

Enhancing the Learning Environment through Student-Led Mathematical Discussions

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Teachers establish and nurture an environment conducive to learning mathematics through the decisions they make, the conversations they orchestrate, and the physical setting they create. Teachers' actions are what encourage students to think, question, solve problems, and discuss their ideas, strategies, and solutions. The teacher is responsible for creating an intellectual environment where serious mathematical thinking is the norm. More than just a physical setting with desks, bulletin boards, and posters, the classroom environment communicates subtle messages about what is valued in learning and doing mathematics. Are students' discussion and collaboration encouraged? Are students expected to justify their thinking? If students are to learn to make conjectures, experiment with various approaches to solving problems, construct mathematical arguments and respond to others' arguments, then creating an environment that fosters these kinds of activities is essential.

—National Council of Teachers of Mathematics

THIS passage from Principles and Standards for School Mathematics (NCTM 2000, p. 18) suggests a mathematics classroom in which the teacher creates opportunities for students to talk about mathematics in a meaningful way. The authors of Principles and Standards argue that this type of classroom supports all students in learning meaningful mathematics. However, as many teachers know, achieving this kind of environment is often a challenging endeavor. When working with high school mathematics teachers during professional development, we often hear such comments as, "I know that I am supposed to be creating a supportive environment for my students, but I'm just not sure what to do differently."

In this article, we describe a facet of Patty's (coauthor and high school mathematics teacher) classroom practice that fosters students' discourse. We first set the stage with a short description of Patty's school and classroom. We then present an episode from Patty's classroom during which her high school mathematics students take charge of orchestrating whole-class discussion around a mathematical task.

Patty's Classroom

Patty is a high school mathematics teacher for Columbia (Missouri) Public Schools, a district that has approximately 16,000 students in grades K–12. Rock Bridge High School, where Patty is on faculty, has approximately 1,800 students in grades 10–12. Patty is a veteran teacher, having taught mathematics in grades 7–12 for more than thirty years. Patty has been active in many professional development activities throughout her teaching career, most recently as a participant in a professional development project directed by Fran Arbaugh (the other coauthor).

The classroom episode described in this article took place in Patty's high school mathematics class in which the students' textbook is *Contemporary Mathematics in Context*, part of the Core-Plus curriculum. The students enrolled in this Course 3 class were mostly sophomores. Prior to this classroom episode, students had been working in small groups on the task presented in figure 1.1.

The following table shows
the average monthly Fahrenheit temperatures
for Des Moines, Iowa.

Month	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Temperature	19	25	37	51	62	72	77	74	65	54	39	24

- Plot the (month, temperature) data using 0 for January, 1 for February, and so on. On the same plot, sketch a graph that fits the data.
- What function family best fits the data?
- What is the amplitude of the modeling function?
- What is the period of the modeling function?
- Do the data indicate a horizontal shift from the basic toolkit function for this function's family? Do the data indicate a vertical shift?
- Write a symbolic rule that gives average monthly temperature as a function of the month.
- Compare your function rule with the rules of other groups. Resolve any differences.
- Use your function model to estimate the average monthly temperature for the month of April. Compare the temperature as reported in the table with your estimate.

Fig. 1.1 Temperatures task used as basis for student-led discussion
Adapted from *Contemporary Mathematics in Context*, Course 3, Part B (Coxford et al. 2003, p. 467).
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Over the past few years, Patty has been striving to find ways to encourage students' greater involvement in whole-class discussions. She said,

After much exploration, study, and collaboration with others, I finally decided to make a commitment to cultivate more student-centered mathematical authority in my classroom and to hold myself as well as my students accountable for the success of this change. I knew I had to say less and allow my students to process the concepts. I wanted them to think mathematically, not just complete procedures by rote. This meant providing more time for them to collaborate in small groups and allowing them the authority and opportunity to lead whole-class discussions with less input from me. Only by turning over to them the entire experience of learning, would they learn how to learn.

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On the basis of her motivation to strive to include all students in her classroom, Patty developed an environment in which her students took responsibility for leading whole-class discussions. In the next section, we describe an episode from Patty's classroom.

A Peek inside Patty's Classroom

On the day of this episode, Katie and Susan volunteered to lead the discussion for the problem shown in figure 1.1. As they moved to the front of the room, Patty found a place in the back of the room from which to observe and contribute to the subsequent conversation. In the following piece of classroom transcript,¹ we visited Patty's classroom during the student-led discussion about parts b, c, d, and e of the problem. Katie enters data on the overhead graphing calculator and sets the dimensions for the graph to be displayed.

Susan (L):	What function family best fits the data?
Ashley:	It's a periodic function.
Susan (L):	Good job! Does everybody agree it's a periodic function? What kind?
Ashley:	Sine.
Patty (T):	What do you guys think about that so far? It's a periodic function, and it'll be sine?
Zack:	It could be cosine.
Patty (T):	Oh! It could be cosine? (Some chatter ripples around the room discussing this.) Sometimes one will lend itself to an easier match than the other. I'm hearing that we've got votes for sine and cosine. Ladies, part c?

1. In this transcript, discussion leaders are designated with "L" and Patty (the teacher), with "T."

Susan (L): What is the amplitude of the modeling function, and how did you find that? [The question actually reads: What is the amplitude of the modeling function? Susan took the opportunity here to extend the question from part c.]

Lauren: The amplitude equals 29, and I found it because I took the highest point, which is 77, minus the lowest point, which is 19, and I divided it by 2 and that equals 29.

Susan (L): OK! Good job! What about the period?
[Susan is asking about part d: What is the period of the modeling function?] Scott?

Scott: 4π .

Katie (L): How'd you do that?

Scott: Uh, k equals 2π , and I divided that by 0.5

Page: Wouldn't you divide by 2?

Katie (L): That would be for finding the amplitude.

Page: Oh, right. I got it.

This exchange was followed by small-group discussion of period, with students seeking to make sense of the difference between amplitude and period. As discussion slows, Katie went back to Scott's answer of dividing by 0.5.

Katie (L): Scott, can you tell us where you got the 0.5 you used to divide?

Mark: [Mark is Scott's partner] I think we took that 0.47 and kinda rounded up. And well, yeah, we didn't know.

Katie (L): Does anyone know where the 0.5 came from?

At this point, Patty realized that instead of analyzing the graph, Scott and Mark used the regression feature of their graphing calculator to find the amplitude and period, reading from the regression equation.

Patty (T): We're going to back off from the regression equation for the time being. We'll use it as a check later on.

During small-group work on this problem, Patty noticed that the members of Allison's group realized how to find the period from reading the graph.

Patty (T): Allison, I am going to call on you, because when we asked a minute ago about the period, you had a big, sheepish grin on your face.

Allison: It's kind of embarrassing, because I was making it harder than it was. Because you are doing average temperature, and Sneha pointed out that it was twelve months. So that would be the amplitude.

Patty (T): Careful!

Allison: Oh, sorry. Period.

[Some small group discussion follows this exchange.]

Susan (L): [Reading part e of the problem] Do the data indicate a horizontal shift from the basic toolkit function for this function's family? Do the data indicate a vertical shift?

Zack: I said "no" for horizontal and "yes" for vertical.

Matt: It could be both.

Patty (T): You say it could be both?

Matt: If you use the sine, there is horizontal shift.

Patty (T): That was "if"? You said "if" to qualify your answer? What if it is not sine?

Matt: Then it's only a vertical shift.

Patty (T): What do you all think about a vertical shift?

At this point, Katie took over as discussion leader, and referred to the graph showing on the overhead graphing calculator screen as she explained.

Katie (L): If you look at the graph, you see that this is your y-axis, this is your x-axis, [and] this is your origin. So if you look at it, no matter what you do, you will see that there is always a horizontal shift because even if this was cosine, it would still be a horizontal shift because the peak would be over there [pointing to the y-intercept of $y = \cosine(x)$].

Lindsay: What?

Katie removed the graphing calculator projection unit from the overhead and sketched a coordinate plane. On that graph, she sketched the function from this problem.

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| Katie (L): | Like, OK. This is (0, 0). If you move up, then this is the y-intercept of the regular cosine graph. So our graph is shifting. |
| David: | It could be the opposite of a cosine function. |
| Patty (T): | You bet. It could be the opposite. I heard Angela saying that, too. |
| Angela: | It could be a reflection. |
| Patty (T): | Are you getting the feeling that either answer you give, you could make it work for sine or cosine? |
| Class: | [Murmurs around the room – “yes.”] |
| Patty (T): | What about the second part of question e? |
| Susan (L): | If we’re using sine, we’ve already said that there’s vertical shift. There is vertical shift; well, you could do either sine or cosine. |
| Hayden: | There is vertical shift. |
| Katie (L): | Why is there vertical shift? |
| Hayden: | Well, for basic sine, the midline would be at zero. |
| Patty (T): | Where is our midline? |

At this point, Patty sensed that many students needed help in thinking about the vertical shift. She moved on to a short discussion of part e, using one group’s incomplete work on part e to stimulate discussion about vertical shift. Patty asked many questions during this time, prompting students to make connections among what they knew about amplitude, vertical shift, and midline.

Several interesting things happen in this episode, a few of which we note below:

- This discussion involved many students. The environment that Patty established in her classroom encouraged engagement by all students, making it safe to contribute in front of peers.
- Patty did not fade into the background completely during this discussion. Her role as teacher was especially important in helping her students connect important mathematical concepts.
- Katie and Susan took back the lead in the discussion with no prompting from Patty.
- Katie and Susan did not engage in “show and tell.” They prompted reflection, asked open-ended questions, and asked for explanations.

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The roles that the student leaders and Patty played in this classroom are different from teacher and student roles typically found in U. S. mathematics classrooms. Patty, as the teacher, expected Katie and Susan to facilitate discussion, not tell their peers how to solve the problem, and Katie and Susan met that expectation.

Susan met that expectation. In turn, Patty had to yield total authority in the classroom, trusting that her student leaders would facilitate competently.

Incorporating Student-Led Discussions into Your Instruction

If you decide to incorporate student-led discussions in your classroom, an important point to keep in mind is that Patty's and her students' practice did not develop overnight. Patty has worked hard at learning new instructional strategies that improve her classroom environment. In turn, her students have responded to Patty's efforts with cooperation and enthusiasm. "How," you might be wondering, "does Patty get her students to do this?" Patty articulates that process below, and then offers advice for teachers who want to incorporate student-led discussion into their instruction.

Patty starts by explaining how she begins the school year with a new group of students:

In the beginning, this process of change involves me modeling the process of leading discussions. In the first weeks of a new school year, I invite a small group to the front of the room to lead along side of me. I hand over the marker to one of my students to record all thoughts suggested from the class, promising the volunteer group and the class that being in the front of the class is the safest place to be. I tell them, "You only write down what others tell you." Then I begin modeling how to ask questions. I have one of the student leaders read the problem. (Remember, the class members have already had the opportunity to work on the assigned problem in their small groups.) I then ask the class where they started with the information and what they did to solve it. Over the course of the next couple of weeks, I begin to move away from the front of the room when students are leading discussions, removing myself from the focal point of the class. I direct less and less each class period. My contributions to the conversations from then on are more questioning and encouraging, specifically highlighting what the students have done well—pointing out that they do not need my input, that they have all they need to solve the problems.

A straight path does not exist to the point where your students engage in the types of discussion described previously in this article. Some days your students will do well with this organization; other days they may be a bit contentious about the whole idea. Be patient! Remember that your goal is to include more and more students in whole-class discussions, and that change—for you and your students—takes time.

On the basis of her experiences, Patty has some encouraging words of advice for teachers who try this strategy in their classrooms:

- Just try it! Start by choosing a problem for which the answer is not readily evident, that requires some problem-solving strategies and is not simply procedural. Have students work in small groups on the

Remember that your goal is to include more and more students in whole-class discussions, and that change—for you and your students—takes time.

Develop high expectations for students. They can assume more responsibility for their own learning, be actively involved, learn to collaborate with others in small and large groups, and lead large-group discussions.

When students lead class discussions, they justify their own thinking, examine the ideas of others very carefully, and become critical friends to one another. They learn to rely on themselves as the mathematical authority.

problem, and then invite a group to the front to lead the discussion as described in the foregoing.

- You might have to change your mindset and let go of expectations that hold you back. For example, I had to let go of my desire for total control over the pace and direction of discussion. As I let go, I realized that my students sometimes took us to places mathematically that we would not have gone had I maintained total control.
- This type of student-centered learning is messy and slow. When students explore a situation, really grappling with the mathematics, you may find that the pace of the class slows. I have learned to do the same work we did before, but in a different fashion.
- Be patient, and keep trying. Have patience with yourself and with your students. This process will always be a work-in-progress, at times messy, time-consuming, and noisy.
- Develop high expectations for students. They can assume more responsibility for their own learning, be actively involved, learn to collaborate with others in small and large groups, and lead large-group discussions.
- Welcome wrong ideas and answers—making errors is not failing; those errors create a great opportunity during a lesson. If the leaders make a mistake, their classmates will usually notice; allow students to find the leaders' error and suggest corrections.

As Patty incorporated more student-led discussions into her mathematics classroom, she noted changes in her students over the years. The following statements reflect the positive outcomes you may see from your students.

- Students learn to lead large-class discussion by asking questions, not by doing a “show and tell” act.
- Students learn leadership skills. They learn to think on their feet, improve their public-speaking skills and their interpersonal skills, display a certain amount of courage, develop confidence in their preparation for leading, learn how to prepare, and develop empathy for the mathematical struggles of their classmates.
- Overall, students in a class with this type of environment learn to be self-reliant and to collaborate in small groups. They learn to articulate and justify their own thoughts, listen to others, communicate and reflect on their thinking orally and in writing, read technical material, articulate their own questions, and be advocates for their own learning.

When students lead class discussions, they justify their own thinking, examine the ideas of others very carefully, and become critical friends to one another. They learn to rely on themselves as the mathematical authority.

Having students lead whole-group discussion is just one of the ways that Patty seeks to include all learners in engaging in mathematics during class. One other strategy Patty uses is to have students work in small groups regularly, encouraging all members of each group to contribute to

the small-group discussion and thinking. Patty believes that this practice opens up space for students who might be shy when speaking in a whole-group format. For example, Patty remembers a student named Madison, whose first language was Spanish. At the time that Madison was in Patty's class, she had lived in United States for two years and was continuing to develop fluency in English. Madison was shy and reserved in the class. Patty observed that, during whole-class discussions, Madison was most often an observer. "However," Patty says,

I noticed Madison being much more open and involved in her small group than when we worked as a large group. At times after students had had time to process with their group partners, I even noticed her speaking out in large-class discussion with confidence. She had already had the chance to process her thinking with her group and to share her ideas and receive feedback from her partners. This small-group time gave Madison the opportunity to see how the language was used and to adapt it to her own thinking. She was a sponge with her partners, and this type of collaborative learning strengthened her language skills as she became accustomed to the language usage.

Patty believes that small-group work is supportive not just of English language learners but of any student who may have difficulty talking out in large-group discussion. An example is Kayle, a student who had serious brain surgery a few years before she was in Patty's class. Patty describes Kayle as "a bright student, but one who felt an extreme amount of stress when in the spotlight." Because of this feeling of stress, Kayle would not come to the front of the room to lead discussions. However, as Patty describes,

Kayle was very comfortable in her "home group" and was a strong and very trusted contributor to their work. At times during whole-class discussion, I would hear Kayle make a quiet comment, sharing insight or an answer. When this happened, I would highlight her good ideas in the whole-class discussion. For Kayle, having the familiar group of buddies to try out her own thoughts was a good starting point.

Patty finds this tendency to be true of all her students, most of whom need to build their mathematical confidence with the support of others.

Conclusion

Professional Standards for Teaching Mathematics (NCTM 1991) contains recommendations for teachers and students with regard to classroom discourse (p. 45):

The teacher of mathematics should promote classroom discourse in which students —

- listen to, respond to, and question the teacher and one another;

Small-group work is supportive not just of English language learners but of any student who may have difficulty talking out in large-group discussion.

- use a variety of tools to reason, make connections, solve problems, and communicate;
- initiate problems and questions;
- make conjectures and present solutions;
- explore examples and counterexamples to investigate a conjecture;
- try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers;
- rely on mathematical evidence and argument to determine validity.

Incorporating student-led discussions into your mathematics instruction can be an effective instructional strategy for realizing this type of environment—one in which every student has the opportunity to contribute and learn.

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