

The NYC Math Lab: Lessons from a Turn toward *Teaching Evaluation*

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A teacher's day is packed with responsibilities. There are children to console and counsel, field trip forms to collect, and up to five lessons per day to prepare and teach. As a result, teachers rarely have time to collaborate, let alone see each other in action. Yet research suggests that having time to discuss and analyze teaching may be an effective lever for improving teaching and learning (Gallimore et al. 2009; Ronfeldt et al. 2015). Indeed, teachers in countries whose students perform better than the United States on Programme for International Student Assessment (PISA) examinations (such as Japan) often have more time for collaboration than do teachers in the United States. (LeTendre et al. 2001; Kane 1994). Having the opportunity to study live teaching may also be beneficial, as it offers teachers images of new teaching methods and fosters a professional community in which teaching is regularly analyzed and improved.

Instead of focusing on creating opportunities for greater collaboration and collective study of teaching practice, however, the recent conversation in the public sphere about improving teaching and learning has centered on teacher evaluation (Baker, Oluwole, and Green 2013). Over the past decade, most states have developed new teacher evaluation systems (Doherty and Jacobs 2015). These evaluation systems often mandate more observations and new rubrics for assessing teaching, but systematic plans for illustrating the types of teaching skills outlined in these rubrics remain nonexistent. The need to provide images of effective teaching is particularly acute in mathematics education, where procedural instruction persists despite decades of research (Hiebert 2013) and widespread consensus on effective practice (National Council of Teachers of Mathematics [NCTM] 2014).

This chapter describes the NYC Math Lab, a weeklong professional development institute created by three teachers, in which participants observed, analyzed, and engaged in the teaching of twenty rising fifth-grade students attending a summer program at a local nonprofit organization. Teachers unanimously reported that Math Lab had a big impact on their mathematics teaching practice. In this chapter, we describe the rationale for developing the NYC Math Lab, the process through which it was created, and our initial conclusions about the ways in which public teaching might facilitate teacher learning. In providing a detailed portrait of our work, we hope to provoke interest in developing math labs as a form of teaching evaluation, and we offer ideas on the ways one might begin.

■ Teacher Evaluation vs. Teaching Evaluation

Teacher evaluation has played a pivotal role in conversations about improving public education for the first part of the twenty-first century. Significant attention, from both researchers and policy-makers, has been directed toward developing new teacher evaluation formulas and linking teacher evaluation to student achievement data (Baker, Oluwole, and Green 2013). Yet evaluations based on administrator observations and student test scores may not catalyze improvement. According to The New Teacher Project's (TNTP) 2015 report on teacher development, although teachers are being regularly evaluated, "Most teachers in the districts we studied seem to be marching in place when it comes to their development" (Jacob and McGovern 2015, p. 13). Teachers feel the disconnect between evaluative feedback and professional growth, too; when asked to report on the "activity that has helped me learn how to improve the most," only 3 percent of teachers in TNTP's study named observations and feedback (Jacob and McGovern 2015, p. 21).

One problem with using teacher evaluation as a lever for change is that it positions teachers as the passive subjects of review, rather than agents of their own growth. Teachers receive feedback from supervisors a few times a year, with little follow-up to see if that feedback has improved their teaching. This is problematic, as follow-up is consistently proven to be one of the features of successful professional development (Fishman, Davis, and Chan 2014). Without this follow-up, teacher evaluation becomes translated as a teacher rating system rather than a teacher improvement system. When teachers receive the same rating year after year, it reinforces the belief that some teachers will always be successful while others will not, leaving little impetus for growth. In *Building a Better Teacher*, Elizabeth Green exposes the "Myth of the Natural-Born Teacher" (Green 2014, p. 6) and reveals the ways in which this myth has been widely detrimental to the development of the profession.

For elementary school teachers of mathematics, teacher evaluation rarely provides support in the critical area of content knowledge. According to Stipek et al. (2001), "Most American teachers have a conception of mathematics as a static body of knowledge, involving a set of rules and procedures that are applied to yield one right answer" (p. 214). Elementary school teachers may not have robust content knowledge in mathematics, and they often operate from a conception of mathematics teaching that is procedural rather than one focused on developing mathematical understanding (Ball 1990). Therefore, mathematics teachers need professional development and feedback that will enhance their content knowledge. Teacher evaluation systems almost never support this type of work. Because procedural teaching of mathematics is a long-standing tradition in American schools, administrators who have often only learned math and learned to teach math in a procedural way are hard-pressed to move teachers of mathematics toward methods that encourage sense making (Lortie 1975; Steele et al. 2015).

On the other hand, *teaching* evaluation—the study of practice rather than individuals—holds great potential for improving teaching and learning. Lesson study (Lewis and Hurd 2011), peer visitation, and public teaching in a laboratory classroom setting all provide opportunities for teachers to evaluate and implement new teaching practices.

■ What Is Public Teaching?

Public teaching is a new term in the educational literature. In a 2013 American Educational Research Association (AERA) presentation, Deborah Ball and colleagues at the University of Michigan described public teaching as “the work of making one’s own teaching practice studyable by observers.” In doing so, they noted that public teaching might include pre- and post-class analysis and that teaching in public requires attending to both students and observers. Yet this presentation is the only peer-reviewed work that describes public teaching, and it stopped short of defining the concept. In our view, public teaching occurs any time a teacher performs the work of teaching in front of adults who intend to study teacher practice or the resulting student thinking.

We believe that public teaching is an important missing component of current systems of teacher evaluation and professional development. In the pages that follow, we present one approach to using public teaching as a method for improving mathematics instruction: the creation and implementation of a math lab.

■ The NYC Math Lab: A Study in Public Teaching

Background

The NYC Math Lab spawned from the work of the Math Collective, a group of teachers and schools in New York City engaged in inquiry around children’s mathematical thinking. Kate Abell, the founder of the Math Collective, organized a number of lesson study cycles each year. Teachers consistently found lesson study to be a powerful form of professional development. Peter and Kim, the authors of this chapter, were fourth-grade teachers at P.S. 29 who worked regularly with Kate and the Math Collective over several years. As part of our frequent collaboration to plan math instruction, Peter and Kim began to take turns observing a math lesson in each other’s classroom every week. These peer visitations and resulting conversations had a deep impact on our growth as teachers of mathematics. Our peer visitations also made us more aware of the power of public teaching, and as a result we became more willing to teach in front of others. Peter, Kim, and Kate came together in the spring of 2015 and decided to create the NYC Math Lab as a way to bring similar experiences to a broader group of teachers.

The University of Michigan’s Elementary Math Lab (EML), taught by Deborah Ball, also inspired the creation of the NYC Math Lab. Peter attended a two-week session at the EML in the summer of 2014. Ball framed her teaching of the lab class not as model teaching but as a way to provide participants with a shared experience in math teaching and learning. This framing gave us the conviction that making our practice public could benefit teachers in our district as well. Many of the structures described in this chapter came from the EML.

Creating a Partnership

After some deliberation about how to find students willing to participate in a math lab, we decided to partner with the Hudson Guild, a nonprofit organization that offers support services to low-income families in Manhattan’s Chelsea neighborhood. Because we had a negligible budget and no support staff outside of the founding teachers, partnering with the Hudson Guild allowed us to provide two hours of math instruction without needing to develop afternoon programming for students. When approached about the partnership, Hudson Guild staff was excited about the opportunity for their students to have access to summer math instruction within their existing program.

Identifying Values

We began our planning by co-creating a list of instructional values that would guide our work at Math Lab. We decided to orient our lab around the following beliefs:

- Math is about making sense (Hiebert et al. 1997).
- The use of visual representations both deepens students' thinking about math and helps students support their mathematical arguments (NCTM 2014).
- Student ownership is a critical component to teaching mathematical content, and encouraging student talk supports ownership (Yackel and Cobb 1996).
- A mathematical community is one where people can express their ideas and learn from the ideas of others (Hiebert et al. 1997).
- The work of the mathematics teacher is to listen to students as they express their developing ideas and to respond based on their thinking (Hiebert et al. 1997).

Organizing for Instruction

We decided on the following areas of instructional focus: in the first hour of each day's lab class, we would work on (a) developing students' math mindset (Dweck 2006; Boaler 2015) through work with materials from YouCubed's Week of Inspirational Math (fig. 1.1) and (b) growing students' ability to make generalizations through a Number of the Day routine (fig. 1.2), in which students were given a number and asked to write as many number sentences as they could, using any of the operations, that equaled that number (Russell, Schifter, and Bastable 2011, p. 16).



Day 3: Paper Folding



Paper Folding Gr. 3 - 4

Adapted from Driscoll, 2007

Work with a partner. Take turns being the skeptic or the convincer. When you are the convincer your job is to be convincing! Give reasons for all of your statements. Skeptics must be skeptical! Don't be easily convinced. Require reasons and justifications that make sense to you.

For each of the problems below one person should make the shape and then be convincing. Your partner is the skeptic. When you move to the next question switch roles.

Start with a square sheet of paper and make folds to construct a new shape. Then, explain how you know the shape you constructed has the specified area.

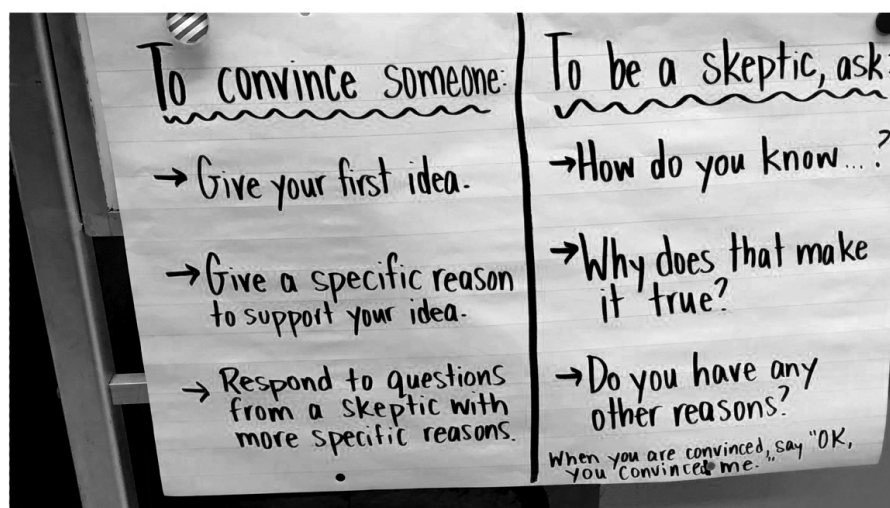


Fig. 1.1. An example of a task from YouCubed's Week of Inspirational Math, and a chart made to accompany this task

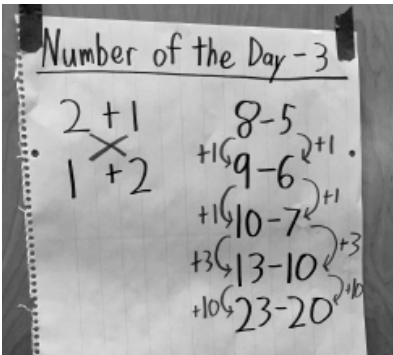


Fig. 1.2. Number of the day routine, with accompanying observations relating to properties of operations

The second hour would be spent focusing on developing students’ conceptual understanding of fractions through a series of inquiry-based, hands-on tasks. We chose fractions for a number of reasons. First, fractions are both vital for future mathematics and often misunderstood by students of all ages (Bezuk and Bieck 1993; Stigler, Givvin, and Thompson 2010; Lortie-Forgues, Tian, and Siegler 2015). Second, fractions are a limited domain within the Common Core State Standards; the Number and Operations—Fractions domain spans grades 3 through 5 only, with foundations in geometry standards in first and second grades and extensions into ratio and proportion in middle school (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA Center and CCSSO] 2010). We hypothesized that teaching fraction content would allow teacher participants to both deepen their understanding of a challenging content area and see the progression of student understanding develop over the relatively short period of time we had for the Math Lab.

About four weeks before Math Lab, we visited the students who would be attending the program to introduce ourselves, get to know them, and tell them about Math Lab. We returned a few days later to conduct one-on-one interviews with each student, in which we gathered information about their understanding of fraction concepts and their attitudes toward mathematics. What we learned informed our planning; for example, after asking a question about the naming of a fractional piece of a rectangle (fig. 1.3), we determined that students would need more work on the concept of the unit fraction and incorporated that into our plan for the first day of the lab.

What fraction of the large rectangle is gray?	

Fig. 1.3. Example of a fraction task used during student interviews

Time + Activity	Task	Details	Commentary/ Anticipations
10:00 – 10:15 Number of the Day – 5	Students will write number sentences. These number sentences may include concepts drawn from previous days' work, such as using multiple terms, the commutative property of addition, constant difference, or a concept from the small group the day before. Participants facilitate small-group discussions.	<ul style="list-style-type: none"> Students will enter the lab class and begin work on 5, the number of the day, in their notebooks. Teacher will positively reinforce this routine. Participants will observe the kids they are working with to glean further insights into their mathematical thinking. Particular items to watch for include— <ul style="list-style-type: none"> → Student uptake of content from previous days → Growth from day 1 of the lab → Areas of continued need 	
10:15 – 10:50 Ant Problem	Students will try equivalence problems and reason about why the pieces are equivalent in those problems.	<ul style="list-style-type: none"> Teacher will introduce the Ant problem. Students will work with their partners to solve the Ant problem and represent their thinking. Teacher will lead a discussion to bring out the proportional relationship (“there are three times as many twelfths, so we need three times as many of them to make the same distance”). Students will generate more equivalent fractions by using Cuisenaire rods. 	This work is directly related to the standard 4.NF.A.1: “Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.”

Fig. 1.4. Part of a lesson plan from day 5 of the NYC Math Lab (following the lesson plan format of the Elementary Math Lab at the University of Michigan)
(Source: Elementary Math Lab, University of Michigan)

Structuring the Day with Teacher Learning in Mind

We started Math Lab with the intention of creating a professional development opportunity for teachers that oriented them toward evaluating and learning from live teaching. The first structure that we incorporated to support this goal was a daily “pre-brief.” In the pre-brief, which took place for the first hour of the day, we presented the participants with the day’s lesson plan (see fig 1.4 on the previous page) and engaged them in a discussion around our instructional considerations and decisions about the enactment of the day’s lessons.

We also gave participants the mathematical tasks students would be working with that day (fig. 1.5), and we had them complete the tasks and discuss what challenges they might present for students.

Name _____
Date _____

The Ant Problem

Two ants were bragging about how far they carried a huge leaf. The red ant ran $\frac{9}{12}$ of a boozle. The black ant ran $\frac{3}{4}$ of a boozle. The red ant said, “I ran further because my numbers are bigger.”

Is the red ant correct? Use rods to decide who carried the leaf the furthest. Use pictures, numbers, and words to explain your thinking so that someone else can understand it.

one boozle

Fig. 1.5. Student task: The Ant Problem

Following the lab class, we held a daily debrief. As part of our focus on teaching evaluation rather than teacher evaluation, we asked participating teachers to comment on the teaching practices and their effects on student understandings, and not on the teachers themselves. Participants were also invited to give feedback and suggestions for future lessons during the debrief, and we regularly implemented those suggestions into the next day’s work with students.

From Observer to Teacher

On the last three days of the week, we also incorporated opportunities for participating teachers themselves to work with students. We paired each teacher with a student and asked him or her to confer with the child about a particular task related to the fraction work. In these one-on-one sessions with students, we first asked teachers to practice just listening to their students and making notes about each child’s understandings in relation to the task. During afternoon sessions, teachers talked with their peers about what they observed and how they might further their students’ understandings.

■ Discussion: Public Teaching as an Engine for Teacher Growth

There are many ways in which professional development centered on teaching evaluation and public teaching can lead to transferable improvements to practice. In this section, we discuss the ways in which Math Lab may have facilitated teacher growth.

Teacher Math Mindset

We believe public teaching helps teachers shift from a “fixed mindset” to a “growth mindset” (see Dweck 2006) about their own teaching. This is an important first step toward improvements to practice. For teachers, a fixed mindset often sounds like “I’m good at teaching x , but not good at teaching y ” or “I never liked math, so I don’t like teaching math.” The traditional model of teacher evaluation, in which teachers are given ratings for different areas of competency, can reinforce the fixed mindset teachers may hold about their areas of strength and weakness.

In contrast, public teaching fosters a growth mindset in teachers by forwarding the belief that teaching can be improved with deliberate analysis. At Math Lab we saw a mindset shift take place for a number of participants over the course of the week. Early in the week, participating teachers were sometimes hesitant to share their ideas on the teaching or ask questions about the math in pre-briefs or debriefs, which we interpreted as possible evidence of a fixed mindset or lack of confidence in their mathematical content or pedagogical knowledge. To encourage teachers to view themselves with a growth mindset, we regularly shared our own doubts and questions about the day’s work, both before and after the lessons were enacted. By modeling vulnerability for our participants, we sought to communicate that (a) teaching is a learnable skill that can be deconstructed and reconstructed collaboratively; and (b) the process of studying teaching is messy and offers tremendous potential for growth. On a feedback chart at the end of the week, one teacher wrote, “I used to think that because I was bad at math I would be horrible at teaching it too. Now I think because of my mistakes I can understand and teach math better” (fig. 1.6). Another teacher reflected, “[Teaching] can be scary. You have to be OK with making mistakes.”

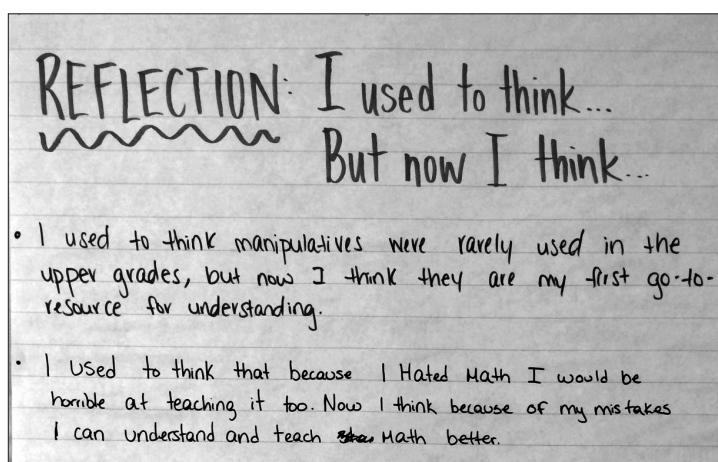


Fig. 1.6. Teachers’ reflections on their mindset shifts over the course of the week

Sharing Students, Sharing Ideas

A shared class of students helped participants shift from a stance of evaluating the teacher to evaluating the teaching. On the first day of Math Lab, participants spontaneously applauded for the teacher who had enacted the day's lessons after the students left the room at the end of the lab class. This act reflected the common view that teaching publicly is akin to performance, a belief that we strongly wanted to challenge in the interest of making teachers more comfortable with sharing their practice and learning from one another. It also reflects the sense teachers had at the beginning of the week that they were observers more than active members of a community of practice. Throughout the week, however, as teachers began to get to know the students, the tenor of conversations in pre-briefs and debriefs changed. Instead of talking about the teacher who had enacted the lesson, participants began to talk about the effect of the teaching on the students, saying things such as, "I thought that Elijah understood the unit fraction when he was working with the Cuisenaire rods yesterday, but today his understanding seemed to fall apart. I wasn't sure what to say to help him connect back to what he seemed to get yesterday." The opportunity to speak so specifically about children, their understandings, and the teaching moves that might further their understandings was a unique element of Math Lab that teachers reported finding meaningful. By the end of the week, one teacher reflected, "The big idea [behind public teaching] is cooperation—teachers united around a shared goal."

Talking about Teaching Practices

Teachers began to use more specific language around teaching practices over the course of the week. We highlighted a number of NCTM's Effective Mathematics Teaching Practices (2014) during the lab. In our pre-briefs and debriefs, we encouraged participants to use language that facilitated the discussion of these practices (table 1.1). By orienting the participating teachers toward a study of practices, we worked to shift their gaze from the teacher to the teaching. As a result, many participants were able to describe new practices they would try out in their own classrooms. One teacher said her biggest takeaway was "allowing students to struggle with problems," while another said, "Determining what children know and can do is crucial to being able to celebrate their current level of success and informs us about what next steps we might take." These reflections demonstrate that teachers began to analyze teaching in ways that would be useful to improving their practice once they returned to their own classrooms.

Table 1.1

Language that facilitates discussion of teaching practices

Instead of Saying . . .	Observers Could Say . . .
You really know how to get students interested.	The problem at the start of the lesson really grabbed students' interest.
You have a great way with the kids.	Asking students to explain their thinking during the discussion seemed to make them feel like their ideas matter to the community.
The way you explained that concept was confusing.	I wonder if explaining the concept by . . . would have helped students to see . . . more clearly.

Teacher-Centered Professional Development

Research suggests that professional development is most meaningful and transferable when it is teacher-centered; that is, when it is born out of something the teacher herself wants to learn, study, or get better at (Schoenfeld 2015; Jacob and McGovern 2015; Baldinger and Hu 2016). Just as hands-on and collaborative experiences often yield the most powerful learning for students, teachers improve when they have both agency over what they are working on and immediate opportunities to try new work in the company of colleagues. Because of the practice-based nature of Math Lab, teachers drove their own growth by raising questions and areas of practice for the group to study and work on together. For example, over the course of the week, teachers attending Math Lab became interested in learning more about math conferring—the practice of having a one-on-one conversation with a student about his or her mathematical thinking.

Co-Constructing Innovations in Teaching: Math Conferring

While conferring practices were not a specific area of focus for us going into the week, it quickly became apparent that participating teachers wanted to know how to interact with students one-on-one and in small groups in ways that supported their sense making. As the week progressed, we collaborated with participating teachers to develop a framework for conferring that offered teachers an approach to talking with students about their ideas (fig. 1.7).

As a result of spending a week evaluating and analyzing teaching, participants reported that they would approach the practice of conferring differently in their classrooms going forward. One teacher said she had learned that “[when conferring] . . . struggle is OK. Do not jump to giving kids the answers or prompts. By struggling, they truly build their own understanding.” A number of teachers said they would spend more time observing, questioning, and listening to their students in their future math conferences. The opportunity to co-construct a conferring framework and immediately try it out with students increased the likelihood that participants would transfer this new practice to their own classrooms.

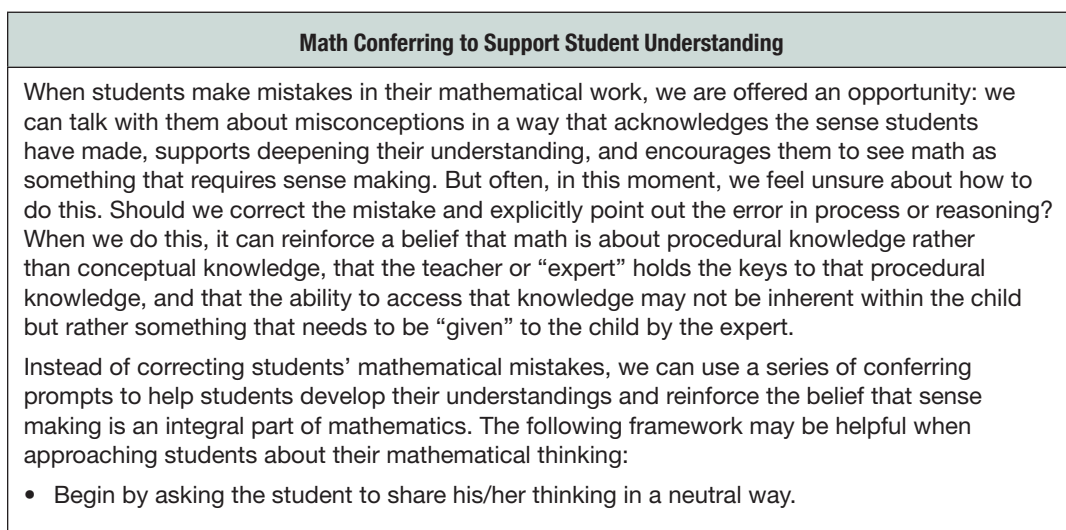


Fig. 1.7. Conferring framework co-constructed by facilitators and participants at NYC Math Lab

Math Conferring to Support Student Understanding
<ul style="list-style-type: none">• Elicit and listen carefully to student thinking, possibly prompting for a visual representation to clarify.• Begin to address a misconception by referring to a shared/commonly agreed upon math idea—this is the common ground on which you can build the sense making.• Ask student to consider this idea alongside his/her solution . . . does the solution make sense in light of this idea?• If a student recognizes that his/her solution doesn't make sense, ask, "What should we do?" This prompts the student to take responsibility for making a new plan to solve the problem.• If a student doesn't recognize that the solution doesn't make sense, it may mean that the shared belief you started with is not secure for the student. You then have some important information about what foundational pieces you need to work on with that student.

Fig. 1.7. Continued

■ Conclusion

Although evaluating teachers has been a major area of focus in recent years for improving American education systems, research does not suggest that simply evaluating teachers more results in improved teaching and learning. We believe that a shift from *teacher* evaluation to *teaching* evaluation, by increasing opportunities to learn from public teaching, holds potential for improving the quality of math instruction in our schools. Professional development structures like the NYC Math Lab position teachers as knowledgeable observers and agents in their own growth. In addition, the fact that teachers themselves created the NYC Math Lab is notable; it suggests that teachers can play a more active role than previously realized in their own professional learning.

To be effective, teachers of mathematics need more than just ratings from their supervisors with limited actionable feedback. Indeed, without access to meaningful, practice-based, teacher-centered professional development opportunities, feedback from supervisors can serve to deepen teachers' fixed mindsets about deficits in their math teaching skills or content knowledge. Having a shared class to work with and a community of teachers to analyze teaching with can stimulate teacher growth. Further, as teachers collaboratively study the craft of teaching mathematics, they often deepen their content knowledge and their ability to view math from a sense-making perspective.

The NYC Math Lab offers an example of how districts, schools, and teachers themselves can create spaces for the study of teacher practice and children's mathematical thinking. Administrators can play a key role in creating these spaces. For example, districtwide leaders can work together to create networks of schools and teachers who periodically come together to study public teaching. At the school level, principals can develop lab settings by encouraging teachers to open up their classrooms and their practice to colleagues for study or by implementing schoolwide lesson study as a form of professional development. Teachers who want to improve their practice can seek out colleagues to participate in regular peer visitations and debriefs.

For those considering creating a math lab, we offer a few lessons learned from our experience. First, the staging of a math lab can pose considerable organizational challenges. The creation and implementation of a math lab involves tasks as varied as establishing a relationship with community-based organizations, handling administrative tasks, planning and writing lessons and

student tasks, teaching the lab classes each day, and recording and collecting artifacts for future study. We suggest recruiting a team to work together on these myriad components of creating a math lab.

Another area for consideration in the creation of a math lab is how to best support teacher learning. As elementary school teachers, we found ourselves primarily focused on the teaching of the students: developing curriculum to use at the lab, creating rich tasks, and working with the individual and collective needs of the students. We found it challenging to also focus our attention on teaching the teachers who participated. For others considering creating a math lab, we suggest that articulating beliefs about teaching teachers or working with university partners might enhance teacher learning at the lab.

A third area of consideration in the creation of a math lab is supporting sustained teacher growth beyond the lab. As noted, research supports the idea that the best professional development involves meaningful follow-up (Fishman, Davis, and Chan 2014) and that the transfer of new teaching practices into the classroom is more likely when teachers are part of a team rather than working in isolation. We selected participants with this in mind: we recruited teachers who would attend the NYC Math Lab in teams, selected teachers working in schools that we knew were supportive of this work, and met throughout the school year to follow up on the work we did in the lab. Yet logistical considerations precluded us from sustaining meaningful collaborations with all participating teachers. We also had little interaction with students in our lab class after the one-week workshop. Those interested in developing a math lab would be wise to build structures from the outset that provide for continued teacher learning and student growth.

Based on the preliminary evidence presented in this article, we believe that observation and analysis of teaching in real time is a powerful learning experience for teachers. Our hope is that initiatives that privilege teaching evaluation over teacher evaluation will spread and that they will play a role in the improvement of mathematics teaching and learning at scale.

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