Express mathematical relationships using equations

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This activity addresses the notions of variables as unknowns and as quantities that vary.

Algebra Scales

Grades 3–4

Goals

The students will—

- determine if expressions constitute an equation (balanced scale) or an inequality (unbalanced scale);
- understand that quantities on both sides of an equation must be equal;
- use logical thinking to find a replacement set to solve equations.

Prior Knowledge

Students should have had experiences with replacement sets for variables.

Materials and Equipment

Use a pan-balance scale and small wooden cubes to introduce the activity. You can model the exercises at the beginning of the activity. Have available one copy of the "Algebra Scales" blackline master for each student or pair of students.

Classroom Environment

Encourage the students to work in pairs, although some of them will prefer to work individually. Be sure to allow time for the students to show and model their solutions.

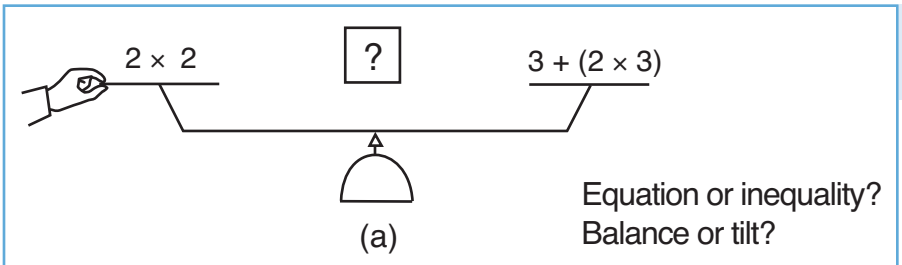
Activity

Engage

Draw the representation in figure 2.4a on the board and ask the students if it makes an equation (represented by the balanced scale in fig. 2.4a) or an inequality (represented by the tilted scale in fig. 2.4b). Ask the following questions to help the students determine that the scale shows an inequality:

- Does the scale show the correct balance? (no) Why not? (2×2 is less than $3 + (2 \times 3)$.)
- How should the scale be redrawn? (The side with $3 + (2 \times 3)$ should be lower, or "heavier.")
- How could we make the scale balanced? (We need to find an expression that is equal to 4 or 9; for example, $3 + 1$ and 2×2 will give a balanced scale; 3×3 and $3 + (2 \times 3)$ will also give a balanced scale.)

The extension in this activity has been adapted from Linda Holden Charles, *Algebraic Thinking: First Experiences—Collections* (Mountain View, Calif.: Creative Publications, 1990).



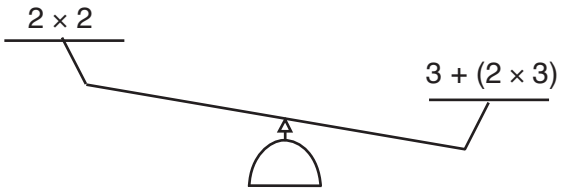


Fig. **2.4.**
A pan-balance representation

Remind the students to apply the order-of-operations rule.

So $2 \times 2 < 3 + (2 \times 3)$.

Help the students make the transition from this representation to a more symbolic form by asking them to rewrite the expression using the symbols =, >, or <. Exercises such as this one also address order of operations, which is an important prealgebra concept.

You can also use a balance scale to reinforce the notion of a replacement set for variables in algebraic expressions and equations. See, for example, the representation in figure 2.5. The students should see that a number of solutions will keep the scale balanced. For example, 5×2 balances 2×5, 5×4 balances 2×10, and so on. Have the students create a table in which they can record the replacement-set values (see the table in fig. 2.6). Ask, "Do you see any patterns in the numbers we use in the square and the triangle? What are the patterns?"

Possible answers: The numbers used in the squares are even numbers. The numbers used in the triangle are multiples of 5.

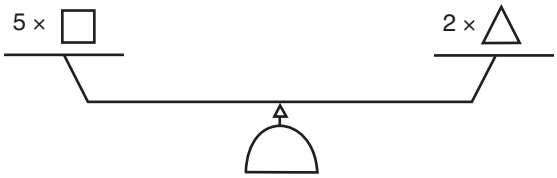
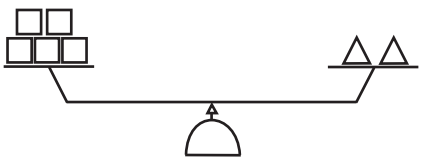


Fig. **2.5.**
A pan-balance representation of the notion of a replacement set for variables

Explore

In the first set of exercises on the "Algebra Scales" blackline master, the students will be able to determine if the expressions shown on the scale form an equation or an inequality. The second set of exercises deals with finding replacement values for the variables.

The scale in figure 2.5 could also be presented in this way:



□	△
2	5
4	10
6	15
8	20

Fig. **2.6.**
A chart of replacement-set values

Solutions to the Blackline Master

1a: The scale balances;
1b: The right side of the scale tilts down;
1c: the scale balances;
2a, 2b, 2c: Many solutions are possible.

Extend

You can extend this activity in at least two ways. First, have the students create their own scales with numerical expressions representing equations or inequalities. Ask the students to support their selection. Second, the open-ended expressions in the second set of exercises can be extended to include logical thinking by using a system of equations. For example, have the students solve the problem in figure 2.7.

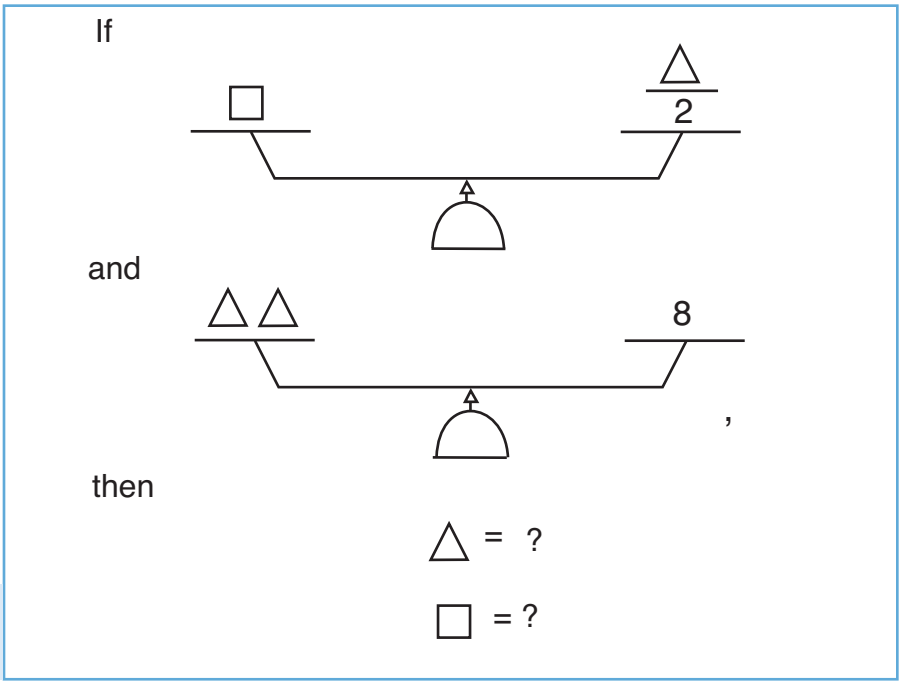


Fig. **2.7.**
A balance problem

In addition to a trial-and-error strategy, the students may use different approaches to this problem. For example,

If

$$\triangle \div 2 = \square,$$

then

$$\triangle = \square\square,$$

and

$$\triangle\triangle = \square\square\square\square.$$

Since

$$\triangle\triangle \text{ balances } 8,$$

$$\square\square\square\square \text{ balance } 8$$

(replacing △△ by □□□□). So 8 must equal four □, or □ = 2. Then △ = 4 because △ ÷ 2 = □.

The following prompts can guide students through this logical approach:

- What information does the first scale give? (A quantity equals half of another quantity.)
- What does the problem ask us to find? (The value of □ and the value of △.)

Pan Balance, an activity that simulates the balance scale can be found on the CD-ROM.

- How can we use this information in the next scale? (△△ = □□□□ because △ = □□, so △△ = □□□□.)
- If we know △△ = □□□□, what does the next scale look like? (Four squares balance the 8.)
- With what quantity could we replace □ to make a balanced scale? (Replace □ with 2.)
- How can we find the value of △? (Replace □ by 2 in the first scale: 2 = △/2. Now we find a quantity that equals 2 when it is divided by 2. 4 ÷ 2 = 2. Therefore, △ = 4.)

Assessment Ideas

Have the students create their own algebra scales, and require that they write statements that support the choice of values for the variables or give their reasons for identifying the expressions as either equations or inequalities. Examine the expressions created and the supporting statements written to determine the students' level of understanding.

Where to Go Next in Instruction?

Given the level of understanding the students have developed, you may wish to make a transition to less graphic representations of equations and inequalities. To do so, present the expressions without the balance scales.

The I Spy Patterns activity, which follows, focuses on the commutative and associative properties of numerical patterns that are derived by partitioning an array into different parts.