessment involves assessment *for* learning—gathering evidence within the stream of instruction about what students are doing, thinking, and learning and then using that evidence to inform decisions that affect teaching and learning.

Many view formative assessment as an essential aspect of effective instruction. In fact, *Principles to Actions: Ensuring Mathematical Success for All* (National Council of Teachers of Mathematics [NCTM] 2014) identifies *eliciting and using evidence of student thinking* as one of eight non-negotiable teaching practices critical for successful implementation of ambitious standards. According to Leahy and colleagues (2005, p. 19), "in a classroom that uses assessment to support learning, the divide between instruction and assessment blurs. Everything students do—such as conversing in groups, completing seatwork, answering and asking questions, working on projects, handing in homework assignments, even sitting silently and looking confused—is a potential source of information about how much they understand." Based on their analysis and synthesis of a number of studies of formative assessment in classroom instruction across a variety of school subjects, Leahy and colleagues (2005) identified several aspects of instruction that characterize effective formative assessment in classrooms, including engineering effective classroom discussions, questions, and learning tasks; promoting students' ownership of their learning; and encouraging students to be learning resources for one another.

Engineering effective classroom discussions, questions, and learning tasks involves at least three interrelated instructional practices: (1) engaging students in tasks and activities that provide insights into their thinking; (2) listening and analyzing student discussions and artifacts interpretatively, not just from an evaluative perspective; and (3) implementing instructional strategies designed to engage all students in tasks, activities, and discussions (Wiliam 2011). For this to work well, instructional tasks and activities should elicit thinking and reasoning, relate to key concepts and skills in the curriculum, and allow students to show what they understand and can do. Also, it is important that teachers and students engage in listening "interpretatively" (Davis 1997); that is, not just listening for the right answers but also listening for evidence about student thinking to inform the next instructional steps. In this way, a teacher can obtain evidence about how well students are learning important mathematical concepts and skills and detect errors or misconceptions that are prevalent in student work, especially those that may interfere with learning new concepts or solving related problems.

Effective formative assessment also means *promoting students' ownership of their learning and encouraging students to be learning resources for one another*. Providing students with challenging mathematical tasks and supporting them to develop persistence in solving such tasks helps students develop a sense of self-efficacy that also supports their motivation to tackle difficult mathematics topics. Also, teachers can engage students in self-assessment and peer-assessment, with an emphasis on listening interpretively as noted above rather than focusing only on right/wrong judgments. Classrooms in which students actively listen to their peers' presentations and explanations can be communities in which each student supports the learning of other students in a mutually enabling manner.

■ Integrating Formative Assessment with the Use of Cognitively Demanding Tasks: The Case of Ms. Dyson

We now return to the question posed earlier in this chapter: *How did Ms. Dyson uncover so much about her students' mathematical understandings during one lesson?*

We think the answer lies in the interplay between the two perspectives just reviewed, namely, cognitively demanding tasks and formative assessment. First, Ms. Dyson selected a mathematical task for her students to work on during the lesson that was—

- cognitively demanding;
- accessible to all students, whether they preferred to work with words, numbers, graphs, or equations;
- aligned with her goals for student learning (e.g., use concepts of slope and *y*-intercept in a problem context; write an equation to represent the relationship between a dependent and independent variable; gain facility in recognizing and expressing a linear function in a table, graph, and equation);
- motivating to students—presenting a familiar context and a question that could not be immediately answered (even if you went to the Domino's website!); and
- capable of revealing students' understanding and thinking, especially by including the requirement that students explain *how* they solved the problem and *why* their solution made sense.

Hence by selecting this particular task, Ms. Dyson took an important first step toward engineering an effective classroom discussion.

Second, she carefully planned the lesson prior to instruction—anticipating the ways in which students might approach or solve the task and generating questions she could to ask to assess what the students understood and to advance their understandings. Here is another part of her post-lesson reflection:

> I learned a lot from listening as students worked in groups on the task, using the monitoring charts I created yesterday. The charts recorded my expectations about what students were likely to do and what I could say to get them to think more deeply. I was free to watch and listen carefully and then to jot notes about what I saw and heard and to flag things that might need follow-up.

The monitoring charts (see appendices 1.A and 1.B) assisted Ms. Dyson both in preparing to teach the lesson and in allowing her to be attentive to students' thinking as they tried to solve the problem.

In each chart she listed solution strategies that she anticipated students might use, obstacles she expected they might encounter, and questions she intended to ask about their methods to highlight key mathematical issues or ways she intended to help them navigate around or through the obstacles (see the first two columns of the tables in appendices 1.A and 1.B). In so doing she illustrated the kind of lesson preparation that is crucial both to using cognitively demanding tasks effectively and to supporting a classroom discussion that clarifies and shares learning intentions and outcomes. Also, by carefully thinking in advance about likely solution methods and questions she might want to pose, Ms. Dyson was preparing herself to support students to persist in solving a challenging problem in the face of obstacles they might encounter rather than telling them explicitly how to solve the problem and thereby lowering the cognitive demand.

Next, while students worked in groups on the task, she used her monitoring chart to remind herself of the questions she wanted to ask students about their solution methods and to keep track of what students were doing (see column 3 of the tables in appendices 1.A and 1.B). Her recordings on the monitoring chart helped her decide which solutions should be presented during the discussion and in what order, key aspects of a solution she wanted to highlight, and who would be asked to present each one to the class (see column 4 of the tables in appendices 1.A and 1.B). In this way, she increased the opportunities that students would be able to learn from each other during the whole-class discussion. The monitoring chart also helped her identify concepts that students were struggling with, to reassign group members so that students have the opportunity to work with peers with different strengths, and to keep track of which students had an opportunity to present their work to the class. Hence the information on the monitoring chart would be useful to Ms. Dyson in making instructional decisions in the current lesson as well as in future lessons.

Finally, the lesson provided an opportunity for students to take ownership of their learning. Students initially visited the website and determined that this was an authentic problem. Later, by carefully selecting the solutions that would be presented and the students who would do the presenting, Ms. Dyson built the lesson upon the thinking of her students and allowed them to be authors of their own ideas. Through their discussion of varied solution methods, students compared responses to identify the strengths and weaknesses of different approaches to or explanations of a solution, rather than simply relying on the teacher to identify them as right or wrong. Students were thus held accountable for reasoning about and understanding key ideas.

Ms. Dyson's instructional practice embodies effective use of formative assessment as well as what Smith and Stein (2011) have referred to as the five practices for orchestrating a productive mathematics discussion. The practices (Anticipating, Monitoring, Selecting, Sequencing, and Connecting) are intended to help teachers maintain the cognitive demands of high-level tasks through thoughtful and thorough planning prior to a lesson, thereby limiting the amount of improvisation needed during the lesson. A lesson enacted using the five practices is similar to what Suurtamm (2012, p. 31) describes as a formative assessment approach called the "Math Forum," in which a teacher gains "a strong sense of individual students' as well as the whole class's understanding of mathematical concepts."

■ Coda

We think Ms. Dyson's lesson offers a vivid example of how formative assessment and the use of cognitively demanding mathematics tasks in instruction can be seamlessly integrated. Moreover, given recent arguments for the importance of taking a disciplinary perspective when thinking about formative assessment (e.g., Bennett 2011; Coffey et al. 2011), we see the integration of these perspectives as one way to accomplish that goal. By connecting these two lines of research in her own practice, Ms. Dyson provided her students with the opportunity to learn mathematical content and to engage in a set of practices that are the hallmark of the discipline, and she also gave herself a window into her students' thinking and a mechanism for instructional decision making and improvement.

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Appendix 1.A

Monitoring Chart for Parts *a* and *b* of the *Building a Pizza* Task

Strategy	Questions	Who/What	Order
Graph – Plot ordered pairs on a coordinate plane and draw a slope triangle or determine the ratio of rise to run between the two points. Connect the points with a line that includes (0,10).	• *Where is the price per topping represented on your graph?* • *What does this mean in terms of the graph?* • *What does the y-intercept mean in terms of the problem?* • *How could you use your graph to find the cost of any pizza?*	Rashard, Hala, Michael, Candace (G4) Used slope triangle; Talked about the "rate" of \$6 for 3 toppings; saw equivalence to \$2 for 1 topping; found *y*-intercept ***Had trouble explaining the meaning of y-intercept*** but could explain that the graph could be extended to find any cost.	Hala (G4) – 3rd Ask students how G4's solution connects with the solution that G3 had presented.
Table – Make a table that has toppings 0 through 5 (or more) and fill in the cost for 2 and 5 toppings. Then determine that since there is \$6 separating the cost of 2 and 5 toppings, you must just add 2 each time. You can then use this "difference of 2" to complete the table. 0 toppings would then be \$10.	• *Where is the price per topping represented in your table?* • *Where is the price of a plain pizza represented in your table?* • *What do you think the graph of your points would look like?* • *How could you use your table to find the cost of any pizza?*	No one used this approach.	Present Teacher Created Table – 2nd Ask students if the reasoning is sound and how it connects to the first solution.
Reasoning with Arithmetic – Determine that if the 5-topping pizza costs \$20 and the 2-topping pizza costs \$14, then the difference in cost is \$6 and the difference in the number of toppings is 3. 6 ÷ 3 = 2, so each topping is \$2.00. If you subtract 2 from 14 you get \$12, which would be for 1 topping, so 0 toppings would be \$10.	• *How did you find the cost of a plain pizza?* • *How could you find out the cost of any pizza?* • *Could you use your method to find the cost per topping given the price of any two medium pizzas?*	Chris, Ashley, Tyronne, Mirah (G1) Jennifer, Marko, Delmar, Shawna (G3) G2 and G5 later used this approach. Found the cost per topping but ***initially had trouble finding the cost of a pizza with no toppings.*** After getting on track, they were working on a generalization to find cost for any pizza.	Shawna (G3)- 1st
Subtract the Two Amounts – Note that one pizza cost \$20 and the other cost \$14, so subtract and calculate the cost per topping as 6/3 = 2.	• *How much more is a 5-topping pizza than a 2-topping pizza?* • *How many toppings were added?* • *How much do you think it would cost for a 3-topping pizza? Why?*	Aaron, Amber, Sheere, Tamika (G5) ***Initially stated the cost per topping to be \$6*** but after I asked a few questions realized that this would be the cost for 3 toppings.	
Can't Get Started	• *What are you trying to find?* • *What is different about these two pizzas?* • *What happens to the price when you increase from 2 to 5 toppings?* • *How much do you think a 2-topping pizza would cost?*	Yolanda, Jared, Mick, Leslie (G2) Were able to answer questions about what the differences are and had some ideas on how to proceed. When I checked back in they were using a "Reasoning with Arithmetic Approach."	

Appendix 1.B

Monitoring Chart for Part *c* of the *Building a Pizza* Task

Strategy	Questions	Who/What	Order
Symbolic – Cost = \$2t + \$10 t = number of toppings	• *What does each part of equation mean in terms of the context of the problem?* • *If the cost per topping increased, what would change in your equation?* • *If the cost of a cheese pizza increased, what would change in your equation?* • *What do you think your equation will look like when graphed? Why?*	Jennifer, Marko, Delmar, Shawna (G3) Explained each part of equations in terms of context; knew what would change if different pricing. They could describe what the equation looked like when graphed. G4 got this after revising their initial work.	Hala (G4) – 2nd Ask class how G3's equation relates to what G5 described. Ask class to explain each part of equation in terms of context. Relate table, graph, equation and context. All but G3 and G4 had trouble relating the graph and the context.
Narrative – You take the number of toppings times 2 and add \$10.	• *How would you figure out the cost of a pizza with 3 toppings? 10 toppings?* • *Can you write this as an equation? How could you represent the number of toppings? How are the number of toppings and cost per topping related? What happens to the \$10?*	Chris, Ashley, Tyronne, Mirah (G1) Yolanda, Jared, Mick, Leslie (G2) ***Had trouble using variables to express the relationship symbolically.*** G5 got to this after revising their initial work.	Aaron (G5) – 1st
Algebraic – incorrect C = \$10x + \$2	• *How much does a 1-topping pizza cost using your rule? How much does a 2-topping pizza cost?* • *What changes and what remains the same each time?*	Rashard, Hala, Michael, Candace (G4) Once they tried their rule for specific values they found that it did not give them the same information that they found with their graph. They fixed it.	
Can't Get Started –	• *How much would a pizza cost with 1 topping?* • *How much would a pizza cost with 2 toppings?* • *What changes when you add more toppings? What remains the same no matter how many toppings you have? Can you write down what you did?*	Aaron, Amber, Sheere, Tamika (G5) Found cost per topping but were ***not sure how to generalize a rule.*** Ended up writing a rule in narrative form.	