

Introduction

As a high school mathematics teacher, you are ever seeking new ways to engage your students in developing reasoning habits. This book is for you. We culled from the National Council of Teachers of Mathematics (NCTM) journal *Mathematics Teacher* articles and activities rooted in a wide variety of content areas and levels of mathematics (e.g., maximizing volume, linear regression). We hope that, when looking for mathematics activities to interest and engage your students, you turn to this compilation of activities and that it will become your reliable resource.

Focus in High School Mathematics: Reasoning and Sense Making (NCTM 2009) guided us and served as the screen through which we sifted our choices. We chose activities to highlight the reasoning habits in *Focus in High School Mathematics*. Reasoning is “the process of drawing conclusions on the basis of evidence or stated assumptions,” and sense making means “developing understanding of a situation, context, or concept by connecting it with existing knowledge” (NCTM 2009, p. 4). The *Focus* authors emphasize developing these two mathematical processes for high school students: “At the high school level, reasoning and sense making are of particular importance, but historically ‘reasoning’ has been limited to very select areas of the high school curriculum, and sense making is in many instances not present at all. However, an emphasis on students’ reasoning and sense making can help students organize their knowledge in ways that enhance the development of number sense, algebraic fluency, functional relationships, geometric reasoning, and statistical thinking” (NCTM 2009, p. 4). To this end, *Focus in High School Mathematics* presents four reasoning habits as productive ways to practice reasoning and sense making in the mathematics classroom:

1. Analyzing a problem (e.g., seeking patterns and relationships, considering special cases or simpler analogs)
2. Implementing a strategy (e.g., making purposeful use of procedures, making logical deductions)
3. Seeking and using connections (e.g., across mathematical domains, different representations)
4. Reflecting on a solution (e.g., considering the reasonableness of a solution, revisiting initial assumptions)

As mentioned, our goal here is to give you activities to engage your students in these reasoning habits. For example, Olson’s (1991) activity in chapter 1 lets students *seek and use connections across mathematical domains* by studying the greatest common divisor in rectangular arrays.

By itself, though, a great activity does not ensure that students develop mathematical reasoning and sense-making habits. *Focus in High School Mathematics: Reasoning and Sense Making* offers several tips for you help your students develop these habits (NCTM 2009, p. 11):

- Provide tasks that require students to figure things out for themselves.
- Ask students to restate the problem in their own words, including any assumptions they have made.
- Give students time to analyze a problem intuitively, explore the problem further by using models, and then proceed to a more formal approach.
- Resist the urge to tell students how to solve a problem when they become frustrated; find other ways to support students as they think and work.
- Ask students questions that will prompt their thinking—for example, “Why does this work?” or “How do you know?”
- Provide adequate wait time after a question for students to formulate their own reasoning.
- Encourage students to ask probing questions of themselves and one another.
- Expect students to communicate their reasoning to their classmates and the teacher, orally and in writing, through using proper mathematical vocabulary.
- Highlight exemplary explanations, and have students reflect on what makes them effective.
- Establish a classroom climate in which students feel comfortable sharing their mathematical arguments and critiquing the arguments of others in a productive manner.

Finding appropriate activities involved several groups of people. We began by soliciting recommendations from secondary school mathematics educators, including teachers, teacher educators, curriculum developers,

TABLE 1**Student Activities Appearing in *Mathematics Teacher* by Content Area and Decade**

Content area	No. of activities in decade beginning:								
	1920	1930	1940	1950	1960	1970	1980	1990	2000
Number and Measurement	2	3	3	7	3	23	34	30	31
Algebraic Symbols	1	5	3	1	3	6	13	18	3
Functions	2	1	1	2	1	10	13	40	55
Geometry	4	6	8	4	8	45	32	67	46
Statistics and Probability	0	0	1	0	0	7	17	24	34
Total	9	15	16	14	15	91	109	179	169

and researchers. At the same time we searched past volumes of *Mathematics Teacher* from 1908 to 2009 to identify all appropriate activities. From the recommendations we received and our own examination of the activities, we compiled 43 activities for this publication. When an activity did not include student activity sheets, we contacted the authors of the original activities and asked them to create one, or we created one ourselves. This collection of activities represents diverse reasoning habits, content areas, and years of publication, but it isn't meant to embody the "best" activities published in *Mathematics Teacher*; this book would not have had enough space to include such a scope.

The number of student activities published in *Mathematics Teacher* increased over the past century, and the content areas represented by the activities changed. Table 1, organized into the *Focus in High School Mathematics* content areas, summarizes these changes by decade beginning in 1920, when activities began to appear in earnest in the journal.

Student activities in *Mathematics Teacher* increased sixfold in the 1970s. For example, before 1970 only seven published activities addressed functions, whereas the number of function activities increased from 10 in the 1970s to 55 between 2000 and 2009. The increase in function activities in the 1990s may have been a result of the inclusion of a function standard in NCTM's 1989 *Curriculum and Evaluation Standards for School Mathematics*. Also notable is the recent inclusion of statistics and probability, with more than twice the activities in the 1990s and 2000s than were present from 1920 to 1980. The number of geometry activities increased markedly beginning in the 1970s, when almost half the activities published addressed geometry. Although not as strikingly as the other content areas, the number of published activities in algebraic symbols also increased beginning in the 1970s.

This book's chapters are similarly organized by the content areas in *Focus in High School Mathematics: Reasoning and Sense Making*: (1) Number and Measurement, (2) Algebraic Symbols (3) Functions, (4) Geometry, and (5) Statistics and Probability. Each chapter contains the following:

- an introduction;
- a table listing related articles and activities from *Mathematics Teacher*, summarizing their features; and
- the articles and activities themselves.

The articles and activities represent a range of difficulty level and mathematical focus. Additional articles and activities are available as PDF downloads through NCTM's More4U online resource center. Simply go to www.nctm.org/more4u and enter the access code that is on the title page of this book.



REFERENCES

- National Council of Teachers of Mathematics (NCTM). *Curriculum and Evaluation Standards for School Mathematics*. Reston, Va.: NCTM, 1989.
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- Olson, Melfried. "A Geometric Look at Greatest Common Divisor." *Mathematics Teacher* 84 (March 1991): 202–8.