

School Mathematics

Preface

Principles and Standards for School Mathematics is intended to be a resource and guide for all who make decisions that affect the mathematics education of students in prekindergarten through grade 12. The recommendations in it are grounded in the belief that all students should learn important mathematical concepts and processes with understanding. Principles and Standards makes an argument for the importance of such understanding and describes ways students can attain it. Its audience includes mathematics teachers; teacher-leaders in schools and districts; developers of instructional materials and frameworks; districtlevel curriculum directors and professional development leaders; those responsible for educating mathematics teachers; preservice teachers; school, state, and provincial administrators; and policymakers. In addition, the document can serve as a resource for researchers, mathematicians, and others with an interest in school mathematics. Principles and Standards has been produced by the National Council of Teachers of Mathematics (NCTM), an international professional organization committed to excellence in mathematics teaching and learning for all

The NCTM had previously produced a landmark trio of Standards documents—Curriculum and Evaluation Standards for School Mathematics (1989), Professional Standards for Teaching Mathematics (1991), and Assessment Standards for School Mathematics (1995). These three documents represented a historically important first attempt by a professional organization to develop and articulate explicit and extensive goals for teachers and policymakers. Since their release, they have given focus, coherence, and new ideas to efforts to improve mathematics education.

From the beginning of its involvement in proposing education standards, NCTM has viewed its efforts as part of an ongoing process of improving mathematics education. For standards to remain viable, the goals and visions they embody must periodically be examined, evaluated, tested by practitioners, and revised. In the early 1990s, the Council began discussing the need for monitoring and updating the existing NCTM *Standards*. These discussions culminated in the appointment of the Commission on the Future of the Standards in 1995. In April 1996, the NCTM Board of Directors approved a process for revising and updating the original *Standards* documents. This project, which was dubbed "Standards 2000," illustrates how the setting of standards can serve as a reflective and consensus-building mechanism for all those interested in mathematics education.

A number of structures were established within NCTM to initiate Standards 2000. First, the Commission on the Future of the Standards was appointed in 1995 and charged to—

- oversee the Standards 2000 project and related projects;
- collect and synthesize information and advice from within and outside NCTM throughout the development of the project;
- develop a plan for the dissemination, interpretation, implementation, evaluation, and subsequent revision of future *Standards* documents.

The Standards 2000 Writing Group and the Standards 2000 Electronic Format Group were appointed by spring 1997. Each included individuals—teachers, teacher educators, administrators, researchers, and mathematicians—with a wide range of expertise. The Writing Group was charged to establish standards that—

- build on the foundation of the original *Standards* documents;
- integrate the classroom-related portions of Curriculum and Evaluation Standards for School Mathematics, Professional Standards for Teaching Mathematics, and Assessment Standards for School Mathematics;
- are organized into four grade bands: prekindergarten through grade 2, grades 3–5, grades 6–8, and grades 9–12.

The Electronic Format Group was charged to—

- think of alternative ways to present and distribute the document that would result;
- envision ways in which technology-based materials could be incorporated in the Standards;
- keep the Standards 2000 Writing Group up-to-date on uses of technology;
- assist in the work of the Standards 2000 Writing Group by finding examples of appropriate uses of technology.

The primary work of the Writing Group was carried out in sessions during the summers of 1997, 1998, and 1999. Extensive efforts were undertaken to ensure that the Writing Group was informed by the best of research and current practice. The writers had access to collections of instructional materials, state and province curriculum documents, research publications, policy documents, and international frameworks and curriculum materials.

Additional input was sought for the Writing Group through a series of activities orchestrated by the Commission on the Future of the Standards. In February 1997, invitations were extended by the NCTM president to all the member societies of the Conference Board of the Mathematical Sciences to form Association Review Groups (ARGs) that would "provide sustained advice and information as it reflects on K–12 mathematics from the perspective of your organization." Over the course of the project, fourteen Association Review Groups were formed, and five sets of questions were formulated and submitted to these groups for their responses.

NCTM's Research Advisory Committee commissioned a set of "white papers" summarizing the current state of education research in eight areas of mathematics teaching and learning to serve as background for the Writing Group. In addition, the Conference on Foundations for School Mathematics, held in Atlanta in March 1999 with support from the National Science Foundation, provided background to the writers concerning theoretical perspectives about teaching and learning. The papers written for this conference, along with the "white papers," are being published by NCTM as *A Research Companion to the NCTM Standards*. Two conferences, supported in part by the Eisenhower National Clearinghouse, also were held to inform the Writing Group about technology and advise it on the development of the electronic version of *Principles and Standards*.

A draft version of the Standards, entitled Principles and Standards for School Mathematics: Discussion Draft, was produced in October 1998 and circulated widely for reaction and discussion. Nearly 30 000 copies of the draft were furnished to persons interested in reading it, and many tens of thousands more accessed it from NCTM's Web site. Presentations and discussion sessions were held at all NCTM regional conferences in 1998–99, presentations were held at the conferences of many other organizations, and articles inviting feedback appeared in NCTM publications. In addition, 25 people were commissioned to review the draft from the perspective of their particular areas of interest. In total, reactions were submitted by more than 650 individuals and more than 70 groups (ranging from school study groups to graduate seminars to sessions held by NCTM Affiliates). The reactions were coded and entered into a qualitative database, resulting in the identification of a series of major issues for consideration. A synthesis of the issues and sample responses, as well as printouts of detailed feedback, was made available to the Writing Group for its work during summer 1999. Arguments on all sides of the issues were examined in the feedback. In light of the feedback, using the writers' best judgment, the Writing Group made careful decisions about the stance that Principles and Standards would take on each of the issues.

In response to a request from the NCTM Board of Directors and with funding from the National Science Foundation, the National Research Council formed a committee of experts from diverse backgrounds to review the process of gathering and analyzing reactions to the discussion draft, the plan to respond to the issues raised in the reactions, and the work of the Writing Group in carrying out that plan in the final document. The Writing Group was able to benefit greatly from the committee's guidance on responding to the comments and suggestions from reviewers and the field, and the document was improved as a result.

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Principles and Standards reflects input and influence from many different sources. Educational research serves as a basis for many of the proposals and claims made throughout this document about what it is possible for students to learn about certain content areas at certain levels and under certain pedagogical conditions. The content and processes emphasized in Principles and Standards also reflect society's needs for mathematical literacy, past practice in mathematics education, and the values and expectations held by teachers, mathematics educators, mathematicians, and the general public. Finally, much of the content included here is based on the experiences and observations of the class-room teachers, teacher educators, educational researchers, and mathematicians in the Writing Group and on the input the Writing Group received throughout the drafting of the document.

Principles and Standards includes a number of classroom examples, instances of students' work, and episodes that illustrate points made in the text. If drawn from another published source, the example or episode includes a citation to that source. If an episode does not have a citation and is written in the past tense, it is drawn from the experiences of a Writing Group member or a teacher colleague, with an indication of its source (such as unpublished observation notes) where appropriate. Episodes written in the present tense are hypothetical examples based on the experiences of the writers and are identified as such.

This document presents a vision of school mathematics—a set of goals toward which to strive. Throughout the document, this vision for mathematics education is expressed using words like *should*, *will*, *can*, and *must* to convey to readers the kind of mathematics teaching and learning that NCTM proposes. In no sense is this language meant to convey an assurance of some predetermined outcome; it is, rather, a means of describing the vision NCTM has constructed.

Principles and Standards is available in print, PDF, and electronic hypertext formats. The electronic edition of Principles and Standards for School Mathematics (E-Standards) includes tools to enhance navigation of the document, as well as a more extensive set of electronic examples (e-examples) to illuminate and enlarge the ideas in the text. It has also made possible the inclusion of links to resource and background material to enhance the messages of Principles and Standards. The e-examples are keyed to particular passages in the text and are signaled by an icon in the margin. The E-Standards is available both on CD-ROM and currently on the World Wide Web at standards.nctm.org.

In the coming years, *Principles and Standards for School Mathematics* will provide focus and direction to the work of the Council. A number of initiatives related to *Principles and Standards* have already begun. The Council has established a task force to develop a series of materials, both print and electronic, with the working title Navigations to assist and support teachers as they work to realize the vision of the *Principles and Standards* in their classrooms, much as the Addenda series did following the release of the *Curriculum and Evaluation Standards for School Mathematics*. A series of institutes, organized by NCTM's new Academy for Professional Development, will give leaders a concentrated introduction to *Principles and Standards* and explore in depth various Standards or themes in the document. Yet another task force is developing a plan and materials to help the Council effectively reach out to education administrators. The Standards Impact Research Group has been established to consider how the overall

process of Standards-based education improvement set forth in the document can be better understood and subsequently refined in order to meet the goal of improving students' learning. The E-Standards Task Force is considering ways to expand and improve future electronic versions (both Web and CD) of *Principles and Standards*, and the Illuminations project is providing Web-based resources to "illuminate" the messages of the document, with funding provided by MarcoPolo Education Foundation. These activities (and many others that will inevitably emerge in the coming years) build on the solid foundation of *Principles and Standards*, ensuring that the National Council of Teachers of Mathematics will continue to provide leadership in improving the mathematics education of all students. For current information on these and other efforts and for other information about the document, visit www.nctm.org.

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