Planning for Student Learning of Mathematics in High School

Mathematics is a conceptual domain. It is not, as many people think, a list of facts and methods to be remembered.

—Jo Boaler

The first critical question of a PLC is, What do we expect all students to know and be able to do? (DuFour et al., 2016). As your collaborative team successfully answers this question for each unit of study, you build a common understanding of the mathematics students will learn in your course. What is the mathematics story that unfolds as student learning progresses from one mathematics unit to the next? How do the units fit together and build upon one another within and across the first three years of high school mathematics?

Guaranteed and Viable Curriculum

Your high school team effectively backward plans the school year by grouping essential mathematics standards into units to create the guaranteed and viable mathematics curriculum students must learn (Marzano, Warrick, Rains, & DuFour, 2018; Wilkins, 1997). The order you teach the units provides the framework for your course's mathematics story. Within each unit, your daily lessons create the beginning, middle, and end for that part of the story.

Thus, evidence of your team's guaranteed and viable curriculum includes (1) semester- and yearlong pacing plans of standards (proficiency maps or pacing guides), (2) unit plans, and (3) daily mathematics lessons. The graphic in figure 1.1 illustrates these areas of team planning for a mathematics guaranteed and viable curriculum. The thick black vertical line down the middle shows the end of semester 1 and the start of semester 2 for your team (keep in mind the number of units in

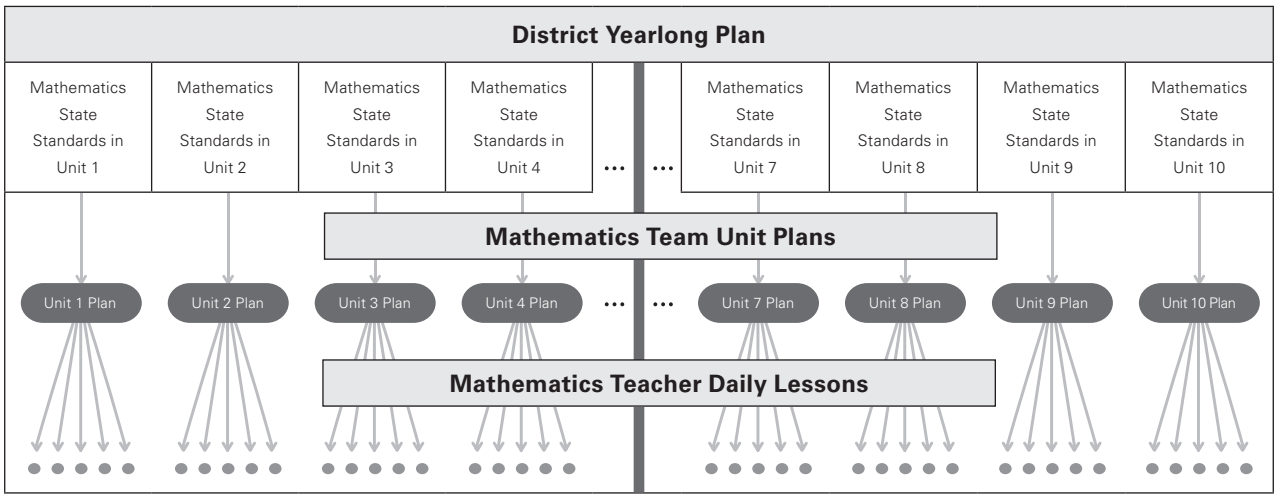

Figure 1.1: Mathematics guaranteed and viable curriculum plan.

each of your semesters may vary from that shown in figure 1.1).

Together, the mathematics units of study tell the story about the course standards students are expected to learn throughout the semester and the academic year, and from one school year to the next.

As figure 1.1 (page 9) shows, a district yearlong pacing guide or proficiency map (showing a timeline for student proficiency with each mathematics standard) first defines your course-based team's guaranteed and viable curriculum. Your team then determines a time frame appropriate for each mathematics unit of study, typically two to four weeks in duration for high school courses. This process eliminates the potential risk of running out of time and skipping end-of-the-year units that address essential standards.

If your collaborative team does not have a yearlong plan with standards in clearly defined units, see appendix A (page 121), "Create a Proficiency Map," for additional support. Helping each teacher on your team become comfortable with the progression of mathematics units throughout the school year will support your students' understanding of the mathematics story arc for various standards.

Mathematics Unit Planner

Once your high school course team determines the mathematics units for your course (detailing the standards and time line for each unit), your team next plans for student learning on a *unit-by-unit* basis (see figure 1.2; Kanold & Schuhl, 2020).

The Mathematics Unit Planner in figure 1.2 provides a template for your course-based team to use as you develop a shared understanding of what students are expected to learn in each unit of study. The numbered sections in the Mathematics Unit Planner correspond with the seven elements of unit planning described in the introduction (page 1). In the upcoming chapters, you will see headings that correspond with these seven areas. You will also find completed examples of unit planners for algebra 1 in figure 3.11 (page 59), geometry in figure 4.10 (page 83), and algebra 2 in figure 5.11 (page 113).

In *Principles to Actions,* researchers for the National Council of Teachers of Mathematics (NCTM; 2014) note, "Effective mathematics teaching begins with a shared understanding among teachers of the mathematics that students are learning and how this mathematics develops along learning progressions" (p. 12). Therefore, as your mathematics department and course-based teams develop unit plans for the year, be sure to make sense of the mathematical content standards students are learning in each semester in your course. Additionally, make sense of the mathematical content trajectories (progressions) students are learning throughout your high school courses.

Mathematics Concepts and Skills in High School

Students in high school deepen their understanding of number, algebra, functions, geometry and measurement, and statistics and probability situations using content from these domains in order to develop a conceptual understanding of mathematics. Mathematical modeling should permeate the content students learn *and* support teaching mathematical process standards that develop the necessary habits of mind for successful student mathematical reasoning. These *mathematical process standards* for learning are an important aspect of instruction, a component of higher-level tasks, and embedded in the strategies and tools students use to problem solve.

Students develop a conceptual understanding of high school mathematics from prior knowledge learned in middle school related to ratio and proportional reasoning, linear equations, expressions and equations with rational numbers, geometry, and statistics and probability. Connections from one course to the next throughout high school prepare students to be college and career ready by the time they graduate. Students in your courses learn higher-order reasoning skills and how to apply learning in novel and new situations through the use of higher-level cognitive tasks and modeling.

Most current high school state mathematics standards were generated from the recommendations of NCTM's *Principles and Standards for School Mathematics* (2000), the Governors' task force for Common Core State Standards (NGA & CCSSO, 2010), and recommendations from *Catalyzing Change in High School Mathematics* (NCTM, 2018). Table 1.1 (page 12) shows some of the key mathematics concepts students are expected to learn in high school over the

Unit: _____

Start Date: _____ End Date: _____ Total Number of Days: _____

Unit Planning	
1 Essential Learning Standards	List the essential learning standards for this unit.
3 Prior Knowledge	List standards from a previous unit or grade students will access in this unit.
4 Vocabulary and Notations	List the mathematical academic vocabulary and notations for this unit.
5 Possible Resources or Activities	List the possible resources or activities to use when teaching the essential learning standards.
6 Tools and Technology	List the essential tools, manipulatives, and technology needed for this unit.
7 Reflection and Notes	After the unit, reflect and list what to do again, revise, or change.

2 Unit Calendar

	Monday	Tuesday	Wednesday	Thursday	Friday
Week 1					
Week 2					
Week 3					
Week 4					
Week 5					

Source: Adapted from Kanold & Schuhl, 2020, p. 30.

Figure 1.2: Mathematics Unit Planner.

Visit **go.SolutionTree.com/MathematicsatWork** *for a free reproducible version of this figure.*

Table 1.1: Mathematics Concepts and Skills in High School

	High School: Algebra 1, Geometry, and Algebra 2
Number	• Use units to understand problems and model with mathematics. • Extend an understanding of number to include irrational numbers and rational numbers. • Extend the properties of exponents to rational exponents. • Perform arithmetic operations involving complex numbers.
Algebra	• Interpret the structure of expressions and use structure to rewrite equations in different forms. • Create equations and inequalities and use them to solve problems. • Understand the process of solving equations and inequalities, including systems of equations, and explain the reasoning used for each. • Understand operations (addition, subtraction, multiplication, division) and apply them to polynomials. • Analyze and make predictions using expressions, equations, and inequalities.
Functions	• Understand the concept of a function and use function notation. • Understand that functions within the same family have distinguishing attributes. • Interpret key features of functions and relate the domain and range to context, if applicable (linear, quadratic, absolute value, exponential, square root, cube root, piecewise, step, polynomial, logarithmic, trigonometric). • Analyze and compare functions represented symbolically, graphically, numerically in tables, or verbally in descriptions. • Build a new function that represents a relationship between two quantities or one from existing functions using transformations. • Interpret expressions for functions based on the situations they model. • Find inverse functions. • Model periodic phenomena with trigonometric functions.
Geometry and Measurement	• Define, represent, describe, and draw transformations that preserve distance and angles and those that do not (rotation, reflection, translation, dilation). • Create geometric proofs related to congruence by first understanding congruence as the result of rigid motions. • Prove geometric theorems algebraically using coordinates. • Prove theorems about similarity by first understanding similarity in terms of similarity transformations. • Solve problems involving right triangles using trigonometric ratios and the Pythagorean theorem. • Understand and apply theorems about circles, including the equation for circles. • Explain volume formulas and use them to solve problems. • Apply geometric concepts in modeling situations.
Statistics and Probability	• Summarize, represent, and interpret data about a single variable (line plot, box plot, histogram, normal curve, measures of center, measures of spread). • Summarize, represent, and interpret data about two categorical or quantitative variables (two-way frequency tables, scatter plots, lines and functions of best fit). • Understand independence and conditional probability and use them to interpret data and solve problems. • Compute probabilities of compound events in terms of a uniform probability model. • Understand and evaluate statistical experiments used to make inferences. • Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Source: Adapted from NCTM, 2018.

course of three years, regardless of how your school or district determines and organizes the courses in which students learn.

Your students in high school extend learning from middle school to understand, analyze, interpret, and create equations and new functions with key features, grow numbers to include irrational numbers, deepen geometric reasoning with proofs, analyze and interpret statistical data and probability, and create and explain mathematical models. Learning is focused on an understanding of each concept and between concepts in order to develop critical reasoning and proof within a consistent system of mathematics and for application within each course and future experiences.

Your course-based team and mathematics department may want to explore mathematics learning progressions as defined by your state standards or reference mathematics learning progression documents online, such as those developed by the Common Core Standards Writing Team (2013) at www.math.arizona.edu/~ime/progressions, or Student Achievement Partners' (n.d.) coherence map. Your team may also want to engage in a book study, perhaps referencing NCTM resources related to understanding the essential content and skills needed in high school.

With so much mathematics content to learn, your mathematics department's and course-based team's unit planning helps ensure a guaranteed and viable mathematics curriculum within your course and across your high school course sequence. Planning the units together to learn your own course content in greater depth and its importance in the vertical high school trajectory builds your teacher team self-efficacy.

Connections Between Mathematics Content and Unit Planning

For each unit in your high school course, you support your team's progress toward better understanding the standards that support the guaranteed and viable mathematics curriculum. Together, you and your team use the Mathematics Unit Planner template in figure 1.2 (page 11) to record answers to the following questions.

- What exactly do students need to know and be able to do in this unit?
- Which mathematics standards should we commonly assess? When?
- How does the mathematics learning in this unit connect to the standards students must learn in previous or future units?
- Which academic mathematics vocabulary and notations must students learn to read, write, and speak in the unit to be proficient with the standards?
- What are examples of higher- and lower-level-cognitive-demand mathematical tasks students should demonstrate proficiency with if they have learned the standards?
- Which mathematical tools or technology should students learn or utilize to demonstrate an understanding of the standards in the unit?

Answering these questions as a team creates more equitable student learning experiences from one teacher to the next. Teachers come to a consensus on what needs to be taught and how to teach it, ensuring all students, regardless of instructor, receive the same mathematics education. Additionally, developing teacher efficacy strengthens your instructional practices. Consequently, student learning improves because your entire team is working to ensure each student *learns* the organized mathematics content from one unit to the next.

Chapter 2 (page 15) provides tools and protocols that help your high school course-based mathematics team unwrap unit standards. These tools and protocols will also assist you in intentionally addressing each unit-planning element as your mathematics story arc develops for the school year.