Chapter 1

Effectively Teaching to the CCSSM Standards

In this introductory chapter, we lay out a philosophy for the most effective way of teaching to the Common Core State Standards for Mathematics (CCSSM). This philosophy includes our beliefs about the importance of (*a*) an inclusive classroom environment, where the contributions of all students are valued; (*b*) the use of problem solving as a vehicle for engaging students in content within the structure of a three-part lesson format; and (*c*) engaging students with mathematics. The three sections of this chapter discuss these points in depth; the five chapters of the book that follow will show how to implement this philosophy in the classroom through thirty-eight tasks designed to meet specific CCSSM standards.

An Inclusive Classroom Environment

Even an engaging and approachable mathematics task is not sufficient to support student achievement, if it is not also accompanied by a classroom atmosphere that values all students, their contributions, and the knowledge they bring. Along with providing interesting and appropriately challenging tasks, it is critical to engage learners in the examination and discussion of mathematical ideas and processes. Students who do not participate are unlikely to learn (Lave and Wenger 1991).

One essential aspect of participation within the problem-solving lesson is the exchange of mathematical ideas through discussion. In order for students to be comfortable presenting their ideas to others, either in small groups or within whole-class discussions, the teacher must create a classroom environment where students feel able to take risks when presenting their thinking, even if their ideas are not fully developed. A classroom should be a place where it is safe to present this emergent thinking and acceptable to revise your thinking when other work that challenges any misconceptions is presented. Emergent thinking might include, for example, a misapplication of principles of operations of addition (add ones and tens) to multiplication of two-digit numbers (not applying the distributive property and thus only multiplying ones with ones and tens with tens). Misconceptions or erroneous answers can be positioned as an opportunity for learning by all, as they often reveal common ways that students make sense of complicated mathematics (e.g., see tasks 3.1 and 4.5) and show the importance of being open to revision (e.g., see task 3.6). Teachers need to support students in developing this willingness to revise thinking, as this process leads to more sophisticated understandings of various topics. In such an open classroom environment, students feel validated and appreciate that the contributions they bring to exploring and discussing mathematics are valued.

As we all know, learning does not happen just in school; it also occurs in other places that students inhabit, such as their homes and communities. In addition to respecting classroom contributions, valuing the knowledge that students bring with them to the classroom—and building on that knowledge—can also support student success. When teachers incorporate familiar home or community contexts into problem solving, this can support students' development toward meeting the CCSSM standards as they grapple with situations in which they are able to draw on their own knowledge (Moll et al. 1992; González et al. 2001). This placing of mathematics within a familiar context also increases the likelihood of equitable learning outcomes in the classroom. In this volume we have attempted to create problem contexts that are accessible to all children, but teachers should use their knowledge of their own students to modify problem contexts as necessary in order to insure that students are able to understand the tasks and find entry points to solve them (e.g., see task 3.6).

A Problem-Solving Approach

Building on the work of Van de Walle, Karp, and Bay-Williams (2010), all of the mathematics tasks in this book employ a problem-solving approach, which follows a three-part lesson format:

Part 1: Launching the Task

Part 2: Task Exploration

Part 3: Summarizing Discussion

In **Launching the Task,** students are presented with a mathematical task that provides them with an opportunity to explore and engage with a particular CCSSM domain. During the launch, the teacher orients students to the task by focusing on three elements: (*a*) *task:* how students understand what a task is asking; (*b*) *engagement:* how students will engage in the exploration; and (*c*) *accountability:* how and in what capacity students will be accountable for demonstrating their thinking and problem solving. Problems can be launched in a variety of ways, such as discussing aspects of the problem that students might not understand fully without some whole-class conversation, reading a book that links to and supports understanding of the problem, brainstorming what students know or think about a particular topic, or linking the problem to some previous experience with mathematics.

In the **Task Exploration** part of the lesson, students have an opportunity to explore the target mathematics using relevant representations and working most often in pairs or in small groups. Students engage in working on (or exploring) the problem while the teacher circulates among groups or individuals, helping students to get started, conducting informal observations and assessments, and/or supporting student thinking with pertinent questions and comments. During this exploration period, teachers seek to understand the problem-solving approaches and mathematical thinking their students are using. In this way, teachers have an opportunity to prepare themselves for the summarizing discussion, during which several students or groups of students will present their thinking. Teachers should come to the exploration portion of the lesson having anticipated both correct and incorrect student responses to the task, and representations that students might use in their problem solving. In this way, they are better prepared to consider the range of ideas and representations that students will use during the exploration. Although they may not have anticipated each student response, through this advanced consideration of student responses, teachers are better prepared to interact thoughtfully with students to support their task exploration.

In the **Summarizing Discussion** that concludes the lesson, various solution paths that have been used to approach the problem are discussed. Connections are made between the mathematics and student thinking. It is during this time that more emergent strategies, such as direct modeling, can be linked to more sophisticated numeric strategies, thus supporting conceptual development. The summarizing discussion should be seen as a non-negotiable norm when it comes to a problem-solving lesson. Many times, teachers spend time on the launch and the exploration but then find the discussion portion of the lesson more difficult to include in the time allotted. This is unfortunate, as the discussion is a critically important part of the lesson. This is the part during which students make connections and then confirm, revise, or correct their thinking, including confronting misconceptions. These discussions are valuable in that they (*a*) support students in developing their metacognitive skills—that is, how to monitor their thinking in relation to situations; (*b*) allow students to understand how others can think differently, thus deepening their understanding that there are a variety of approaches possible to any given problem; and (*c*) support students in expanding their repertoire of problem-solving strategies. Not having a summarizing discussion can be detrimental to students' growth, as they may leave a problem-solving experience with misunderstandings that could have been addressed during discussion.

It is important that before students present their thinking in front of others, the teacher, during the earlier exploration section, has reviewed their work. Not all student work that is presented needs to be correct, as there is value in discussing emergent thinking on a topic. As mentioned previously, the teacher needs to have created a classroom environment in which it is safe to present emergent thinking and acceptable to revise thinking when other work that challenges misconceptions is presented. Exploring misconceptions must be positioned as a worthwhile enterprise. When students present work that the teacher cannot position as valuable and contributory, this can be embarrassing and consequently demoralizing for them, something that can seriously impact how they feel towards mathematics.

Fosnot and Dolk (2001) discuss how "contextual problems" can be a good way to "develop children's mathematical modeling of the real world" (p. 24). A problem-solving approach can be brought to bear on both contextualized problems (or what are often referred to as word problems) and bare number problems, or problems that are not set within a context. Problems that may have mathematical merit from an instructional perspective but might appear dry to students can often be given a simple context to make the problem more accessible to students. See, for example, how sample task B gives a context to the problem included in sample task A.

Sample Task A

Figure out what this point is called on the number line.

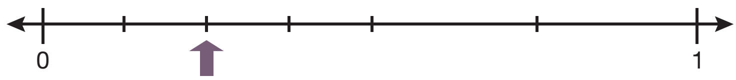

Fig. 1.1. Number line for sample tasks A and B

Circle the correct answer:

$^2/_6$ $^2/_7$ $^1/_4$ 2 $^2/_4$

How did you figure out the answer?

Sample Task B

A runner is running a one-mile race. He is currently where the arrow is pointing to on the number line in figure 1.1. How far along the way do you think he is?

(*a*) two-sixths (*b*) two-sevenths (*c*) one-fourth (*d*) two (*e*) two-fourths

Please explain why you chose your answer.

There are times, however, when it may be more practical or even advantageous to present students with a non-contextualized or bare number problem such as the one in sample task A (Saxe et al. 2007). Well-designed problems involving only bare numbers have the potential to engage students in puzzling over mathematical arguments, structure, and representations by inspiring their curiosity. In the case of sample task A, for example, the unequal intervals in the problem contradict students' expectations of number lines, provoking them to puzzle over the principles or rules that underlie a representation they may see every day. Within this volume, you will find both types of problems.

Engaging with Mathematics

As well as delineating the mathematical content to be addressed at each grade level, CCSSM also offers eight key mathematical practices in which students should engage. These eight Standards for Mathematical Practices build on the earlier work of the NCTM Process Standards (NCTM 2000) and the Strands of Mathematical Proficiency from *Adding It Up* (National Research Council 2001). In this book, we will highlight particular practice standards that are addressed by the thirty-eight included tasks. Not all practice standards will be highlighted within particular tasks, but within the volume as a whole all eight practice standards are discussed in relation to specific problems.

In the various content chapters, we have often taken an "across the grades" perspective in developing and explicating tasks for this band of grades 3–5. This is for a number of reasons. Not only are there teachers who are teaching multigrade classes, but it is also typical for teachers to have a wide range of abilities among students even in a single-grade class. As we know, development is uneven, and mathematical tasks are at their best when they have multiple entry points for students of varying levels. This also means that it is helpful if tasks can be solved using a variety of strategies in addition to numeric approaches. These can include direct modeling, which can take the form of using manipulatives or representing thinking through diagrams or drawings. This method of presentation makes the tasks accessible to a wider range of students, and it makes them more suitable for multiage settings as well as for classrooms with a range of learners. One important result of this is more equitable participation. Furthermore, we hope that our problems are such that they can be adapted by teachers for use at different grades. Although content standards shift from grade to grade, mathematical ideas are revisited from year to year as more complicated mathematical ideas are linked to ones learned previously.

Chapter 2

Operations and Algebraic Thinking

The word *algebra* has its roots in Arabic. Although much of what we today call algebra can be traced to Arabic/Islamic mathematics, algebra itself has roots that go back to the Babylonians. Algebraic thinking was initially expressed with prose statements that did not include symbolic notation. This was true until symbolic notation became more widely used in the sixteenth century.

The Operations and Algebraic Thinking domain of the Common Core State Standards for Mathematics (CCSSM) is featured in elementary grades beginning in kindergarten and continuing through grade 5. The standards in this domain underscore the critical work of exploring and recording mathematical patterns in operations. Noticing, exploring, and representing patterns support a robust understanding of number and operations and prepare students for formal algebra work in later grades. In the kindergarten–grade 2 band, the Operations and Algebraic Thinking standards focus on the properties and structure of addition and subtraction; the standards for grades 3–5 build upon this work as they emphasize the properties and structures of multiplication and division through critical analysis and reflection. Grade 6 introduces the Expressions and Equations domain, in which students begin to work with numeric expressions as entities unto themselves. A strong foundation in Operations and Algebraic Thinking supports students in their work with this domain in middle school and with the domains of algebra and functions in high school.

In this chapter, we draw upon a problem-solving approach that supports explorations of patterns through the strategic use of representations. Representations that well support this work include numeric expressions, arrays, tables, letter notation, and concrete materials and manipulatives. In order to develop algebraic and relational thinking, students must be able to make connections across representations and ask questions of representations. Opportunities to notice, express, and then record patterns using different representations including manipulatives allow students to explore and communicate the properties and structure of the operations.

The Common Core mathematics standards for grades 3–5 are organized by grade. As with other chapters in this book, we present tasks with an *across grade-level organization*, by which we mean that these tasks may be used in different grades. To coordinate the organization of the Common Core State Standards with the purpose of this book, we indicate specific standards we believe a task may address well, though we note that tasks are not restricted to the standards we have identified. With our across grade-level perspective, we hope to support teachers of students who have different ability levels or who are in multiage or multigrade contexts.