

“ We use the five practices as a framework to help teachers see how they can they stop the stand-and-deliver traditional method and really be able to implement a structure in their classes so that the discourse can come out and come alive. ”

—LAURA MALY, MATHEMATICS SPECIALIST,  
MILWAUKEE PUBLIC SCHOOLS

“ The five practices is really the centerpiece of our professional development. We frame our discussions around how we can create good conversations so that all kids are advancing in their thinking about a problem. ”

—JOE GIERA, MATHEMATICS COACH,  
SCHOOL DISTRICT OF SOUTH MILWAUKEE

# CHAPTER 1

# Introduction



At the heart of efforts to help students in Grades 9–12 learn mathematics is the idea of *ambitious teaching*. It's referred to as *ambitious* because of the substantial student learning goals that it encompasses—that all students have opportunities “to understand and use knowledge ... [to] solve authentic problems” (Lampert & Graziani, 2009, p. 492). The Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) provide a powerful vision of these goals through their description of grade-level, domain-specific content standards and the cross-cutting Standards for Mathematical Practice.

We believe that the phrase *ambitious teaching* is also appropriate because teaching in ways that align with these goals is a formidable task! To help you and other teachers understand what this looks like, *Principles to Actions: Ensuring Mathematical Success for All* (National Council of Teachers of Mathematics, 2014) describes a set of eight teaching practices that serve as a foundation for ambitious teaching (Figure 1.1). These practices are based on what we know from research about how to effectively support students' learning of mathematics.

## Figure 1.1 • Eight effective mathematics teaching practices

<b>Establish mathematics goals to focus learning.</b> Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
<b>Implement tasks that promote reasoning and problem solving.</b> Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
<b>Use and connect mathematical representations.</b> Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
<b>Facilitate meaningful mathematical discourse.</b> Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
<b>Pose purposeful questions.</b> Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense-making about important mathematical ideas and relationships.
<b>Build procedural fluency from conceptual understanding.</b> Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
<b>Support productive struggle in learning mathematics.</b> Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
<b>Elicit and use evidence of student thinking.</b> Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Source: Reprinted with permission from *Principles to Actions: Ensuring Mathematical Success for All*, copyright 2014, by the National Council of Teachers of Mathematics. All rights reserved.

Ambitious teaching also requires attention to equity. Mathematics has long been considered a gatekeeper, limiting opportunities for some students while promoting opportunities for others (Martin, Gholson, & Leonard, 2010). These differences in opportunities are apparent in high schools where students who are Black, Latinx, American Indian, emergent multilingual, or living in poverty are overrepresented in classrooms that focus on learning and practicing procedures (Oakes, 2008), while Asian and White students are more likely to be assigned to higher-level classes where they have the opportunity to engage in thinking, reasoning, and problem solving around cognitively challenging tasks (White, Fernandes, & Civil, 2018). Ambitious teaching requires you to challenge these long-standing practices and provide access and opportunity for all students so that they can develop strong positive identities as learners of mathematics (Aguirre, Mayfield-Ingram, & Martin, 2013).

In their most recent policy statement, the National Council of Teachers of Mathematics (NCTM, 2018) stated that the practice of tracking high school students into “qualitatively different or dead-end course pathways” should be discontinued. Eliminating tracking has been successful in strengthening overall mathematics outcomes and providing

stronger mathematics pathways for historically underrepresented groups in places such as San Francisco and Escondido, California. Moreover, NCTM states that the practice of teacher tracking—assigning the most experienced teachers primarily to courses with higher-achieving students—should also end. All students should have opportunities to learn high-quality, grade level–appropriate mathematics in classrooms where teachers are enacting research-informed ambitious teaching practices. If your school is engaged in the practice of tracking, now is the time to begin conversations with school and district administrators on how to ensure that each and every student has access to high-quality mathematics instruction. (For information on how to begin such conversations and current efforts to eliminate high school tracking, go to <https://www.nctm.org/change/>.)

At the center of ambitious teaching is a focus on classroom discourse. As you facilitate meaningful discussions with students, you will typically engage in several of the effective mathematics teaching practices, including asking purposeful questions, eliciting and using evidence of student thinking, connecting to various mathematical representations, and supporting productive struggle among students as they learn mathematics (Figure 1.1). In addition, allowing students to share their thinking with the class can help to position all students as valuable resources for learning and promote an equitable learning environment. In these ways, organizing discussions around students’ ideas becomes critical for successfully enacting ambitious instruction.

What does it take then to organize and implement effective discussions? In this book, we present guidelines for using the five practices described by Smith and Stein (2018) in their book *5 Practices for Orchestrating Productive Mathematics Discussions*.

## The Five Practices in Practice: An Overview

The five practices are a set of related instructional routines that can help you design and implement lessons that address important mathematical content in ways that build on students’ thinking (Figure 1.2). Warning: There is actually a Practice 0, which serves as a foundation for the remaining practices—yup, this means there are six practices in total, but for historical reasons, we will still call the set “the five practices.” (In case you are wondering how this could have happened, here is the scoop: After some early articles about the five practices were published, a mathematics coach with whom Peg was working suggested to her that a practice was missing—that before teachers could engage with the five practices, they needed to set goals and select a task. Though this idea was already implied in the five practices, the coach persuaded Peg to make it explicit, and hence Practice 0 was born!)

**Figure 1.2 • The five practices in practice**

Practices that take place while planning for instruction		<b>Practice 0: Setting goals and selecting tasks (Chapter 2)</b> Specifying learning goals and choosing a high-level task that aligns with those goals
		<b>Practice 1: Anticipating student responses (Chapter 3)</b> Exploring how you expect students to solve the task and preparing questions to ask them about their thinking
Practices that take place during instruction but are considered while planning	Students work individually or in small groups	<b>Practice 2: Monitoring student work (Chapter 4)</b> Looking closely as students work on the task and asking questions to assess their understanding and move their thinking forward
	As you move from small group work to whole class discussion	<b>Practice 3: Selecting student solutions (Chapter 5)</b> Choosing solutions for students to share that highlight key mathematical ideas that will help you achieve lesson goals
		<b>Practice 4: Sequencing student solutions (Chapter 5)</b> Determining the order in which to share solutions to create a coherent storyline for the lesson
Whole class discussion	<b>Practice 5: Connecting student solutions (Chapter 6)</b> Identifying connections among student solutions and to the goals of the lesson that you want to bring out during discussion	

Teachers often think that ambitious teaching requires you to make all your instructional decisions during instruction based on what students say and do in class. The five practices, however, help you think through all aspects of the lesson *in advance* of teaching, thus limiting the number of in-the-moment decisions you have to make during a lesson. Careful planning prior to a lesson reduces what you need to think about during instruction, allowing you time to listen more actively, question more thoughtfully, and respond more acutely.

Practice zero, *setting goals and selecting tasks*, lays the groundwork for the remaining five practices. It is essential to be clear on what you want students to learn and to choose a cognitively demanding task that aligns with those goals. Once you have the task in mind, you can move to *anticipating student responses*. Here, the purpose is to think about how students might solve the problem, what challenges they might face, and how you will respond to their thinking. One benefit to doing so is that you can develop—before class—targeted questions you might want to ask students about these different approaches.

Although the next four practices take place during instruction, you will also want to think them through carefully during planning. *Monitoring student work* involves giving students time—usually in groups—to work on the task, while you circulate among them. As you look closely at how students are progressing, you can use the questions you developed earlier to assess what students understand and to try to move their thinking forward. As you prepare to transition students into a whole class discussion,

you will engage in *selecting student solutions*—deciding which solutions you want to have shared in the discussion and who should present those solutions—as well as *sequencing student solutions*—deciding how you want to order the presentation of the solutions. Selecting and sequencing require close attention to the mathematical ideas that are highlighted in different solutions and to helping all students have access to the ideas shared in the discussion. As you plan the lesson you will consider what you want to be on the lookout for as you monitor students’ work, what solutions will help you surface the mathematical ideas you are targeting, and what order of solutions will provide access to all students.

The final practice, *connecting student solutions*, takes place as the discussion unfolds in your classroom. The purpose is to make explicit the connections between students’ solutions and the mathematical goals of the lesson. Drawing out these connections for students is essential to ensure that students take away from the discussion what you intended. This too is something you can consider as you plan the lesson!

Together, the five practices can help you prepare for and carry out meaningful discussions with your students, discussions that revolve around the thinking of your students. And that is the essence of ambitious instruction!

## Purpose and Content

The purpose of this book is to deepen your understanding of the five practices as described by Smith and Stein (2018). Toward that end, Chapters 2 to 6 comprise two parts: unpacking the practice (Part One) and challenges teachers face in enacting the practice (Part Two). In Part One, we describe in some detail what is involved in engaging in the practice, provide questions that you should ask yourself as you undertake the practice, and use an example from a high school classroom to illustrate the components of the practice. In Part Two, we highlight aspects of the practice that have proven to be challenging for teachers, suggest ways you can address the challenge, and provide examples of how teachers are overcoming the challenge.

Throughout these chapters, we encourage you to actively engage with the content. Toward this end, we have created three types of opportunities for engagement: *Pause and Consider* questions (reflection), *Analyzing the Work of Teaching* activities (analysis), and *Linking the Five Practices to Your Own Instruction* assignments (implementation). The Pause and Consider questions give you the opportunity to think about an issue, in some cases drawing on your own classroom experience, prior to reading more about it. The Analyzing the Work of Teaching activities engage you in analyzing aspects of teachers’ planning for and enacting of grade-level lessons. The Linking the Five Practices to Your Own Instruction assignments provide you with the opportunity to put the ideas discussed in the chapter to work in your own classroom.

Throughout the book we have included a range of different types of examples drawn from high school classrooms to illustrate aspects of the five practices and the associated challenges. The video excerpts and related classroom artifacts—featuring the three teachers who are introduced later in the chapter—provide vivid images of real teachers using the five practices in their efforts to orchestrate productive discussions. These three teachers illustrate the practices and challenges in each of the chapters that follow. In addition, in Chapter 5 we provide authentic descriptions and artifacts from the work of two additional teachers with whom we have worked—Travis Lemon and Martha Mulligan. Other narrative examples that appear in the book are based on our experiences working with high school teachers through professional development initiatives and teacher education courses. These examples are intended to provide insights into specific challenges teachers face when engaging in the five practices and are not exact representations of a specific teacher’s practice. Each of these teachers has been given a pseudonym (e.g., Grace Sullivan, George Jacobson, Kendra Nelson, and Denise Hansen featured in Chapter 2). The video and narrative examples are not intended as exemplars to be copied but rather as opportunities for analysis, discussion, and new learning.

If you are coming to the five practices for the first time, you might find it helpful to start with *5 Practices for Orchestrating Productive Mathematics Discussions* by Peg Smith and Mary Kay Stein (2018). Smith and Stein’s book offers a wonderful, easy-to-read introduction to and overview of the five practices. The book you are reading now takes a much deeper dive into the five practices, asking you to stop and think, watch videos of the practices in action, and consider what is challenging about each practice. While you can certainly start here, the overview of the five practices provided by Smith and Stein (2018) may help you get the big picture before taking a deeper dive!

This book will be a valuable resource for looking closely at what it takes to be successful with the five practices. For each practice, we offer key questions, which identify the essential components of the practice. We suspect these questions will enhance your understanding of the practices and perhaps provide new information about the goals and expectations for each practice. This book also describes challenges associated with each practice that teachers we have worked with have encountered, as well as specific suggestions for successfully addressing these challenges. If you have already been using the five practices, we suspect that some of these challenges may be familiar to you and that these discussions will be particularly useful.

## Classroom Video Context

In identifying teachers to feature on video, we felt that it was important to select teachers who were working hard to improve mathematics teaching and learning in their high school classrooms. We selected teachers who

were participating in the Milwaukee Master Teacher Partnership for several reasons. Michael Steele, the second author of this book, is the director and principal investigator of this five-year project funded by the National Science Foundation and works closely with the 24 teacher-participants, all of whom work in urban high schools and are committed to improving learning outcomes for their students.

The greater Milwaukee area has many features of a typical large urban education system but is also unique in a number of ways. From the 1900s through the late 1960s, the City of Milwaukee has geographically shifted from a single municipality to a series of smaller towns surrounding the central city. This factor and others have resulted in Milwaukee being one of the most racially segregated cities in the United States (Brookings Institution, 2018). The region has also been a central focus of the charter and voucher school movements, which have fragmented both the student and teacher populations in significant ways. One of the responses to this privatization has been a push to specialize high schools in Milwaukee as thematic magnet schools with specialized curricular opportunities. Changes to labor and union laws have contributed to more significant teacher mobility between schools and districts. Despite these challenges, the region has a strong legacy in mathematics education. Major research and professional development partnerships have served the region since the early 1990s, including the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) project and a two-decade series of teacher professional development projects focused on ambitious teaching through the University of Wisconsin-Milwaukee and their partners. Building on a base of content, pedagogy, and leadership learning related to elementary and middle grades mathematics, recent efforts in the past six years in Milwaukee have bridged into serving high school teachers and their students.

One such project, the Milwaukee Master Teacher Partnership, is a five-year partnership between the University of Wisconsin-Milwaukee, Milwaukee Public Schools, and the School District of South Milwaukee.<sup>1</sup> The goal of the partnership is to enhance teachers' content knowledge, to strengthen their pedagogical skills, and to develop their capacity for leadership. Toward that end, teachers across schools and districts work together to engage in inquiry about their classroom practice. Teachers' work in the partnership is centered on repeated cycles of action research, focusing on some small pieces of their classroom practice such as the tasks they choose to use in the classroom or the questions they decide to ask students. For example, teachers may decide to use different types of tasks or ask different kinds of questions and then investigate the impact the changes they have made had on student learning. Teachers then come

---

<sup>1</sup> The Milwaukee Master Teacher Partnership project is funded by a grant from the National Science Foundation (NSF), awards 1540840 and 1557397, to the University of Wisconsin-Milwaukee. The content of this book does not necessarily reflect the views of the NSF.

together and share what they have tried and what they have learned. According to Steele,

*We equip teachers with the skills to rigorously conduct that inquiry in the classroom—not just say, “Well, you know, I noticed that more kids seem to be paying attention,” but to really start to capture some data that they can analyze and say, “Yeah, this is changing the way my students learn.”*

In the video clip that follows, Dr. Steele discusses how he has woven the five practices into the professional development in which project teachers participate and the impact of this work on student learning and engagement.



## A Professional Community Engages in the Five Practices



### Video Clip 1.1

In Video Clip 1.1, Michael Steele, director and principal investigator of the Milwaukee Master Teacher Partnership, explains how the five practices align with the work of the project and the impact they are having on teaching and learning.



To read a QR code, you must have a smartphone or tablet with a camera. We recommend that you download a QR code reader app that is made specifically for your phone or tablet brand.



Videos may also be accessed at [resources.corwin.com/5practices-highschool](https://resources.corwin.com/5practices-highschool)

Dr. Steele suggests that the five practices can also have an impact more broadly on issues of equity and identity. He explains:

*One of the persistent challenges of high school mathematics has been that we've tended to sort and track kids into the haves and the have-nots. And one of our big initiatives as a profession is to really help teachers and school districts understand that that practice has actually been detrimental in promoting stronger student achievement for all kids. So when we think about putting diverse groups of students together, who have different mathematical identities, who may have or may have not struggled with mathematics in the past, the five practices helps a teacher think about “How do I provide multiple entry points for a student? How do I elevate student voices for all*

*the students in my classroom? How do I focus less on students' past performance and achievement and more on the mathematical ideas that my students who are in here today are doing and saying, and then how do I make use of those mathematical ideas in advancing the goals of my lesson?" It actually equips the teachers with a broader range of tools because now, as a teacher, I'm not just relying on the good thinking that I know three or four of my kids who are high achievers are walking in with. It opens the door to making use of a broader range of student thinking, of elevating more student voices and positioning each and every student in the classroom as a capable mathematical learner.*

## Meet the Teachers

The video recordings and related classroom artifacts featured in this book are drawn from the work of three high school teachers who are participating in the Milwaukee Master Teacher Partnership—Matthew Harmon, Michael Moore, and Cori Moran. The lesson taught by Cori Moran will be used in Part One of Chapters 2 to 6 to unpack the focal practice. By focusing on the same teacher across chapters, you will have a coherent picture of instruction in her classroom and a better understanding of how the practices provide synergy. The lessons taught by Matthew Harmon and Michael Moore will be used in Part Two of Chapters 2 to 6 to provide illustrations of how specific challenges can be addressed.



**Matthew Harmon** has been teaching mathematics at Rufus King International High School in the Milwaukee Public Schools district since he began his career 11 years ago. He has a bachelor's degree in sports management and a master's of education in educational policy and leadership with a focus on curriculum, and he is certified to teach secondary mathematics. He

became a teacher because he liked working with kids and he wanted a job that had a meaningful purpose. He has great passion for his job and as he explains, "It makes everything associated with it a little easier."

Matthew wants all of his students to have a positive experience with mathematics education, gaining confidence in their skills and seeing the usefulness of the mathematics they are learning. Toward that end, he has been selecting rich tasks with multiple entry points so that students can access a task and experience success at some level. Then he can move a student forward from wherever they are in their understanding.

Matthew sees the five practices as playing a vital role in preparing for instruction. He explains:

*As I incorporate the five practices into my instruction, I've learned that having a clear learning target in mind in the planning process*

*really allows you to come up with questions that drive precisely towards the goal of the lesson. It just really makes a lot of sense to be able to plan for anticipated responses and think about questions that you're going to use to move students towards the goals. And then basically, putting students up there, displaying their work, discussing it, being challenged by other students—that's really when the depth of their understanding is on display. So the five practices gets me to just be much more thoughtful and plan about what I intend to do.*

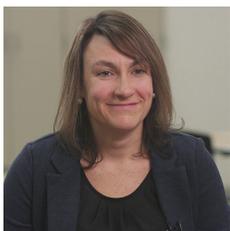


**Michael Moore** has been teaching at Ronald Wilson Reagan College Preparatory High School in the Milwaukee Public Schools district since he started teaching 11 years ago. He has a bachelor's degree in secondary mathematics education and a master's of education in teaching, learning, and leadership. He became a teacher because he sees it as both rewarding and challenging. He enjoys how quickly each day goes when he is engaged in conversations with students about their ideas.

Michael feels that learning mathematics in his classroom should be fun and engaging for students. He wants students to be challenged to justify their work by sharing their thoughts publicly, which he feels will in turn help other students understand different ways of thinking. Michael explains that task selection is critical in achieving this outcome because it allows students to buy into the idea that mathematics is exciting.

Michael explains how the five practices can make you more aware of your actions as a teacher:

*I think every single teacher on the face of the earth struggles with not telling, right? That's really, really, really hard. I think that working with the five practices definitely makes you aware of how often you're leading students, how much you're wanting to control the work, and it really makes you take a step back and value what they have to say and makes you value the fact that they can learn probably more from each other than they can learn from you just talking at the front of the room. If you try to make it a point every day to let the students do the explaining, it becomes routine.*



**Cori Moran** teaches at South Milwaukee High School in the School District of South Milwaukee, a position she has held for the past two years. She previously taught in Milwaukee Public Schools for nine years. She has undergraduate degrees in secondary mathematics education and American sign language and a graduate degree in curriculum and instruction. Cori enjoyed the challenges of mathematics and appreciated the support she received from caring mathematics teachers. Because of her love for learning, her love of

helping others, and her connections with her own teachers, she always felt that teaching was her path.

Cori's goal as a teacher is to develop a mathematics community in her classroom where students respect and support each other using Restorative Practice Circles as a framework. She takes her role as a mathematics teacher seriously because mathematics has been shown to be a gatekeeper for student potential and she sees it as a privilege to guide students on their journey in life. She wants to ensure that "the gate is wide open for them and I have added no barriers, and even better, removed some." In her view, the key to accomplishing this is to build trusting relationships with her students.

The five practices have helped Cori feel like a more prepared teacher. As she explains,

*One of the things I have learned is that prior to using the five practices, I did not plan enough. I feel like I'm now a prepared teacher and that my students get a well-prepared lesson. The five practices gave me a tool that helped me direct my planning towards my goals for the lesson. I can now see that I can sequence student solutions in a way that really reaches the goal. And for me, I really enjoy that whole storytelling and connecting the students with the work so that they feel they are part of it. And they also still get to that goal and get to share what they are thinking. I think that the most important part of using the five practices is that you really can see what students can do when they're given that opportunity.*

These three teachers are making their teaching practice public so that others can learn from their efforts. Hiebert, Gallimore, and Stigler (2003) argue that we must respect teachers for being "brave enough to open their classroom doors" (p. 56). To honor their courage, as you read about and view excerpts from their classrooms, we encourage you to avoid critiquing what you see or discussing what the teacher "should have done." Instead, our goal is to use the access we have been given as an opportunity for learning—for serious reflection and analysis—in an effort to improve our own teaching in ways that open up new opportunities for our students to learn.

## Using This Book

You will likely get the most out of this book if you are committed to ambitious teaching that provides students with increased opportunities to engage in productive discussions in mathematics classrooms. Through engaging with the ideas in the book, you will learn much about how to increase students' engagement in and learning from classroom discussions.

This book can be used in several different ways. You might read through the book on your own, stopping to engage with the questions, activities, and assignments as suggested. Alternatively, and perhaps more powerfully, you can work through the book with colleagues in professional learning communities, department meetings, or when time permits. The book would also be a good choice for a book study with a group of peers interested in improving the quality of their classroom discussions. You might also encounter this book in college or university education courses for practicing or preservice teachers or in professional development workshops during the summer or school year. We will explore more ideas about ways to make the five practices central to your instruction in Chapter 7.

## Norms for Video Viewing

The video excerpts that accompany this book are intended to provide authentic examples from high school classrooms on which to base discussions of the five practices. To take full advantage of these examples, we encourage you to consider the following three norms for video viewing. These norms are based on recent research that documents how video can support teacher learning and reflection (Sherin & Dyer, 2017; Sherin & van Es, 2009).

**Focus on student sense-making.** The majority of the video clips that you will watch in this book focus on students. That is intentional. While the five practices describe actions that you as the teacher will take, this work involves looking closely at what students do and say. The videos thus provide an opportunity for you to do just that outside of the immediate demands of teaching.

As you explore students' actions in the videos, we encourage you to look beyond simply whether a student's idea is correct or incorrect. Instead, examine what it is that the student understands. What is the student's idea? Where does it come from? Why is it sensible, given what the student understands? Focus on what it is that makes sense about the students' thinking.

**Be specific about what you notice.** Much of the value of video viewing is the sense that you can slow down classroom interactions and have the time to notice what is taking place in a detailed way. In addition, with video you can often focus on just a subset of events and look closely, for example, at what a particular student is saying and to whom, what gestures or drawings the student is making, and more.

As you view the video excerpts, we encourage you to be specific about what you notice. Provide detailed evidence to support your claims about what is happening. Explain what it is you see in the video that leads you to a particular interpretation.

**Consider alternative interpretations.** As you watch the video, you may find yourself quickly making assumptions about what is taking place and why. As teachers, we must often respond quickly, diagnosing student confusions, responding to student questions, and making changes in the direction of a lesson. Video, however, provides the luxury of time. Use this to your advantage!

Once you have an idea of what you think is taking place in the video, look for alternatives. How else might you understand what is happening? This is particularly important when examining students' ideas. Rather than assume you know the reason behind a student's strategy or statement, look for alternatives. Considering alternate interpretations is important because when we assume we understand what a student means, we often limit ourselves to what we have heard from students previously.

## Getting Started!

You are now ready to begin a deep dive into the five practices. In the next five chapters, you will learn more about the practices. We encourage you to keep a journal or notebook in which you can respond to questions that are posed and make note of questions you have. Such a journal can be helpful in conversations with other teachers or in reflecting from time to time about how your thinking is evolving and changing.