Transitioning to Common Mathematics Standards:
Computational Fluency in the K–5 Curriculum

Dawn Teuscher, *Brigham Young University, Provo, Utah*
Barbara J. Reys, *University of Missouri, Columbia*
Shannon Dingman, *University of Arkansas, Fayetteville*
Amanda Thomas, *Penn State Harrisburg, Middletown, Pennsylvania*

The Common Core State Standards for Mathematics (CCSSM; National Governors Association Center for Best Practices [NGA Center] and Council of Chief State School Officers [CCSSO] 2010) is a "common" set of learning expectations that forty-five states, the District of Columbia, Department of Defense Education Activity (DoDEA), American Samoa Islands, U.S. Virgin Islands, Guam, and Northern Mariana Islands have adopted. The widespread adoption of CCSSM affects multiple stakeholders (e.g., curriculum developers, district and school leaders). However, a crucial constituency—teachers—is affected the most, as teachers will implement the new standards. Unfortunately, a recent national survey found that approximately 80 percent of teachers felt that CCSSM is "pretty much the same" as their previous state standards (Schmidt 2012). While similarities do exist between CCSSM and pre-CCSSM state standards, CCSSM represents a significant departure from *when* and, in some cases, *how* certain content is developed (Dingman et al. 2013).

In this chapter, we build on prior analyses of the number and operation strand of pre-CCSSM state-level mathematics standards (Reys et al. 2006; Reys and Thomas 2011) to compare and contrast the approach and progression of computational fluency with whole numbers, fractions, and decimals within CCSSM. We present this comparison (1) to help teachers understand that CCSSM differs significantly from pre-CCSSM standards and (2) to point out important issues that teachers will face as they prepare to address computational fluency in their instruction. We first define computational fluency and then describe changes in approach and grade-level emphasis of computational fluency in CCSSM. We conclude with implications for teachers and district leaders as they continue implementing CCSSM.

Computational Fluency

In *Principles and Standards for School Mathematics,* the National Council of Teachers of Mathematics (NCTM 2000) defines *computational fluency* as "efficient and accurate methods that are supported by an understanding of numbers and operations" (p. 35). Russell (2000) elaborated on this definition:

> Fluency demands more of students than does memorization of a single procedure. Fluency rests on well-built mathematical foundations with three parts: (1) an understanding of the meaning of the operations and their relationships to each other; (2) the knowledge of a large repertoire of number relationships; and (3) a thorough understanding of the base-ten number system, how numbers are structured in this system, and how the place value system of numbers behaves in different operations. (pp. 154–55)

The Common Core Learning Progressions (Common Core Standards Writing Team 2011), a series of background documents commissioned and used by the authors of CCSSM, indicate that fluency "involves a mixture of just knowing some answers, knowing some answers from patterns (e.g., 'adding 0 yields the same number'), and knowing some answers from the use of strategies" (p. 18). *Principles and Standards,* CCSSM, and Russell's definitions describe a common goal for students—efficiency and accuracy and use of strategies based on number properties. In this chapter we refer to computational fluency as the ability of students, at the appropriate grade levels, to compute efficiently and accurately, with understanding.

Grade-Level Emphasis on Computational Fluency

We summarize here differences between pre-CCSSM state standards and CCSSM for basic facts, whole numbers, fractions, and decimals with emphasis on (1) the trajectory of development, that is, the number of grade levels that students spend developing computational fluency and (2) the grade level when students are expected to attain fluency, that is, have and use efficient and accurate methods for computing (Fuson and Beckmann 2013; NCTM 2000; Common Core Standards Writing Team 2011). We also highlight, where appropriate, expectations for use of invented strategies and standard algorithms. The progression from informal invented strategies based on number properties to standard algorithms for computation as outlined in CCSSM is carefully described elsewhere (see Fuson and Beckmann 2013).

Basic Facts

In pre-CCSSM state standards, instruction with single-digit addition and related subtraction facts was typically spread over two years, beginning in grade 1, with fluency expected by the end of grade 2 (twenty-eight state standards). In CCSSM this topic is extended by one grade—introduced in kindergarten with fluency expected in grade 2: "Fluently add and subtract with 20 using mental strategies. By the end of Grade 2, know from memory all sums of two one-digit numbers" (NGA Center and CCSSO 2010, 2.0A.2). The CCSSM learning progression document (Common Core Standards Writing Team 2011) details how students are to become fluent with basic facts by the end of grade 2, having spent time in kindergarten and grade 1 using various strategies (e.g., counting all, adding on, decomposing or composing easier numbers). As students work toward fluency in grade 2 they should be—

> allowed to fall back on earlier strategies when needed. By the end of the K–2 grade span, students have sufficient experience with addition and subtraction to know single-digit sums from memory; . . . this is not a matter of instilling facts divorced from their meanings, but rather as an outcome of a multi-year process that heavily involves the interplay of practice and reasoning. (p. 19)

Prior to CCSSM, single-digit multiplication and related division facts were typically developed over a two-year period, with fluency expected at grade 4 in twenty-two states. In CCSSM, this topic is developed over one year, introduced in grade 3 with fluency expected by the end of grade 3. In pre-CCSSM state standards, students were expected to demonstrate fluency with basic facts in division by the end of grade 4 in twenty states. CCSSM does not include an explicit standard for fluency with basic facts in division, yet in grade 3 students are expected to perform division of two whole numbers with whole number answers and with the dividend in the range of zero to one hundred (NGA Center and CCSSO 2010, 3.0A.7).

Multi-Ddigit Whole-Number Computation

In pre-CCSSM state standards, the grade level at which computational fluency with addition and subtraction of multi-digit whole numbers was expected varied across grades 1 through 6, with the majority of states (twenty-nine) requiring fluency by the end of grades 3 or 4 (Reys et al. 2006). Furthermore, the development of computational fluency for addition and subtraction with multi-digit whole numbers varied from one to six years with a mean of three years across states (see table 1.1). In CCSSM, fluency with multi-digit whole number addition and subtraction is expected at the end of grade 4, which is similar to fifteen states' pre-CCSSM documents. In eight states this is earlier than what was previously expected, whereas in seventeen states this expectation is at a later grade level, allowing students more time to learn and develop strategies for multi-digit addition and subtraction. Furthermore, the progression of developing fluency with addition and subtraction of multi-digit whole numbers begins in grade 1 and continues through grade 4 in CCSSM. At grade 4, students are expected to "fluently add and subtract multi-digit whole numbers using the standard algorithm" (NGA Center and CCSSO 2010, 4.NBT.4). Again, fluency with the standard algorithm is not meant to be the goal; rather, the development over four years (grades 1 through 4) from students' use of multiples of ten in grade 1 to relationships between operations and understanding of place value and properties of operations in grades 2 through 4 should help students understand, use, and apply the standard algorithm.

CCSSM also includes specific references for students to connect multi-digit computation to place value and properties of operations while using concrete models to demonstrate their reasoning and understanding. For example, in grade 2 students should be able to

> add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. (2.NBT.7)

Table 1.1
Grade level at which pre-CCSSM state standards and CCSSM expect fluency with whole number and fraction computation, by operation

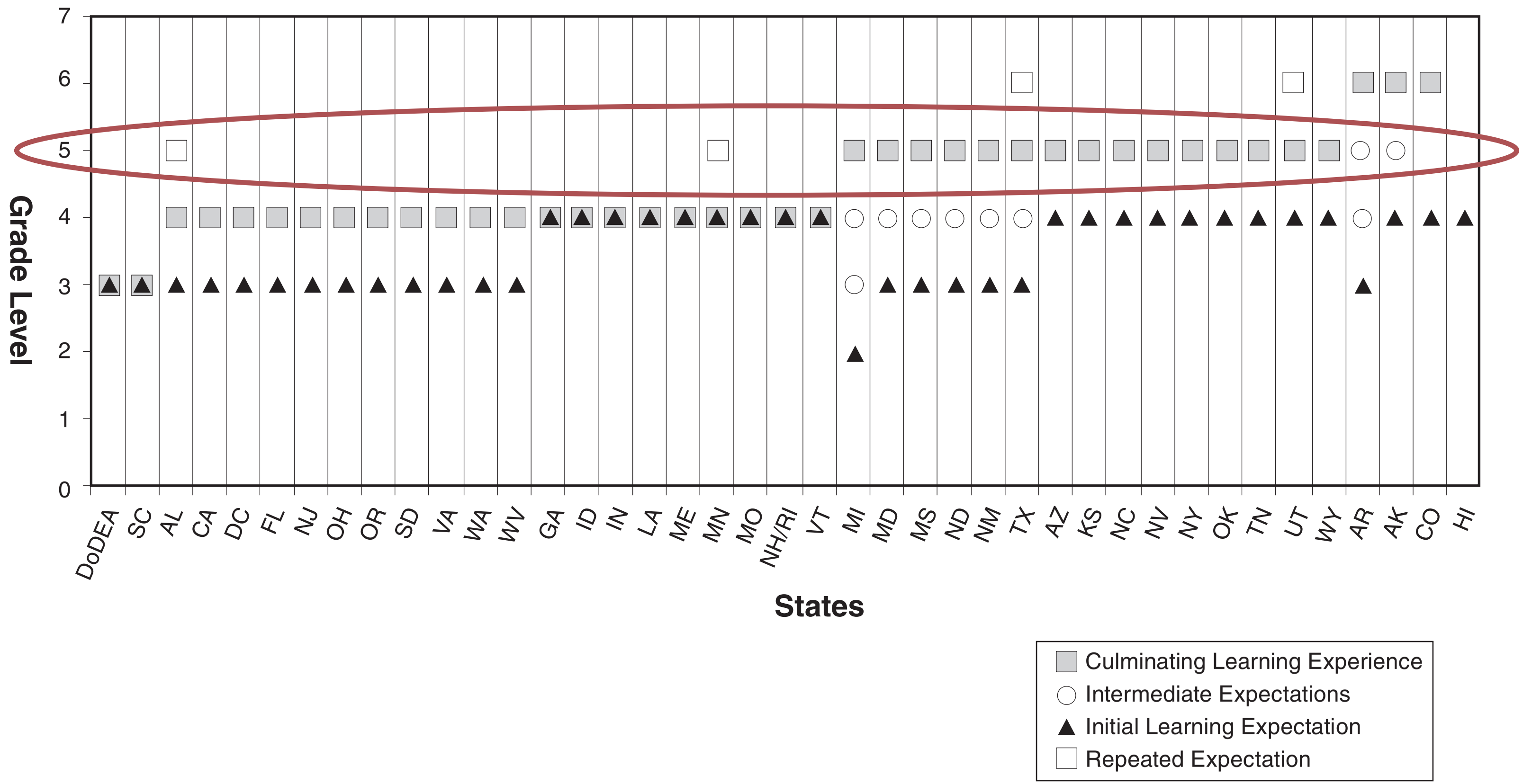
Grade level	Addition: Whole Numbers	Addition: Fractions	Subtraction: Whole Numbers	Subtraction: Fractions	Multiplication: Whole Numbers	Multiplication: Fractions	Division: Whole Numbers	Division: Fractions
	Number of States		Number of States		Number of States		Number of States	
1	1		1					
2	3		2					
3	14		15		2			
4	**15**[a]	1	**15**[a]	1	21		12	
5	5	**15**[a]	5	**15**[a]	**15**[a]	**2**[a]	23	1
6	3	20	3	20	3	26	**6**[a]	**25**[a]
7		6		6		13		14
8						1		1
Unspecified	1		1		1		1	1

[a] Grade level when CCSSM expects fluency

Therefore, teachers must understand different ways students might think about adding and subtracting whole numbers and how these strategies relate to place value and properties of operations.

For example, if students are asked to add 394 + 508, some students may decompose the 8 ones in 508 to (6 + 2) so they can then use the associative property to easily add (394 + 6) + 2 + 500 and then add 400 + 2 + 500, which equals 902. Yet other students may focus more on place value by adding the hundreds, the tens, and the ones separately using partial sums. These students may add the hundreds (300 + 500) to get 800, then they add the tens (90 + 0) to get 90, and then they add the ones (4 + 8) to get 12. From here students would add up the three numbers 800 + 90 + 12, resulting in the answer of 902. Although different strategies were used, the answers are equivalent, and a powerful discussion could be held in the classroom to compare these two strategies. Potential questions teachers could ask to further connections for students include the following: How are these strategies similar and different? Will these strategies work for all numbers? Why or why not?

As illustrated in figure 1.1, fluency with multiplication of multi-digit whole numbers varied across states prior to CCSSM. (*Note:* Figs. 1.1 and 1.2 include DoDEA, and they combine New Hampshire and Rhode Island, which worked together on the standards. The states are arranged by grade level of culminating expectation.) As noted, a few state standards indicated fluency as early as the end of grade 3, with other state standards calling for fluency by the end of grade 6. CCSSM includes standards on this topic over a three-year span (grades 3 through 5) with fluency expected at the end of grade 5. As seen in figure 1.1, fifteen states have similar expectations for fluency as CCSSM (oval on fig. 1.1 at grade 5); yet, in most states (twenty-seven) fluency with



5.NBT.5. Fluently multiply multi-digit whole numbers using the standard algorithm. (NGA Center and CCSSO 2010)

Fig. 1.1. Progression of computational fluency for multiplication of multi-digit whole numbers across pre-CCSSM state standards and culminating expectation in CCSSM

multi-digit whole number multiplication at grade 5 represents a change from past practice. In particular, fluency with multiplication of multi-digit whole numbers is expected at one grade level later and, in a few cases, two grade levels later than pre-CCSSM state standards, allowing students more time to develop understanding and become fluent with multi-digit whole number multiplication.

As illustrated in table 1.1, the majority of pre-CCSSM state standards (twenty-three) included an expectation of fluency with division of multi-digit whole numbers by the end of grade 5 with the majority of states introducing the topic two years before (in grade 3). With CCSSM, teachers will find the development of this topic is extended over three years (grades 4 through 6), with the expectation for fluency at the end of grade 6. As noted, this represents a shift to a later grade level for fluency with division as well as an extension in the number of grades that students will focus on learning and developing this topic than was typical in most pre-CCSSM standards.

Fraction Computation

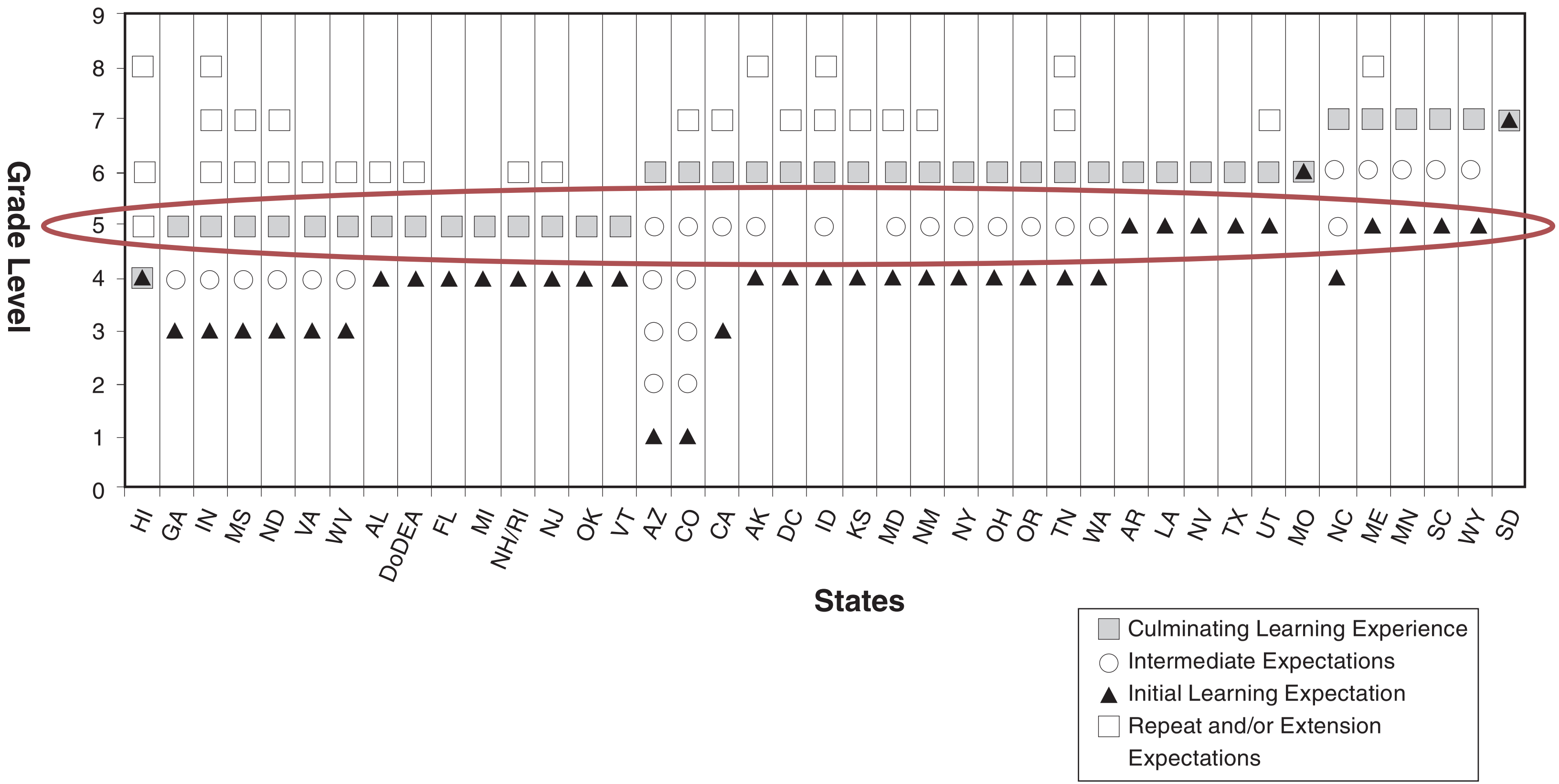
The term *fraction computation* is defined, for the purposes of this discussion, as computations with proper fractions, improper fractions, or mixed numbers for the four arithmetic operations. As with multi-digit whole number computation, there was considerable variation across pre-CCSSM state standards regarding when fraction computation was first introduced and when fluency was expected (see table 1.1). As illustrated in figure 1.2, the initial expectations for addition and subtraction of fractions ranged from grades 1 through 7, with the majority of states (twenty-two) initiating this topic in grade 4. Additionally, the grade level at which fluency with addition and subtraction of fractions was expected varied across grades 4 through 7 in pre-CCSSM state standards.

A similar picture emerged for the progression of standards concerning multiplication and division of fractions (Reys et al. 2006). In contrast, the progression of CCSSM standards related to fraction computation is condensed into a smaller grade span. CCSSM indicates that students are expected to add and subtract with fractions and mixed numbers that contain common denominators in grade 4, while in grade 5 (oval on fig. 1.2) they are expected to add and subtract fractions with like and unlike denominators. This represents a compression of grade spans for thirty-two states that included adding and subtracting fractions across three or more grade levels in their pre-CCSSM standards. Many states expected fluency at either the same grade or at a later grade level compared to CCSSM (see fig. 1.2).

Unlike the standards for computation with whole numbers, CCSSM standards related to addition and subtraction of fractions do not use the term *standard algorithm.* However, a specific strategy, including the general form, is included in the statement of the standard:

> Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $^2/_3 + {}^5/_4 = {}^8/_{12} + {}^{15}/_{12} = {}^{23}/_{12}$. (In general, $a/b + c/d = (ad + bc)/bd$.) (5.NF.1)

In CCSSM, multiplying fractions by whole numbers is initiated in grade 4, with fluency in multiplying fractions by fractions expected in grade 5—at an earlier grade level for forty states. For division of fractions in CCSSM, students are to divide whole numbers by unit fractions (or vice versa) in grade 5, while fluency in dividing with all fractions is expected in grade 6. This is earlier than what was typical in pre-CCSSM state standards by at least one year for fifteen states.



5.NF.1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* (NGA Center and CCSSO 2010)

Fig. 1.2. Progression of computational fluency for addition and subtraction of fractions across pre-CCSSM state standards and culminating expectation in CCSSM

Another difference that will affect teachers is the emphasis in CCSSM on use of specific models to develop computational fluency with fractions. For example, the initial development of fractions focuses on students understanding a fraction as a part of a whole that is partitioned into equal parts. Then students extend their understanding by representing fractions on a number line:

> Understand a fraction as a number on the number line; represent fractions on a number line diagram.
>
> a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.
>
> b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line. (NGA Center and CCSSO 2010, 3.NF.2)

While the number line is found in most primary-grade textbooks, it is generally offered as a model to locate and compare whole numbers. Some teachers (and textbooks) return to the number line as a tool for comparing whole numbers and fractions. However, it is likely that teachers will need to seek CCSSM-aligned curriculum materials to support using number lines to develop the fraction ideas outlined in CCSSM.

Decimal Computation

In pre-CCSSM standards, decimal computation was most often introduced in grade 4 (twenty-six states) with the majority of states (thirty-five) spending three to four years developing decimal computation (Reys and Thomas 2011). In fact, only seven states spent two years or less developing decimal computation. In CCSSM, decimal computation is developed over two years, spanning grades 5 and 6. This is a compression from pre-CCSSM state standards, and decimal computation is introduced at a later grade level than what was typically expected in most states. In addition, pre-CCSSM state standards frequently initiated decimal computation in the context of money, which is not the case in CCSSM.

In pre-CCSSM state standards, there existed variation not only in the grade levels for when decimal computation was introduced and developed, but also in the order in which fraction and decimal computational fluency was developed. Analysis of pre-CCSSM state standards found twenty-three states introduced decimal and fraction computation during the same grade level, sixteen states introduced decimal computation first, and seven states introduced fraction computation prior to decimal computation. In CCSSM, decimal computation is introduced in grade 5, after fraction computation is initiated in grade 4. This will be a change for 84 percent of the states (thirty-nine).

Implications

While the majority of teachers may believe that CCSSM is “pretty much the same” as their pre-CCSSM standards (Schmidt 2012), the analysis reported in this article runs counter to that belief. In fact, the differences between pre-CCSSM standards and CCSSM related to computational fluency will require teachers, across multiple grade levels, to revise the timeline and emphasis on computational fluency. Specifically, the immediate work for elementary teachers should include preparing different content for the grade level they teach, understanding and preparing lessons using models (number lines) with which they may not be familiar, and preparing to identify gaps in student learning as individual states transition to CCSSM. To do this work, teachers must be aware of the changes in emphasis regarding when certain computational fluency is expected. Teachers will need to compress some material—that is, focus on particular topics over fewer grade levels than in the past—and will likely need to increase the emphasis on these topics at particular grade levels.

Given the analysis presented here, kindergarten through grade 6 textbooks published prior to 2010 are not likely to be aligned to CCSSM with respect to computational fluency involving basic facts, whole numbers, fractions, and decimals. Therefore, teachers need to take a more active role in identifying and deciding when and how they will teach these topics. It is also critical to understand the emphasis in CCSSM on particular topics and models. For example, the initial development of fractions focuses on unit fractions of $1/b$ with fractions of the form a/b developed as a parts of size $1/b$ (NGA Center and CCSSO 2010, 3.NF.1). Although unit fractions are introduced in elementary mathematics, they are generally not presented as the basis for all fractions. Another example is the heavy emphasis on the number line as a model for fractions. Most textbooks do not currently use this model for learning fractions; in fact, there is more emphasis on the area model (Watanabe 2002).

District leaders have an important role in providing professional development that focuses on major changes in CCSSM, such as those we have illustrated in this chapter. Given current economic conditions, districts initially may not be able to purchase new textbooks aligned to CCSSM; therefore, it is important to provide teachers with supplemental materials that are connected and aligned with CCSSM, with the goal being students' mathematical understanding in a connected and meaningful way.

Although we focused this chapter on changes in the standards related to computational fluency, this is just one area in the kindergarten through grade 6 mathematics curriculum that is different in CCSSM. Teachers need to be aware of changes in other areas of the curriculum as well (Dingman et al. 2013). Given the unprecedented nature of CCSSM reform, the increased attention to mathematical practices reflected in the Common Core State Standards for Mathematical Practice, and the changes in content that are the focus of instruction at specific grade levels, the implementation of CCSSM will challenge elementary grades mathematics teachers to align their teaching with the vision of CCSSM. It is therefore vital that teachers are knowledgeable about the changes that are occurring and are provided the necessary support needed to enact CCSSM with the diligence it requires.

References

Common Core Standards Writing Team. *Progressions for the Common Core State Standards in Mathematics: K, Counting and Cardinality; K–5 Operations and Algebraic Thinking* (draft), 2011. http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf.

Dingman, Shannon, Dawn Teuscher, Jill Newton, and Lisa Kasmer. "Common Mathematics Standards in the United States: A Comparison of K–8 State and Common Core Standards." *The Elementary School Journal* 113, no. 4 (2013): 541–64.

Fuson, Karen C., and Sybilla Beckmann. "Standard Algorithms in the Common Core State Standards." *NCSM Journal of Mathematics Education Leadership* 14, no. 2 (2013): 14–30.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, Va.: NCTM, 2000.

National Governors Association Center for Best Practices and Council of Chief State School Officers (NGA Center and CCSSO). *Common Core State Standards for Mathematics.* Washington, D.C.: NGA Center and CCSSO, 2010. http://www.corestandards.org.

Reys, Barbara J., Shannon Dingman, Travis Olson, Angela Sutter, Dawn Teuscher, and Kathryn Chval. "Analysis of Number and Operation Grade-Level Learning Expectations in State Standards Documents." In *The Intended Mathematics Curriculum as Represented in State-Level Curriculum Standards: Consensus or Confusion?,* edited by Barbara J. Reys, pp. 15–57. Charlotte, N.C.: Information Age Publishing, 2006.

Reys, Barbara J., and Amanda Thomas. "Standards for Computational Fluency: A Comparison of State and CCSS-M Expectations." *Journal of Mathematics Education Leadership* 13 (Fall 2011): 21–32.

Russell, Susan Jo. "Developing Computational Fluency with Whole Numbers." *Teaching Children Mathematics* 7, no. 3 (2000): 154–58.

Schmidt, William. *The Common Core State Standards for Mathematics Math: The Relationship Between High Standards, Systemic Implementation and Student Achievement* [PowerPoint presentation], 2012. www.achieve.org/CCSS-schmidt-research.

Watanabe, Tad. "Representations in Teaching and Learning Fractions." *Teaching Children Mathematics* 8, no. 8 (2002): 457–63.