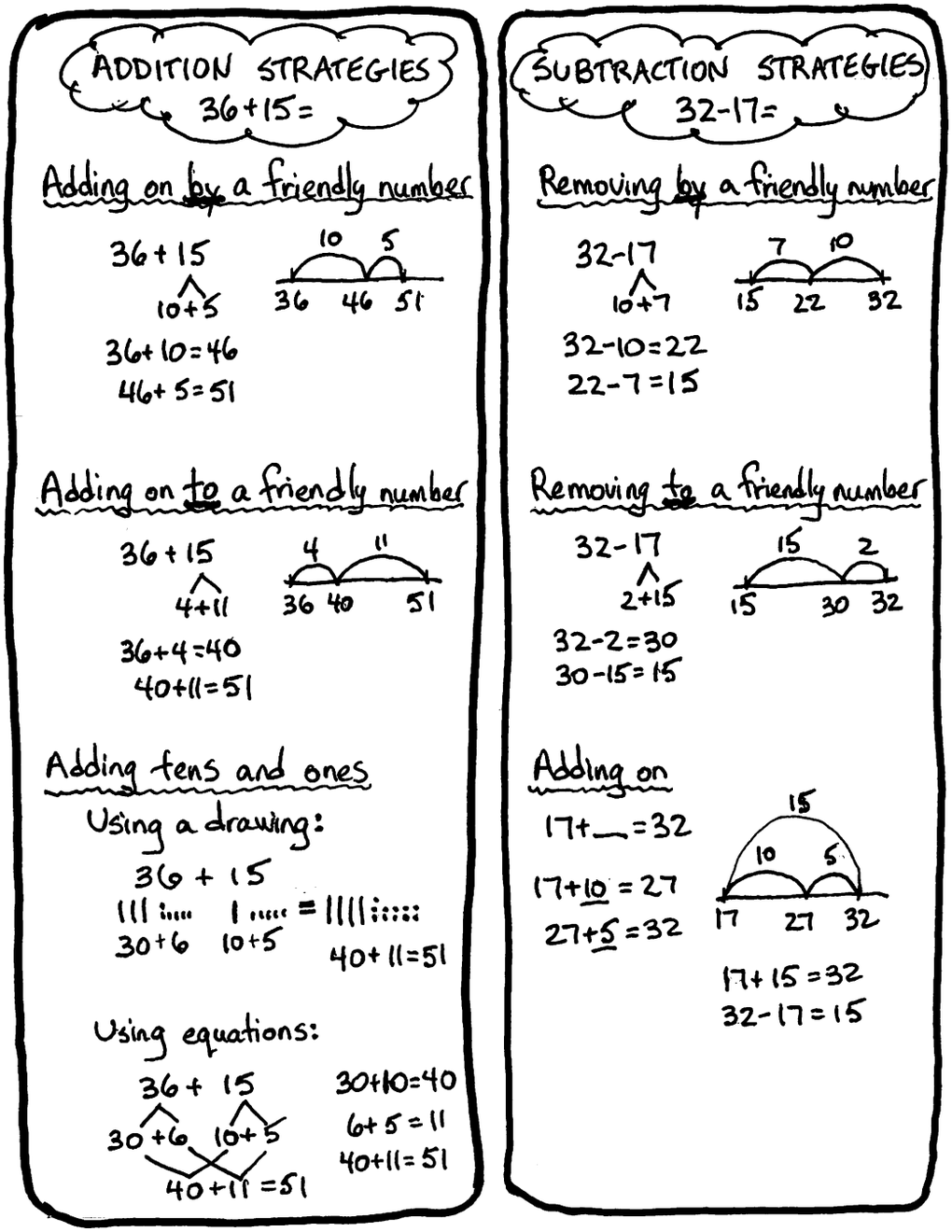
chapter one

How Should Elementary Math Class Look and Sound?

Picture a second-grade classroom. It is November, and students are working to make sense of and solve addition story problems. They are involved in doing mathematics, and the teacher is working skillfully and intentionally to support them. Read and reflect on several excerpts from one lesson to examine the work of students and teachers in classrooms where students learn math with understanding.

For the past two weeks, these second-grade students have been solving addition and subtraction story problems with two-digit addends. During each class period, students solve problems on their own, share their solutions with a partner, and engage in a ten- to twenty-minute whole-class discussion of a few student strategies. These students have not been taught particular procedures for adding and subtracting two-digit numbers, nor have they been taught to recognize "key words" for deciding which operations to use to solve story problems. However, these students have studied place value and have been using what they know about two-digit numbers to develop increasingly efficient and sophisticated strategies for addition and subtraction. At the front of the room hang charts of addition and subtraction strategies that the students have developed.



The teacher, Ms. Davis, has assessed her students informally throughout the unit. Each day during class, she circulates as students work, observing what strategies students use, taking notes, and questioning students both to better assess what they know and to support and extend their thinking. Today, she focuses her questioning and instruction on addition strategies. At this point in the unit, many students have developed the strategy of breaking both numbers into tens and ones to add them (adding tens and ones). However, many students still rely on drawings to break the numbers into tens and ones. Ms. Davis wants to encourage more students to try using just equations when solving the problems. But she wants to do so such

that the new strategy makes sense to students and that they see the connection between the drawings and the equations.

As you read these vignettes, pay attention to what the students do and say and what that reveals about what they know and are learning about mathematics. Also, pay attention to what the teacher does and says and how her words and actions are likely to support students' learning.

Read through each vignette once, paying attention to the students, and then read it again, paying attention to the teacher. Then read the commentary provided.

INTRODUCING THE LESSON

Ms. Davis has written this problem on the board:

> Sandy had 46 baseball cards. Her brother gave her 37 baseball cards for her birthday. Now how many baseball cards does Sandy have?

Ms. Davis Saya, "Today we are going to work on some more story problems, just like you've been doing for the past few days. We'll begin by thinking about the first problem together. Read the problem to yourself and try to picture what is happening in the story. Then decide what the story is asking you to find out." After about a minute, she says, "Raise your hand if you've had enough time to figure out what is happening in this story problem."

Twenty of the twenty-five students raise their hands.

Ms. Davis says, "I'm going to read the problem out loud, and I want everyone to try to picture what is happening in this problem." She reads the problem. Ms. Davis then turns to Isaiah, who had raised his hand previously, and says, "Isaiah, can you tell us what happens first in this story problem?"

Isaiah slowly responds, "Um . . . first Sandy has 46 baseball cards."

Ms. Davis begins making a flowchart. She writes "Sandy has 46 cards" and draws a rectangle around it. "Then what happens?" She pauses for students to think. "Emma, what happens next?"

"Her brother gives her 37 more baseball cards," Emma says.

Ms. Davis draws an arrow and another rectangle. Inside she writes, "Brother gave her 37 cards." Then she asks, "What happens next?" She calls on Marcus.

Marcus says, "How many cards does she have now?"

Ms. Davis records the following:

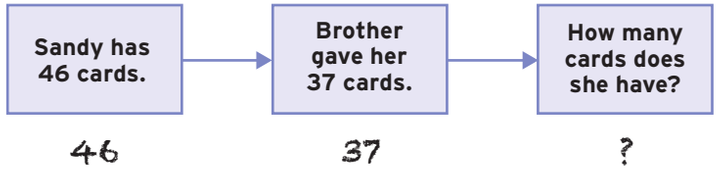
Ms. Davis says, "So let's read our problem together." She leads the class in choral reading of the chart sequencing the action of the story. "What do we have to find out?" She calls on Alia.

Alia says, "We have to find out how many cards she has now."

Ms. Davis asks, "How can we find out how many cards Sandy has now?"

"You have to add 37 cards to the 46 because now she has more," Alia answers.

Ms. Davis asks, "Who agrees with what Alia just said?" All hands go up. Ms. Davis writes the numbers 46 and 37 and a question mark under the boxes in the flowchart.



"How can we show that we're adding 37 to the 46? Sam, how do you think we could do that?"

"Put a plus sign?"

"That's right. We use an addition sign to show that we're adding."

Ms. Davis writes "46 + 37 = ?" on the board. She continues, "Let's read this together." The class reads the equation as Ms. Davis points: "46 plus 37 equals something."

Ms. Davis asks, "What's the 'something' you have to find out?"

The students say, "You have to find out how many baseball cards Sandy has now."

"So now we've translated the story problem into an equation, and Alia just told us we have to find out what 46 cards and 37 cards adds up to. Now I'd like you to think of a strategy that you could use to solve this problem. You can look at the chart of addition strategies to help you. Choose a strategy that makes sense to you and that you think is efficient. When you have a strategy, put a hand on your head. When I know you have a strategy, I will call on you to go back to your seat and get started. You will have about twenty-five minutes to work on these problems."

STOP+ REFLECT

- What did Ms. Davis ask the students to do and think about in the lesson introduction?
- How did Ms. Davis support all students to make sense of the problem?

The lesson begins with a problem, and Ms. Davis does not tell the students how to solve it. Instead she deliberately focuses students on visualizing the sequence of actions in the problem, translating the problem into an equation, determining what the problem asks them to find out, and choosing a strategy to solve it. Instead of telling students key words, Ms. Davis gives them time and focus questions to help them make sense of any problem. Her goal is to have all students leave the meeting

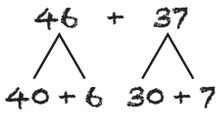
area understanding the first problem and with an idea for a strategy they could use to solve it. She is supporting all her students in becoming problem solvers.

TWO STUDENTS AT WORK

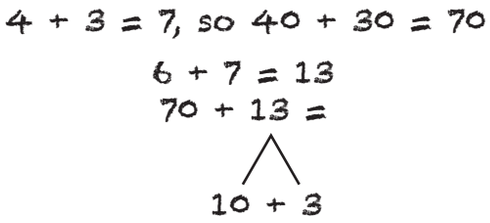
Tiffany and Michael tackle Ms. Davis's problem:

> Sandy had 46 baseball cards. Her brother gave her 37 baseball cards for her birthday. Now how many baseball cards does Sandy have?

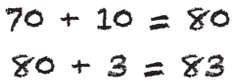
Tiffany sits down at her desk and writes her name on her paper. She rereads the problem, looks up briefly at the strategy charts, and then writes



in the space under the word problem. Under the 46 she writes 40 and 6, showing that 46 can be split into 40 and 6. Under the 37 she writes 30 and 7, showing that 37 can be split into 30 and 7. Then she writes

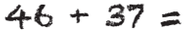


She then writes



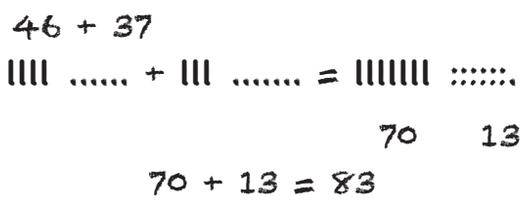
Tiffany moves on to the next problem.

Michael, sitting next to Tiffany, writes



on his paper. He then says "40" as he draws four lines to represent sticks of ten and says "6" as he draws six small dots next to the lines. He writes an addition sign and mouths "30" as he draws three lines and "7" as he draws seven dots. He writes an equals sign and draws four lines, then three more, then a row of six dots and a row of seven dots underneath. His paper now looks like this:

Michael writes "70" under the sticks of ten, counts the dots by ones, and writes "13" under the dots. He writes "70 + 13 =" and then he pauses, looks intently at the hundred chart on the wall for few moments, and then writes "83." His paper now looks like this:



Michael goes on to the next problem.

After about twenty minutes, the teacher stops the class and says, "Now I would like you to share your solutions with your partner. When I finish talking, show your partner your paper and explain how you solved the problem. Then look at your partner's paper and listen to how your partner solved the problem. After that, I want you to discuss ways that your solutions are similar and different. You have three minutes. Please begin."

Tiffany immediately turns to Michael, who does not look up. Michael continues making lines and dots on his paper to represent numbers in the problem he is working on. Tiffany says, "Stop. We need to share."

Michael says, "I know," as he draws four more dots. He then looks up.

Tiffany turns her paper toward Michael and says, "I solved the first one by breaking the 46 into 40 and 6 and the 37 into 30 and 7, and I added the 40 and 30 to get 70 and the 6 and 7 to get 13. Then I added the 70 and 13 and got 83."

Michael says, "I drew sticks of ten. I made four sticks of ten and six ones, and three sticks of ten and seven ones, and then I counted the sticks and counted the ones and added them up. I got 83, too." Michael adds, "We both got the same answer."

Tiffany says, "We both wrote 70 + 13 = 83," and then adds, "But you make sticks of ten and I just used numbers."

STOP+ REFLECT

- What is Tiffany doing and thinking about as she solves the problem?
- What is Michael doing and thinking about as he solves the problem?
- How do Tiffany's and Michael's solutions compare?
- What does the teacher ask them to do?
- What is the teacher's role in this vignette?

As the students complete the problem, record their work, and share strategies, they are problem solving, reasoning, representing their thinking on paper, communicating their thinking to each other, and looking for connections. Tiffany reasons about numbers and number relationships when she writes "4 + 3 = 7, so 40 + 30 = 70." Michael uses a base-ten representation when he records 46 as four ten sticks

and six ones. Tiffany and Michael communicate their thinking orally and in writing, which helps them become aware of the strategies and mathematical relationships that they are using to solve the problem, helps them see connections between their strategies, and will let the teacher assess what each student knows and can do.

The teacher's role here may seem minimal. However, this level of student work and conversation is possible only because the teacher has established a safe and intellectually rigorous classroom environment in which students are expected to solve problems, record their thinking, and communicate clearly and respectfully with one another.

THE TEACHER SUPPORTS A STUDENT HAVING DIFFICULTY

Timmy is sitting at his desk staring at his paper, not writing anything. Ms. Davis walks by and stops. She waits a few minutes, watching Timmy. Timmy writes "46 + 37" on his paper. Then he stops. Ms. Davis squats down at eye level with Timmy and says, "Tell me about this equation that you just wrote down." Ms. Davis points to the 46. "What does the 46 represent?"

Timmy pauses, glancing up at the flowchart Ms. Davis constructed with the class during the lesson launch. "That's how many baseball cards Sandy had."

"And what does the 37 represent?"

"That's the cards her brother gave her for her birthday."

"What is the problem asking you to find out?"

"Um . . . how many baseball cards she has now?"

"How can you figure that out?"

"By adding the 46 cards and the 37 cards."

"What strategy can you use to help you add all the cards?"

[*Pause.*] "I could draw sticks of ten?"

"OK. Why don't you draw sticks of ten? I'll come back in a few minutes to see what you've done."

When Ms. Davis returns, Timmy has drawn a picture similar to Michael's: four sticks of ten and six dots, and three sticks of ten and seven dots. He is counting the dots. She sees him point to the last few dots and say, "180, 190, 200, 300." He writes "300" on his paper and circles it.

Ms. Davis gently asks, "Timmy, can you explain to me how you solved this problem?"

"Um . . . I drew four tens and six ones for 46 and I drew three tens and seven ones for 37. Then I counted the tens: 10, 20, 30, 40, 50, 60, 70. Then I counted the ones: 80, 90 . . . no . . . I mean, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83."

STOP + REFLECT

- What does Timmy understand about the baseball cards problem?
- What does he understand about the numbers 46 and 37?
- What mistake does Timmy make in solving the problem?
- How does the teacher interact with Timmy after she notices him staring at his paper? After she sees him write an equation? After she observes him counting incorrectly?
- How does the teacher support Timmy in problem solving?

Ms. Davis asks, "So how many baseball cards do you think Sandy now has?"

Timmy answers, "83."

Ms. Davis asks, "Why 83 and not 300?"

"Because I counted wrong before."

Ms. Davis says, "Good work! You found your mistake by checking your thinking," and she leaves to observe other students solving the problems.

Timmy is slow to start solving the problem and then makes a common counting mistake. He initially continues counting by tens when he moves from counting the sticks of ten to the ones. However, Timmy does realize that the problem is an addition problem, and he represents 46 as four tens and six ones and 37 as three tens and seven ones. Ms. Davis first observes Timmy staring at his paper but says nothing. Then she asks him about the meaning of his equation and how he plans to solve the problem, and she tells him she will return to check on his progress. Later, after observing him make a mistake, she asks him to explain his thinking, during which he realizes his mistake and fixes it.

Asking students to record their work and explain their solutions helps them become more aware of their own thought processes. Here, explaining how he got his answer enables Timmy to find and fix his own mistake. Ms. Davis supports Timmy in problem solving by initially assessing whether he understands the problem and has a way to solve it, by monitoring and assessing his progress in solving the problem, and by asking him to explain his thinking. Ms. Davis also supports Timmy in problem solving through what she does not do: She does not step in and do any work for him, either to help him fix his mistake or to help him get more work done.

SHARING STUDENT SOLUTIONS WITH THE WHOLE GROUP

Ms. Davis asks students to bring their papers but not their pencils to the meeting area. The students sit in a circle on the rug. Ms. Davis asks them to place their papers on the floor in front of them so that she can see their different solutions. Ms. Davis has already circulated as students worked, so she is aware of most strategies students used and has jotted down names of some students who used each strategy. However, because she talked for ten minutes with Timmy, she could not observe what strategies all students used. Ms. Davis takes a minute to scan the papers, as the students wait patiently.

Ms. Davis says, "We'll discuss solutions to the first problem about the baseball cards and then compare two different ways to solve that problem. Who would like to remind us of the problem? Maya, can you please read it loudly enough for all of us to hear?"

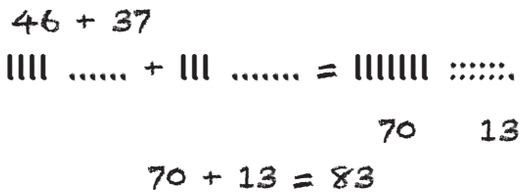
Maya reads, "Sandy had 46 baseball cards. Her brother gave her 37 baseball cards for her birthday. Now how many baseball cards does Sandy have?"

Ms. Davis says, "I first want us to look at Michael's solution. Michael, would you come up and show us how you solved the problem?"

Michael carries his paper with him to the front of the room. "First, I knew it was addition, so I wrote '46 + 37.'" He writes

46 + 37 =

He looks at his paper. "Then I drew tens and ones." Michael draws four tens sticks and six dots, and three tens sticks and seven dots, under the 46 and 37. "Then I added them." Michael writes on the board what is on his paper.



Ms. Davis asks the class, "What do you notice about Michael's solution?" She calls on a few students:

One says, "He used tens and ones."

Another says, "He made drawings."

Someone else says, "He did addition."

Ms. Davis says, "Michael, I have a question. How did you add the 70 and 13 to get 83?"

Michael answers, "I used the hundred chart."

"How did you use the hundred chart—can you show us?"

Michael walks to the hundred chart and points. "I started at 70. I went down one." He moves his finger to 80. "And then I went 81, 82, 83."

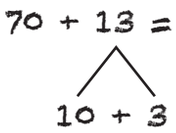
Ms. Davis asks, "Why did you 'go down one'?"

"To add 10."

Ms. Davis waits and then asks, "Where did the 10 come from?"

"The 13."

Ms. Davis summarizes, "So it looks like Michael broke this 13 into a 10 and 3." She writes "10" and "3" below Michael's equation.



"Michael made a jump of ten on the hundred chart to add the 10, and then he moved three spaces horizontally—one, two, three [*pointing to the 81, 82, and 83 on the chart*]—to add the 3." She writes

70 + 10 = 80
80 + 3 = 83

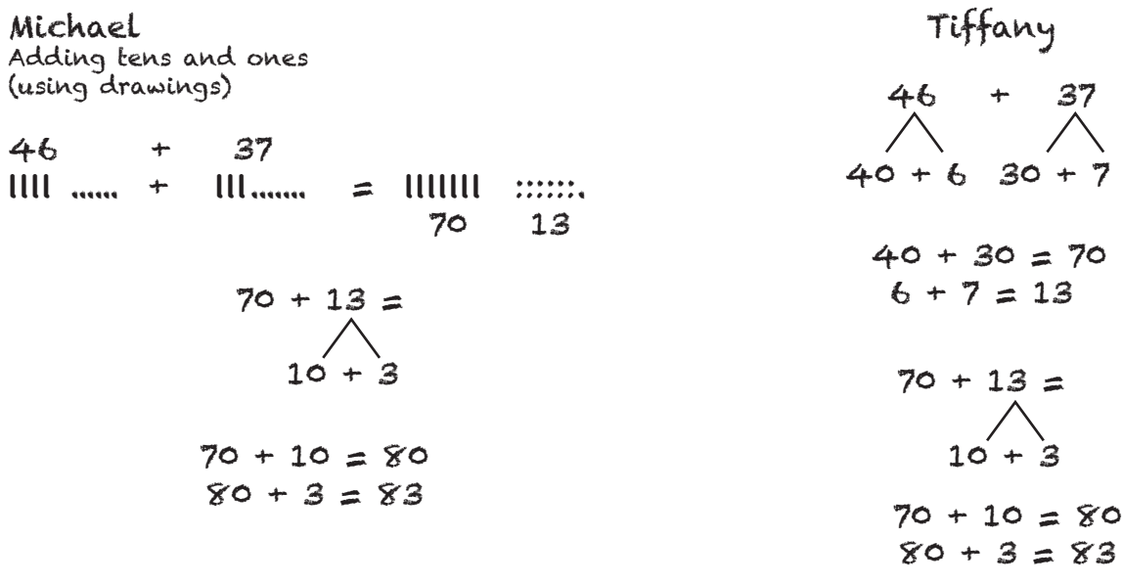
Ms. Davis continues, "Let's look at our chart of addition strategies. What strategy did Michael use?" She pauses to let students think. "Ally, what strategy do you think he used?"

"He added tens and ones . . . and he did drawings."

Ms. Davis writes "Michael" and "Add tens and ones (using drawings)" above Michael's solution.

She then says, "Next I would like Tiffany to share her solution. Tiffany, can you write your solution on the board next to Michael's?" Tiffany carries her paper to the front of the room. First she writes. Then she stops to explain, "I split 46 into 40 and 6 and 37 into 30 and 7." She pauses briefly and then continues: "I added 30 plus 40 and got 70, and I added 6 plus 7 and got 13." She clarifies, "I knew that 6 plus 6 was 12, so one more was 13," but does not record that part. "Then I added 70 plus 13. I split the 13 into 10 and 3, like Michael. I just knew that 70 plus 10 was 80 and then I knew that 80 plus 3 was 83."

The board now has both students' strategies side by side:



Ms. Davis says, "Thank you, Tiffany. You can go back to your seat now." Then Ms. Davis says to the group, "I'd like everyone to look carefully at Tiffany's solution and see whether you understand what she did. Once you understand it, please raise your hand." Ms. Davis waits for one whole minute. By then eighteen hands have gone up. "Explain to your partner what you think Tiffany did to solve the problem."

Students turn and talk. Ms. Davis leans in and listens to a few different pairs. "Now stop talking. I have another question for you. How is Tiffany's solution

similar to Michael's? Think about it first. Then talk to your neighbor about how they are similar."

Ms. Davis again listens in as students talk. Crystal says, "They both did tens and ones. Tiffany used numbers and Michael used pictures."

After giving students another minute to talk, Ms. Davis asks them to stop and calls on Crystal. "Crystal, can you share how you think what Tiffany did is similar to what Michael did?"

Crystal replies, "They both did tens and ones."

"Can you say more about how they both used tens and ones?"

Crystal continues, "They both split numbers into tens and ones. Tiffany wrote 40 and 6 and 30 and 7. Michael drew four ten sticks and six ones and three ten sticks and seven ones."

"How else are the solutions similar? Thomas?"

Thomas answers, "They both used addition."

"Are they similar in any other ways? Andre?"

Andre replies, "I know a way they are different."

"How are they different?"

"Michael used the hundred chart and Tiffany didn't."

"That's true, but let's keep talking about how these are the same. Hannah? How do you think these strategies are the same?"

Hannah responds, "Most of it is the same. They mostly have the same numbers. But Michael drew tens and ones and Tiffany just did numbers."

Ms. Davis asks, "Who heard what Hannah just said?" Ten hands go up. "Hannah, can you say what you just said again? And come show us the parts that you think are mostly the same."

Hannah says, "This part is mostly the same." She waves her hand around the equations under Michael's drawings and the same equations in Tiffany's solution. "Michael just made drawings at the top, and Tiffany didn't."

Ms. Davis says, "This is interesting. Crystal, you said the first part of their solutions were similar. You said that they both broke their numbers into tens and ones. Now Hannah is saying that the next part of their solutions are similar. It sounds like these two solutions are pretty similar. They both used the adding-tens-and-ones strategy. They both split 46 into 40 and 6 and 37 into 30 and 7, and they both added 70 plus 13 to get 83.

"I'm curious, who solved the problem in a way similar to Tiffany's?" Eight hands go up. "Who solved the problem in a way similar to Michael's?" Nine hands go up. "Who solved the problem in a different way?" Six hands go up. Ms. Davis says, "We've been talking about choosing efficient ways to solve problems—ways that make it easy for you to keep track of the problem and all the steps. Which solution was more efficient, adding tens and ones by using drawings or by using equations? [*She points to the two solutions on the board.*] Think, and then talk to your partner about which one you think is most efficient and why."

Ms. Davis listens to conversations the students have with each other. She notices that as they justify their reasoning, all are looking at Michael's and Tiffany's solutions. She hears Somo and Andre discussing the number of steps in each strategy. After three minutes, she says, "I heard Somo say something interesting. Andre, can you tell us what Somo said?"

Andre says, "Somo said that using equations is more efficient because it has fewer steps."

Ms. Davis asked, "Do you know what Somo means about fewer steps?"

"Yeah, you don't have to make the drawings and count the dots. You just have to write the numbers. You can add 6 and 7 in your head instead of counting."

Ms. Davis checks with Somo, "Is that what you said?"

Somo replies, "Uh-huh . . . and . . . I think you might make a mistake when you are drawing. You might draw the wrong number . . . like you might draw eight dots when you were supposed to do seven."

Ms. Davis summarizes, "So you thought of two reasons why using equations is more efficient than using drawings. You said that it has fewer steps. You also said if you use equations you might be less likely to make a mistake."

She tells the class, "I want you to think about whether you agree with what Andre and Somo just said, disagree with it, or are not sure." She gives them time to think. "OK, put a thumb up if you agree with what Andre and Somo said." Most show a thumbs-up sign. "OK, now put your thumb up if you disagree with what Andre and Somo said." Nobody puts a thumb up. "Now put your thumb up if you are not sure about what Andre and Somo said." Three students show thumbs-up. She looks at those students and says, "That's OK if you are not sure. We are going to keep thinking and talking about this."

Then Ms. Davis ends the lesson by saying, "It seems that most of you think that adding tens and ones by using equations is more efficient. But remember, it's efficient for you only if it makes sense and you practice using it. If you've used only drawings, I'd like you to try using equations tomorrow when you solve problems."

STOP + REFLECT

- Why does Ms. Davis ask Michael and Tiffany to share their solutions?
- How does Ms. Davis engage students in thinking and talking about Michael's and Tiffany's solutions? Find specific things that she does.
- Why does Ms. Davis spend class time discussing two solutions, instead of sharing many different solutions or going over the answers to all the problems?
- What might students learn from this discussion? How might they solve future problems differently because of this discussion?

Ms. Davis chooses two students to share their solutions after observing students solving the problems and after scanning papers that students brought to the rug. Ms. Davis chooses these two students because she has a learning goal in mind: to move students from drawing pictures of tens and ones to using equations.

She knows that not all students are ready to stop using drawings. Timmy may continue using them for some time. But other students, like Michael, are probably ready. She wants those students to realize that Tiffany's approach is not so different from theirs. So Ms. Davis focuses on the similarities between the solutions and asks the students to look for connections.

Ms. Davis's decisions support students in understanding representations, making connections, and communicating their thinking and reasoning: She makes room on the board for two solutions to be written side by side. She periodically asks

clarifying questions, such as where the 10 came from. She asks students to show their thinking, as when she asks Michael to show how he added the 13 with the hundred chart or when she asked Hannah to show the parts of the solutions that are "mostly the same." Ms. Davis gives wait time after her questions. She asks students to talk to further process their ideas, and she calls on a range of students, not just the first ones to raise their hands. Finally, Ms. Davis asks students to think about which solution is more efficient and why. Through all these means, Ms. Davis supports broad participation in a mathematically focused discussion.

CONCLUSION

The National Council of Teachers of Mathematics (2000) identified five Process Standards that "highlight ways of acquiring and using content knowledge": problem solving, reasoning and proving, making and using representations, communicating mathematical thinking, and making connections (p. 29). They are similar to the Common Core State Standards for Mathematical Practice. In classrooms such as Ms. Davis's, the teacher engages all students in these processes.