

Contents

Foreword vii

Preface ix

Introduction..... 1

 Pedagogical Content Knowledge 1

 Model of Teacher Knowledge 3

 Characteristics of Tasks 7

 Types of Questions 9

 Conclusion..... 10

Chapter 1

Diagrams and Definitions..... 11

 Identifying Common Misconceptions and Challenges..... 14

 Evaluating Prerequisite Knowledge and Skills with van Hiele Levels..... 14

 Level-to-level progress through experiential phases..... 17

 Van Hiele in the classroom: Hierarchy of Hexagons 18

 Developing Understanding of Diagrams with VPR..... 23

 Perceptual organization (VPR phase 1)..... 24

 Building perceptual organization through activities..... 27

 Recognition (VPR phase 2) 33

 Representation (VPR phase 3)..... 35

 Supporting Connections between Diagrams and Definitions 37

 Developing understanding of mathematical definitions..... 38

 Definitional activities in the classroom 40

 Summative activities for connecting diagrams and definitions..... 49

 Conclusion..... 51

Chapter 2

Transformations..... 53

 Identifying Common Misconceptions 58

 Examining general misconceptions..... 60

 Addressing misconceptions about terms and relationships:
 Eight Transformations..... 60

Examining misconceptions about the impact of a transformation 62

Reinforcing the realization that the plane is transformed:
Match a Transformation..... 65

Supporting Work with Multiple Transformations 68

Extending understanding by tessellating hexagons..... 70

Solidifying ideas about composition by working with the set of isometries .. 73

Applying Understanding to Transformations in the Coordinate Plane 79

Linking transformations and coordinates: A bridging activity 80

Exploring the effects of transformations on points in the coordinate plane .. 81

Building Understanding of Dilations..... 82

Underscoring the relationship between perimeter and area with a
dilation activity..... 85

Extending thinking about linear and area relationships with dilations 85

Using Transformations to Set the Stage for Proof..... 86

Supporting verification through transformations: A reflection proof 88

Using multiple transformations to demonstrate similar triangles..... 90

Conclusion..... 92

Chapter 3

Proof 95

Drawing on van Hiele Levels to Offer Opportunities for Proving 105

Delving into proving at level 2: Hierarchy of Hexagons, revisited..... 106

Supporting proving at level 3: Extending work with the hierarchy
of quadrilaterals 110

Identifying Common Misconceptions and Challenges..... 111

Focusing on Misconceptions about the Character and Meaning of Proof..... 111

Clarifying the nature of proof..... 113

Clarifying the universality of proof..... 116

Clarifying the generality of proof 119

Clarifying counterexamples..... 120

Focusing on Challenges to Creating Proofs..... 121

Helping students grasp the process—in particular, modes of
argumentation..... 122

Addressing visual dependence on diagrams for relationships 134

Ensuring readiness for proof 135

Conclusion..... 143

Chapter 4

Looking Back and Looking Ahead with Geometry..... 145

Looking Back: Geometry in Grades 6–8..... 145

Looking Ahead: Postsecondary Geometry..... 159

Advanced Euclidean geometry..... 159

Isometries and coordinate geometry 163

Dilations and coordinate geometry 168

Isometries as a group..... 170

Non-Euclidean geometry 173

Conclusion..... 177

Appendix 1

The Big Ideas and Essential Understandings for Geometry..... 179

Appendix 2

Resources for Teachers 181

Appendix 3

Tasks..... 187

Altitude of a Triangle: Getting the Picture 188

Altitude of a Triangle: Probing the Definition 189

Altitude of a Triangle: Probing the Definition—*For the Teacher*..... 190

What Is a Square? 191

What Is a Square? *For the Teacher*..... 192

Transformations That Combine Two Transformations 194

Transformations That Combine Two Transformations—*For the Teacher*..... 195

Transform the Figure; Transform the Coordinate Plane 196

Representing Simple Transformations in the Coordinate Plane 198

Proving with Transformations: Congruent Angles..... 199

Proving with Transformations: Congruent Angles—*For the Teacher* 200

References 201

Accompanying Materials at More4U

Links to GeoGebra and Applets Developed by the Authors

Appendix 1

The Big Ideas and Essential Understandings for Geometry

Appendix 2

Resources for Teachers

Appendix 3

Tasks

Hierarchy of Hexagons

Finding Triangles

Decomposing Squares

The Changing Diagram

Four Shapes

Mystery Shape

Altitude of a Triangle: Getting the Picture

Altitude of a Triangle: Probing the Definition

Altitude of a Triangle: Probing the Definition—*For the Teacher*

What Is a Square?

What Is a Square? *For the Teacher*

Eight Transformations

Match a Transformation

Transformations That Combine Two Transformations

Transformations That Combine Two Transformations—*For the Teacher*

Representing a Reflection with Shapes *and* Coordinates

Transform the Figure; Transform the Coordinate Plane

Representing Simple Transformations in the Coordinate Plane

Proving with Transformations: Congruent Angles

Proving with Transformations: Congruent Angles—*For the Teacher*

Proving with Transformations: Similar Triangles

Applying the Logic of $p \rightarrow q$