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The Shifting Landscape of School Algebra in the United States

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ALTHOUGH many critics suggest that the U.S. school system does not change, school algebra is shifting and changing before our eyes. These changes are of different kinds and reflect the complexity of the educational system in our society. These changes, filtered through the institutional pressures associated with compulsory schooling, come from competing sources: the vision articulated by the National Council of Teachers of Mathematics (NCTM) (2000), and the broader movement—also known by the name *standards*—to hold schools accountable for having students meet world-class academic standards relevant for both college and the world of work. This article begins with a description of structural changes in the role of algebra in the U.S. educational system—changes that are shifting the teaching responsibilities of secondary school mathematics teachers. At the same time, and perhaps partially in response to the structural changes to the role of algebra in U.S. schools, there are curricular changes. The article closes by identifying three sets of challenges and opportunities posed by these changes for mathematics educators.

Structural Changes Relevant to School Algebra

This section does not consider curricular issues in school algebra. Rather, it focuses on changes regarding which students study school algebra, when they study algebra, how their learning is assessed, the stakes of these assessments, and the rationales for these changes.

Who Studies School Algebra and Why?

In the not-too-distant past, say, twenty years ago, algebra was seen as abstract mathematics suitable only for students who were developmentally ready and college intending. Proportionally, African American, Hispanic, and white working-class students were underrepresented in courses in algebra (Gamoran 1987). As a result, such students and their teachers are also disproportionately the object of current policies.

Of course, twenty years ago, students had to study mathematics in high school; depending on their district and state, two, three, or, in very rare circumstances, one or four years of mathematics were required as part of a coursework load that would lead to high school graduation (Hawkins, Stancavage, and Dossey 1998, p. 48). However, the content of these years of study was left unspecified. As a consequence, a sizable percentage of students graduated from what some scholars called "the shopping mall high school" (Powell, Farrar, and Cohen 1985) with non-academic training in mathematics, and with no study of algebra or geometry. The National Assessment of Educational Progress (NAEP) tracks course taking. On the 1990 NAEP, 17 percent of the surveyed twelfth-grade students indicated that their terminal mathematics course was prealgebra or arithmetic (Mitchell et al. 1999, p. 225).

The mathematics required of high school students is changing. Increasingly, over the last decade calls have suggested that all students study algebra before they graduate high school (College Board 2000). States are considering policies like those advocated by Achieve's American Diploma Project (ADP) (2004), which are meant to enforce such a course of action and ensure that students study more than first-year algebra. At the end of 2006, eight states had diploma requirements that met Achieve's standards; seven others raised their graduation requirements in the preceding year, but not to the level Achieve suggests; and twelve more were planning to raise graduation requirements. (This information was downloaded from www.achieve.org/node/332#states on November 28, 2006). These calls and policies are not just abstractions; they are changing what happens to children in school. For example, in a comparison of "highest math course taken," as reported by students on NAEP in 1990 and 1996, the percent of twelfth-grade students reporting that they have never taken a course in algebra or above dropped from 17 percent to 8 percent (Mitchell et al. 1999, p. 225); from 1978 to 1999, the percent dropped from 20 percent to 7 percent (Campbell, Hombo, and Mazzeo 2000, p. 63). On a national scale, these percentages represent large numbers of students.

Why Insist That All Students Study What Used to Be Considered Mathematics for Those Intending to Go to College?

The rationale is often couched in two sets of terms, the competitiveness of the United States in the global economy on the one hand, and the access of minority students to high-wage career opportunities on the other hand (ADP 2004; College Board 2000; Moses and Cobb 2002; RAND Mathematics Study Panel 2003). Research has examined much narrower outcomes. When Gamoran and Hannigan (2000) studied the benefits of college-preparatory mathematics, they found that enrolling in algebra, as opposed to a general math course, would lead to improved academic outcomes. (In this study, achievement in mathematics was assessed by the National Educational Longitudinal Study multiple-choice mathematics examination.) However, the researchers were careful to note that their results do not indicate what would happen if all students were required to study algebra (p. 241).

Side effects of policies that result in students' enrolling in an algebra class instead of a different mathematics course, including decreased arithmetic skill or an increase in the number of dropouts, have not been studied extensively. (See Carnoy and Loeb 2003 for an inconclusive attempt to examine changes to dropout levels as standards were raised.)

Yet, critics of such recommendations are on difficult rhetorical terrain. To suggest that not all students need to study algebra seems to be tantamount to suggesting that one does not see all students as capable thinkers or that one is willing to curtail the economic prospects of some members of our society. (See Noddings 1993 for an explication of the challenge of opposing such recommendations.)

When Do Students Study Algebra?

In an article in the *Mathematics Teacher,* Usiskin (1987) suggested that under certain conditions, algebra could and should be an eighth-grade course in the United States. But the very suggestion makes it clear that at that time, algebra was primarily a high school mathematics topic, or, more accurately, a pair of high school mathematics courses.

This also is changing. Now, in line with *Principles and Standards for School Mathematics* (NCTM 2000), algebraic thinking is being infused into arithmetic work at the elementary school level (for a report on the progress of such efforts, see Schliemann, Carraher, and Brizuela 2007). Middle-grade mathematics textbooks often include strands of work that is algebraic or preparatory for algebra. At the high school level, renewed interest has arisen in "integrated" courses. Although in most high schools, the "layer cake" with two algebra courses persists, National Science Foundation (NSF)–funded high school mathematics curricula, like the Interactive Mathematics Project (Fendel et al. 1997) and Core-Plus Mathematics (Coxford et al. 2003), take an integrated approach and offer algebra as a strand of study rather than as a pair of courses.

In line with Usiskin's (1987) call for offering algebra earlier in students' education and with international comparative studies that show that algebra is studied earlier in several countries (Martin et al. 2000), many districts in the United States require students who aspire to attend college to enroll in first-year algebra in grade

8, or even earlier. According to 1978 NAEP data, 16 percent of eighth-grade students were enrolled in courses in algebra; in 1999, the percent had increased to 22 percent (Campbell, Hombo, and Mazzeo 2000, p. 62).

How Is the Learning of Algebra Assessed, and What Are the Stakes of These Assessments?

Here, again, we see changes. In the past, for students who studied high school algebra, assessment was primarily the business of the classroom teacher or the school. The SATs or ACTs, as a hurdle to attaining college admission at particular universities, did not assess algebra directly but rather mathematics more broadly.

At present, in response to the legislation H.R. 1 of 2001, known colloquially as No Child Left Behind (NCLB), all students are expected not only to study algebra but also to demonstrate that during their years in high school, they have learned mathematics well enough to meet benchmarks on state examinations. Students also have to pass examinations developed by their teachers or their districts. In some states, such as Maryland and Texas, to graduate from high school or be awarded a state-certified diploma, students have to pass a state-developed end-of-course test. As states become more familiar with NCLB, the degree of difficulty of the tests is being clarified. It will be illuminating to track how data and political pressures influence the level of difficulty on these tests. Although most of these tests focus on introductory algebra, Achieve is working with a consortium of nine states to explore an exit exam for second-year algebra or its equivalent.

To reiterate, in the past it was possible to graduate from high school without having studied and passed a course that included the study of algebra. Now, in most states, students must not only take an algebra course or two in order to graduate but also pass an examination that tests whether they are able to meet state algebra-specific benchmarks.

The previous discussion focused only on structural changes relevant to the teaching and learning of school algebra. Changes to secondary school mathematics teachers' instructional responsibilities are addressed in the next section.

Changes to Secondary School Mathematics Teachers' Teaching Responsibilities

Taken together, the structural changes to school algebra represent substantial change—change that constitutes a large challenge to mathematics teachers. In the past, first-year algebra was a course in which students often were not successful. Although passing and failing rates are typically difficult to find, Gamoran and Hannigan (2000) note that even the most successful districts participating in the College Board's Equity 2000 program for six years had more than 25 percent of their students failing algebra. As the changes described above are carried out broadly in the United States, mathematics teachers are responsible for helping *all* their students be successful in their study of algebra or risk the severe economic consequences of not having a high school diploma. (In order to understand these consequences, scholars, like Entwisle, Alexander, and Olson [2004], investigate differences between the opportunities of those who drop out temporarily versus permanently.) From the point of view of the mathematics teacher, this is a major and demanding responsibility that has been mandated without a plan for how this new mandate can be achieved. It appears that teachers are being told to insist that their students jump higher, with the notion that students will comply and will then be successful.

The changes described, both in who studies algebra and when they study this content, have many implications for the teaching loads of middle or high school mathematics teachers. As more students take algebra before ninth grade, more middle school teachers with elementary school (K–8) certification are being asked to teach algebra, a domain of mathematics they were not prepared to teach. In systems that track by age, where higher-achieving students start algebra earlier than their peers, elementary-certified teachers teach algebra to those students who are the highest of achievers.

Whereas in middle schools the structural changes described earlier affect what mathematics is taught, at the high school level there are changes both in how many students are taught and what they are taught. NAEP trends suggest that more high school students are taking higher-level mathematics courses than in the past (Hawkins, Stancavage, and Dossey 1998). There are fewer general mathematics courses and more AP Statistics and Calculus courses to teach. At the same time, transcript studies indicate that more students are studying more mathematics. For example, the study by Perkins and colleagues (2004) indicates that from 1990 to 2000, the mean number of mathematics course credits jumped from 3.2 to 3.7 (pp. 2–4). Thus, high school mathematics departments are growing to accommodate the greater percentage of students studying mathematics. A particular challenge is for high school mathematics teachers to help students who have typically had difficulty to be more successful with more-advanced content than they might have been if they had been students a decade earlier. High school teachers, who may have chosen to teach high school for its disciplinary focus, may not be adequately prepared for this challenge.

At the same time, mathematics teachers may have less autonomy in meeting ambitious goals. The use of high-stakes state examinations on algebra content introduces new challenges for schools and districts. Districts and schools have to contend with the possibility that students will take and pass an algebra course but will not pass a state examination mandated for graduation. In large school districts, the pressure to help students succeed on the state's end-of-course examinations can lead to nine-week benchmark tests scored throughout the district to assess students' progress toward meeting the state assessment goals. Such assessments may lead teachers to face stark dilemmas about whether to stay with the pacing chart and the district examination schedule or fall behind in order to address difficulties students have with content previously taught.

There is a tremendous irony to the fact that these changes are being proposed during a time when the U.S. Department of Education seeks randomized controlled trials to justify even small-scale educational innovations. (See the Institute of Education Sciences [2003] *Effective Mathematics Education Research Grants, Request for Applications*.) Where are the randomized trials that show a positive impact for shifting graduation requirements to meet more demanding targets? Where is the evidence to show that such changes won't have unanticipated and unintended consequences, like a greater lack of arithmetic skill on important segments of the school population that will now devote more time to the study of algebra or increases in school dropout rates?

Changes to What School Algebra Is

So far, structural changes relevant to the teaching and learning of school algebra and changes to teachers' teaching responsibilities have been described. Changes to the algebra curriculum itself have not been discussed. Yet, there are substantial changes to the curriculum, perhaps linked to the structural changes described earlier.

This section will focus on shifts in the mathematical perspective on school algebra. Although these shifts may seem less dramatic than the policies described in earlier sections, these shifts are substantial. Perhaps they are what Kilpatrick and Izsák (this volume) describe as the earlier change from algebra as generalized arithmetic to algebra as structure (the story of algebra that highlights the relationship between factoring whole numbers and factoring polynomials over the complex numbers, which is familiar and comfortable to mathematicians like Wu [2001]). In time, these shifts may be as consequential for students as the policy changes described earlier.

Since reasoning with representations (Kilpatrick and Izsák, this volume) has become focal in discussions of school algebra (if not yet necessarily in classrooms) and since teachers struggle to help a wider range of students succeed in algebra, the curriculum's perspective on the x's and y's of school algebra has begun to shift subtly. (For another attempt to capture this shift, see Chazan and Yerushalmy 2003.) To appreciate this shift, it is useful to consider one mathematician's perspective on the real numbers. Weyl (2002, p. 453) writes:

> The system of real numbers is like a Janus head with two oppositely directed faces. In one respect it is the domain of + and • and their inverses, and in another it is a continuous manifold, and the two are continuously related. One is the algebraic and the other is the topological face of numbers.

This Janus-faced aspect of numbers has a particular implication for the conceptualization of the variables used in school algebra to write polynomials over the real numbers. While describing the algebraic properties of polynomials (think abstract algebra!), Weyl implicitly contrasts these algebraic properties with topological ones (think real analysis!) (pp. 455–56):

> The idea that the argument x is a variable that traverses continuously its values is foreign to algebra; it is just an indeterminate, an empty symbol that binds the coefficients of the polynomial into a uniform expression that makes it easier to remember the rules for addition and multiplication. 0 is the polynomial all of whose coefficients are 0 (not the polynomial that takes on the value 0 for all values of the variable x).

To bring Weyl's comment back to school algebra, when a polynomial function is graphed or a table of values is created for an expression, it is the topological, not the algebraic, face of polynomials that is being presented to students. By way of contrast, when one argues that the equation $x - 2 = 0$ has only one solution because of the uniqueness of the additive inverse, the algebraic face is forward.

These two faces can be used to describe how curricula present what an equation is and what representations students can use when working with equations. Of course, curricula intend to teach understandings of equations that will allow students to move fluidly among different notions of what an equation is (see Sfard and Linchevski 1994 for an example of the fluidity required of students). No one view of equations in one variable listed below will identify how a particular curriculum teaches students. But the four possibilities listed below in italics will help concretize what Kilpatrick and Izsák call "reasoning with representations" (related to the topological) and differentiate it from "algebra as structure" (related to the algebraic).

For example, an algebra curriculum might conceptualize the equation in one variable $3x + 2 = 7$ as (1) *a representation of a set;* the solution set to this equation is $\{5/3\}$. In such a curriculum, when an equation in one variable is being solved, the question being asked of students is, "What are the elements of the set of numbers that can be used as replacements for x to make the conditional statement a true statement?" One might then represent this solution set on a number line and conceptualize the equation as another representation of this same set. The focus when solving equations in one variable is on the operations that can be carried out on both sides to arrive at the solution set; the values that can be substituted but that are not members of the solution set are not of interest.

The previous way of thinking seems quite consonant with the algebraic face of polynomials (and with algebra textbooks from the 1970s, like that of Dolciani and Wooton [1970]). There is a range of ways of conceptualizing equations that moves further and further into the realm of the topological and that incorporates other sorts of representations. In contrast to the previous view, an algebra curriculum can conceptualize this same equation, $3x + 2 = 7$, as (2) *a template for producing sentences about numbers*—sentences that can be true or false depending on the values used to