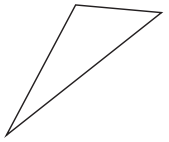
Introduction

> Through the study of geometry, students will learn about geometric shapes and structures and how to analyze their characteristics and relationships. Spatial visualization—building and manipulating mental representations of two- and three-dimensional objects and perceiving an object from different perspectives—is an important aspect of geometric thinking. . . . Geometry is more than definitions; it is about describing relationships and reasoning.
>
> — from *Principles and Standards for School Mathematics*
> National Council of Teachers of Mathematics, 2000

This statement is as true today as it was when it was written. The study of shape has always been a part of the elementary mathematics curriculum. Depending on grade level, children identify familiar shapes, locate right angles, and examine shapes for symmetry. As the statements from the NCTM document indicate, geometry can also be an opportunity for children to explore the relationships among geometric objects and their component parts.

The cases in the *Examining Features of Shape Casebook* present children's ideas about two-dimensional and three-dimensional objects. They provide a window on the complex interaction between understanding geometric ideas and developing the specialized vocabulary of geometry. Because the cases span kindergarten to grade 7, they illustrate how children's geometric thinking becomes increasingly more complex.

For example, children initially describe shapes by comparing them with objects in their everyday world. Consider the ways these two kindergarten children respond to this shape.



"It looks like a sail of a boat."
"It's like the wing of an airplane."

Both of these kindergartners reacted to the shape as a whole. As children continue to work with shape, they begin to add references to the component parts of the shape to their descriptions. A third-grade student wrote "It has 3 sides, 3 corners, 2 slants, 1 strate [straight] side" to explain how he knows something is a triangle. This child has moved from paying attention to what the shape looks like overall to noticing the components (sides and angles) that make up the figure. The children's thinking in the cases provides insight into the process of moving from general descriptions of objects to formulating and using mathematical definitions.

Children's comments as they work with three-dimensional objects illuminate the mental image they hold for such objects. For example, two third-grade students are talking about a cube. As you read their conversation, work to understand how each child is thinking.

"It takes six squares to make a cube."

"I don't think that is right. It takes much more than that. I mean . . . you would need a whole lot of squares."

Both of these students have correct conceptions for a cube. One is considering the squares that make up the faces of the cube. The other has pictured the cube as a stack of squares, one piled on another. One has a view of the surface of the cube; the other has a view of the cube as a solid object. A full understanding of a three-dimensional object incorporates both of these views. This is an example of one issue children must work through as they begin to deepen their understanding of the relationships between 2-D and 3-D objects.

Through the *Examining Features of Shape* seminar, you will explore key ideas of geometric shape and how children in elementary and middle school come to understand them. The cases were written by elementary and middle school teachers recounting episodes from their own classrooms. All had inclusive classrooms; the range represents schools in urban, suburban, and rural communities. The teacher-authors, who were themselves working to understand the "big ideas" of the elementary- and middle-grade mathematics curriculum, wrote these cases as part of their own process of inquiry. They came together on a regular basis to read and discuss one another's developing work.

The cases are grouped to present children in classrooms that are working on related mathematical issues pertaining to shape. In chapter 1, the cases illustrate children as they describe geometric objects in both two and three dimensions. Chapter 2 provides examples of children developing meaning for geometric terms such as *face, edge,* and *side*. In chapter 3, children encounter and sort out multiple conceptions of angle, and the cases in chapter 4 illustrate children who are developing definitions in the context of three- and four-sided polygons. In chapter 5, children work on the mathematical ideas of similarity and congruence. The relationships between 3-D and 2-D objects are explored through the work of children in chapter 6. Chapter 7 presents instances of children employing geometric reasoning in various contexts: locating the center of a room or of a triangle, determining fractional parts of a square, and finding missing lengths in a diagram.

Chapter 8, the last in this casebook, is the essay "Highlights of Related Research," which summarizes some research findings that touch on issues explored in the cases.