

# CONTENTS

<b>Foreword</b> .....	<b>vii</b>
<i>Henry Pollak</i>	
<b>Preface</b> .....	<b>ix</b>
<b>PART I Understanding Models and Modeling</b>	
<b>Introduction</b> .....	<b>1</b>
<i>Judith S. Zawojewski and Elizabeth Difanis Phillips</i>	
1. Perspectives on Modeling in School Mathematics.....	3
<i>Michelle Cirillo, John A. Pelesko, Mathew D. Felton-Koestler, and Laurie Rubel</i>	
2. Different Types of Mathematical Models .....	17
<i>Kimberly Groshong</i>	
<b>PART II Using Models to Represent Mathematics</b>	
<b>Introduction</b> .....	<b>25</b>
<i>Elizabeth Difanis Phillips</i>	
3. TouchCounts: Visual, Auditory, Haptic, and Symbolic Models for Numbers and Operations .....	27
<i>Nathalie Sinclair</i>	
4. Building Conceptual Understanding of Probability Models: Visualizing Chance.....	37
<i>Stephanie Budgett, Maxine Pfannkuch, and Christine Franklin</i>	
<b>PART III Teaching and Learning about Mathematical Modeling</b>	
<b>Introduction</b> .....	<b>51</b>
<i>Judith S. Zawojewski</i>	
5. Moving beyond Context: Challenges in Modeling Instruction .....	53
<i>Sarah K. Bleiler-Baxter, Angela T. Barlow, and D. Christopher Stephens</i>	
6. Advancing the Teaching of Mathematical Modeling: Research-Based Concepts and Examples.....	65
<i>Werner Blum and Rita Borromeo Ferri</i>	
7. Broadening the Landscape of Modeling by Including an Emergent View .....	77
<i>Kevin J. Reins</i>	
8. Teaching Mathematics for Social Justice through Mathematical Modeling.....	87
<i>Michelle Cirillo, Tonya Gau Bartell, and Anita A. Wager</i>	

9. Moving Students from Remembering to Thinking: The Power of Mathematical Modeling ..... 97  
*Cheryl Gann, Tamar Avineri, Julie Graves, Maria Hernandez, and Daniel Teague*
10. Fostering Modeling Competencies for Complex Situations .....107  
*Peter Stender and Gabriele Kaiser*

## **PART IV Mathematical Modeling as a Vehicle for STEM Learning**

<b>Introduction.....</b>	<b>117</b>
<i>Judith S. Zawojewski</i>	
11. A Case for Mathematical Modeling in the Elementary School Classroom .....	121
<i>Mary Alice Carlson, Megan H. Wickstrom, Elizabeth A. Burroughs, and Elizabeth W. Fulton</i>	
12. Learning Secondary School Mathematics through Authentic Mathematical Modeling Tasks .....	131
<i>Laurie O. Cavey and Joe Champion</i>	
13. Modeling Using Data Available on the Internet .....	143
<i>Marcelo de Carvalho Borba, Mónica E. Villarreal, and Débora da Silva Soares</i>	
14. Inspiring Students in Integrated STEM Education through Modeling Activities .....	153
<i>Cathrine Maiorca and Micah Stohlmann</i>	
15. A Bootstrapping Approach to Eliciting Students' Informal Inferential Reasoning through Model Development Sequences.....	163
<i>Jeffrey A. McLean and Helen M. Doerr</i>	

## **PART V Designing Modeling-Oriented Tasks and Curricula**

<b>Introduction.....</b>	<b>175</b>
<i>Elizabeth Difanis Phillips</i>	
16. What a Modeling Task Looks Like .....	179
<i>Heather Gould</i>	
17. Developing Early Foundations through Modeling with Data .....	187
<i>Lyn D. English</i>	
18. Designing Sequences of Model Development Tasks.....	197
<i>Helen M. Doerr</i>	
19. Interpreting Curricula to Find Opportunities for Modeling: Case Studies from Australia and Sweden .....	207
<i>Vincent Geiger, Jonas Bergman Årlebäck, and Peter Frejd</i>	
20. Discrete Mathematical Modeling in the High School Curriculum .....	217
<i>Eric Hart and W. Gary Martin</i>	

**PART VI Assessing Mathematical Modeling**

<b>Introduction.....</b>	<b>227</b>
<i>Judith S. Zarwojewski</i>	

- |   |     |
|---|-----|
| 21. Formative Self-Assessment: A Critical Component<br>of Mathematical Modeling.....      | 229 |
| <i>Cheryl L. Eames, Corey Brady, and Richard Lesh</i>                                     |     |
| 22. The OECD PISA: An Assessment of Mathematical Literacy and<br>Modeling Processes ..... | 239 |
| <i>Kaye Stacey</i>  |     |

**PART VII Supporting Teachers' Learning about Mathematical Modeling**

<b>Introduction.....</b>	<b>249</b>
<i>Elizabeth Difanis Phillips</i>	

- |   |     |
|---|-----|
| 23. The GAIMME Report: Mathematical Modeling in the K–16 Curriculum.....  | 253 |
| <i>Daniel Teague, Rachel Levy, and Kathleen Fowler</i>  |     |
| 24. Supporting Teachers' Development as Modelers and Teachers of Modelers .....                                   | 263 |
| <i>Rose Mary Zbiek</i>  |     |
| 25. Theoretical and Pedagogical Considerations in Promoting Students'<br>Metacognitive Modeling Competencies..... | 273 |
| <i>Katrin Vorhölter and Gabriele Kaiser</i>   |     |