into practice

Chapter 1
Classifying Objects

Big Idea 1
A classification scheme specifies for a space or the objects within it the properties that are relevant to particular goals and intentions.

Essential Understanding 1*a*
Mathematical classification extends and refines everyday categorization by making more precise what we mean by "sides," "angles," "straightness," or other features that we attend to as we categorize mathematical objects.

Essential Understanding 1*b*
We may classify the same collection of objects in different ways.

This chapter addresses the most familiar topic of early elementary school geometry—classifying and naming shapes. Even before entering preschool, most children have been taught to identify circles, triangles (in limited ways; they are usually shown examples of regular triangles and always in the same orientation), and rectangles and squares (typically taught incorrectly as two discrete shapes). Those who assess young children before they enter school to decide whether they are ready for pre-kindergarten or kindergarten often hear parents say, "My child is ready for school because she can count up to 10, and she can identify circles, triangles, squares, and rectangles." This type of knowledge, easily recognized and described by parents and teachers alike, is only a small part of the understanding that Big Idea 1 of geometry and measurement encompasses, and it typically represents knowledge taught only as definitions related to regular shapes.

Working toward Big Idea 1 through Essential Understandings 1*a* and 1*b*

In *Developing Essential Understanding of Geometry and Measurement for Teaching Mathematics in Prekindergarten–Grade 2*, Goldenberg and Clements (2014) approach the essential understanding that lays the foundation for later learning in geometry and measurement by first discussing classifying and the reasons for classification. They emphasize the importance of identifying similarities and differences among geometric figures and connecting informal descriptions with increasingly more precise language as learners progress in their understanding. In this approach, language is clarified, connected, and refocused on the specific vocabulary that defines particular shapes. As the authors state, "A precise specification of the way that we are classifying a collection of objects gives us a definition for the objects in that collection" (p. 13). This approach to classification of shapes marks a major difference from the way in which names of shapes and classification have typically been taught. Rather than defining geometric terms first and then asking students to find examples of shapes that fit that classification, students are given a variety of shapes and asked to sort and classify them by using attributes that they observe. The teacher starts with experiences that her students have had and examples that they have already seen. Then, as students develop their classifications, she poses questions and makes comments on their groupings, first using their words and then progressing to more precise geometric terms.

Classification tasks that require students to sort shapes or solids according to their own systems and then describe their sorting by using their own informal language provide excellent assessment opportunities. This chapter examines five classification tasks that build on one another to help young learners advance in their understanding of, and development of language for, classifying shapes.

Task 1: Sorting and describing shapes and solids of different materials

Figure 1.1 presents task 1, an initial classification task. The materials for this task are varied, and each type provides different classification experiences. Students should work in groups to sort and classify sets of–

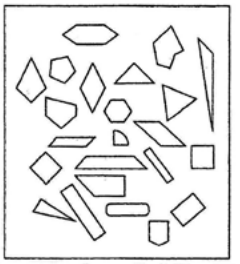
- attribute blocks;
- paper shapes;
- wooden shapes and solids; and
- plastic-straw and pipe-cleaner creations.

Sort and classify the shapes or solids identified below into groups. Use words to tell how you sorted the shapes.

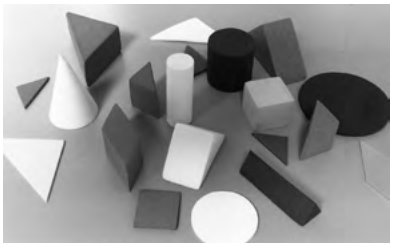
Materials for this task:

- Attribute shapes (three colors—red, blue, yellow; four shapes—rectangle [that is not a square], square, circle, triangle [equilateral only]; two sizes—large and small [only one size is shown here])

- Paper shapes (a variety, as pictured)



- Shapes and solids (a variety, as pictured)



- Shapes created with plastic straws with pipe cleaners threaded through them (a variety, as pictured; straw pieces are 2 inches, 4 inches, and 6 inches in length)

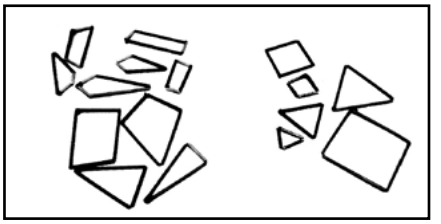


Fig. 1.1. Task 1

Reflect 1.1 asks you to think closely about this task, attending to the way that the different materials might affect the words that students use to label their groups.

Reflect 1.1

Consider the classification task shown as task 1 in figure 1.1.

- **How would you expect students to sort and classify the shapes or solids in task 1?**
- **What words would you expect students to use to label the groups when using each of the four materials?**

The attribute blocks in task 1 provide examples of shapes that are most familiar to students. The properties of color, shape, and size are clearly defined, and young students typically begin to classify these shapes by at least one of these properties. The paper shapes provide students with opportunities to categorize unfamiliar shapes by using numbers of sides, curved or straight lines, or different angles within shapes. Sets of shapes that include both two- and three-dimensional examples introduce students to the differences and similarities between and among these shapes. The plastic-straw and pipe-cleaner creations are limited to three or four straws or pipe cleaners per shape (see Chapter 3 for more information about their composition), and their flexibility adds to the classification possibilities.

Eight students' responses to the various shapes and solids that they encountered in task 1 appear in figures 1.2–1.5. Figure 1.2, shows the work of two students—(a) Colin, a prekindergartner, and (b) Amanda, a kindergartner—in sorting and classifying colored attribute shapes, the first set of materials listed for task 1. When Colin was asked to sort the shapes into groups, he was confused. The teacher then placed the circle divider on the table and told him to "put the shapes that are all alike in this loop." The teacher's conversation with Colin continued:

Teacher: I see that you have lots of shapes inside the loop. Can you tell me about them?

Colin: Red is my favorite color [*pointing to the four red shapes, which appear as gray in figure 1.2*].

Teacher: I see the red shapes. Can you tell me more about this one? [*She points inside the loop at the red triangle—gray in the figure.*]

Colin: It's red.

Teacher: Anything else?

Colin: It's red; I like it.

Teacher: I see some shapes that are not red, and they are in your loop. What are they?

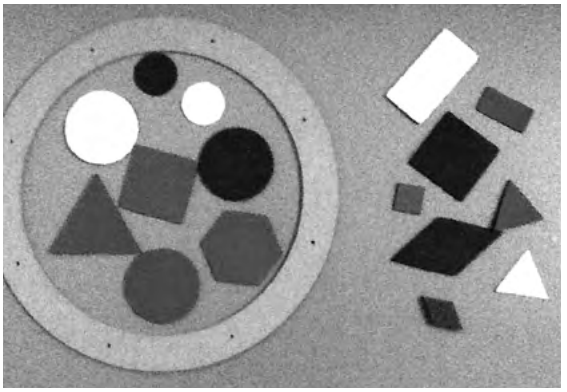
Colin: They are circles [*pointing to each circle, leaving out the red circle—gray in the figure*]. . . I like circles too!

Teacher: Is this one a circle [*pointing to the red circle*]?

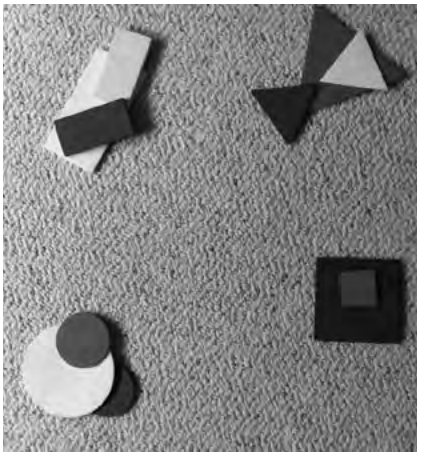
Colin: No, it's red!

Teacher: Oh . . . So how are these shapes all alike [*motioning to the shapes in the loop*]?

Colin: I like 'em!



a. Colin's (prekindergarten) response



b. Amanda's (kindergarten) response

Fig. 1.2. Colin's and Amanda's responses to the attribute shapes in task 1 (fig. 1.1)

When Amanda was asked to sort the shapes into groups, she quickly responded by stacking the shapes as shown in figure 1.2b, and the teacher engaged her in the following discussion:

Teacher: I see that you have stacked your shapes. Use your words and tell me about them.

Amanda: [*Pointing to the circles*] Circles. [*Pointing to the squares*] Squares. [*Pointing to the triangles*] Rectangles . . . No . . . Triangles. [*Pointing to the rectangles*] Rectangles.

Teacher: Thank you. I noticed that you changed your answer on this one [*pointing to the triangles*]. Why did you do that?

Amanda: [*Laughing*] I always get them mixed up!

Teacher: I know a child in a different class who gets mixed up about triangles. Can you tell her why this is a triangle?

Amanda: See [*picking it up and rotating her fingers in a quick clockwise motion around the perimeter*] . . . It has three!

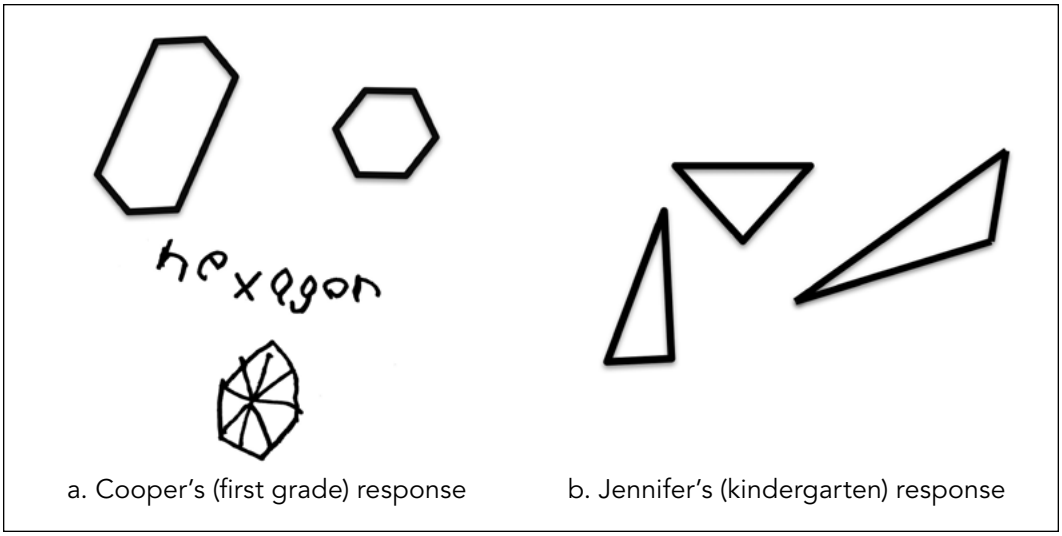
Teacher: Three?

Amanda: You know . . . Three points!

Figure 1.3 shows the work of (a) a first grader, Cooper, and (b) a kindergartner, Jennifer, in sorting and classifying paper shapes, the second set of materials in task 1. Cooper grouped the two paper shapes shown in figure 1.3a and labeled them "hexagon." To illustrate his label, he drew a hexagon of his own. He also said, "The long one isn't a very good hexagon, and the good one should also be colored yellow." When he was asked, "Why yellow?" he responded by showing a yellow pattern block. When he was asked why he had drawn his picture as he did, he said, "'Cause you can put green triangles on the yellow hexagon," obviously making another connection with the yellow pattern block.

Jennifer also explained her work, shown in figure 1.3b: "These are all triangles." She then asked how to spell *triangle*. When the response was, "Spell it how it sounds," she wrote *T-R-I-E-O-N-E-G-L-E*. She also noted as she pointed to the middle triangle, "See . . . Upside-down triangles are still triangles. My teacher said so!"

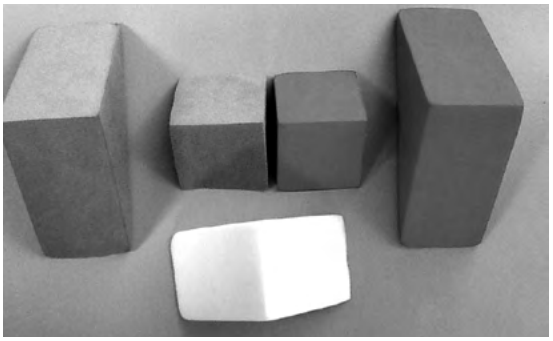
Two first graders, Armando and Juan, worked with shapes and solids in task 1. Their work appears in figure 1.4. Armando selected five triangular prisms and arranged them as shown in figure 1.4a. When the teacher asked him about these solids, he responded, simultaneously demonstrating what he was saying, "They all look alike . . . See! . . . They are fat triangles." Juan worked with his four-year-old brother, and as shown in figure 1.4b, together they classified their shapes as "not flat" (Juan's words) and as "cercles" (his brother's word) that are "flat" (Juan's word).



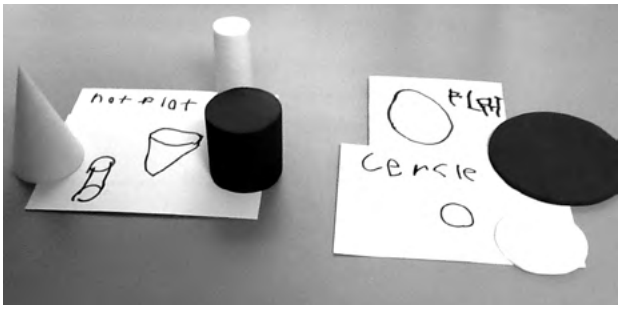
a. Cooper's (first grade) response

b. Jennifer's (kindergarten) response

Fig. 1.3. Cooper's and Jennifer's responses to the paper shapes in task 1 (fig. 1.1)



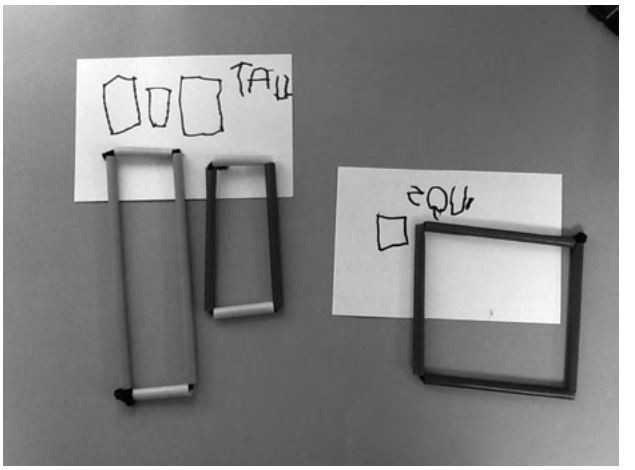
a. Armando's (first grade) response



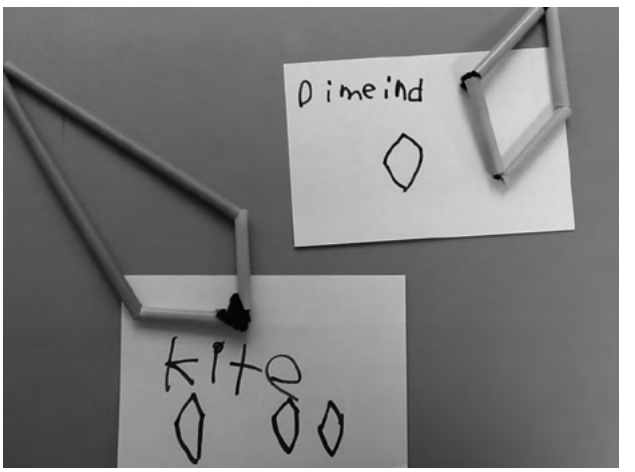
b. Juan's (first grade) response

Fig. 1.4. Armando's and Juan's responses to the shapes and solids in task 1 (fig. 1.1)

Figure 1.5 shows the work of (a) a prekindergartner, Tucker, and (b) a second grader, Hong, in sorting and classifying the plastic-straw and pipe-cleaner creations in task 1. Tucker, whose work appears in figure 1.5a, identified two "tall" shapes and one "squ." He also drew them. When the "tall" shapes were repositioned horizontally, he said they weren't "tall" anymore. Hong identified two shapes, a kite and a diamond (spelled *D-I-M-E-I-N-D*), as shown in figure 1.5b. When the teacher asked whether the diamond shape could be called by any other name, Hong nodded and showed that if it were stretched out, it could be a square. He also demonstrated



a. Tucker's (prekindergarten) response



b. Hong's (second grade) response

Fig. 1.5. Tucker's and Hong's responses to the plastic-straw-and-pipe-cleaner shapes in task 1 (fig. 1.1)

that a rectangle could be "squashed" and then wouldn't be a rectangle anymore. He called it a *quadrilateral*.

Look back at the responses of all eight students, considering both what they did and what they said about their sorting and classification of the various shapes. Use the questions in Reflect 1.2 to guide your examination.

Reflect 1.2

- **What would be your assessment of the students' understanding of shapes or solids on the basis of their work on task 1 shown in figures 1.2–1.5 and revealed in their descriptions?**
- **What questions would you ask or comments would you make to assess their understanding further?**

All eight students whose responses have been shown and discussed demonstrated specific knowledge about objects and their possible classifications by labeling the shapes. Generally, these students labeled and defined shapes according to what they "looked like" rather than stating their properties. In addition, most students were selective about the shapes that they included in their groupings—that is, if the shape didn't fit in their system, they left it out. Except for Juan, who offered his "flat" and "not flat" classifications, the students did not use non-examples to clarify definitions of shapes. Several students sorted shapes according to non-defining properties. For example, Colin and Cooper specifically mentioned the property of color. Orientation came into play for Tucker, who described his "tall" shapes, and for Jennifer, who suggested that orientation was important to her response, according to her teacher's notes.

Equally important, all eight students demonstrated the need to grow in understanding and ability to clarify or expand their classifications of the shapes or solids with defining properties. In most cases, their reasoning indicated a partial understanding of shape classification and, in some cases, a misunderstanding of defining properties of shapes. For example, consider Cooper's (fig. 1.3a) response to the paper hexagons and his remark, "The long one isn't a very good hexagon, and the good one should also be colored yellow." The yellow pattern block is often shown as an example of a hexagon. As a result, students frequently think that a hexagon must be yellow like the pattern block and have six sides ofre equal length. When they are introduced to non-regular hexagons, they learn that the length of the sides is not included among the defining properties of a hexagon.

You can use students' responses to task 1 and your assessments of those responses in a variety of ways to inform instruction. Consider, for example, how you might apply these responses to help students develop more precise vocabulary and address specific geometric standards. Review the students' work in the figures and the descriptions of their responses again, this time with respect to the related expectations for prekindergarten identified in *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (NCTM 2006) and the relevant content standards for K–grade 2 in the Common Core State Standards for Mathematics (CCSSM; National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA Center and CCSSO] 2010), as shown below:

Prekindergarten (*Curriculum Focal Points and Connections* [NCTM 2006, p. 11])

> Children "examine the shapes of objects and inspect their relative positions. They find shapes in their environments and describe them in their own words," and use "objects' attributes that they have identified . . . (e.g., size, quantity, orientation, number of sides or vertices, color) for various purposes, such as describing, sorting, or comparing. For example, children sort geometric figures by shape, compare objects by weight ('heavier,' 'lighter'), or describe sets of objects by the number of objects in each set."

Kindergarten (Geometry [K.G], CCSSM 2010, p. 12)

> **Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).**
>
> 3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").
>
> **Analyze, compare, create, and compose shapes.**
>
> 4. Analyze and compare two-and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).

Grade 1 (Geometry [1.G], CCSSM 2010, p. 16)

> **Reason with shapes and their attributes.**
>
> 1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size), build and draw shapes to possess defining attributes.

Grade 2 (Geometry [2.G], CCSSM 2010, p. 20)

Reason with shapes and their attributes.

1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

Refer to the questions in Reflect 1.3 as you consider the instructional implications of each of the responses and assessments in figures 1.2–1.5. Note that what *Curriculum Focal Points* highlights for prekindergartners can be paraphrased as "Children identify and sort shapes and describe geometric figures in their own words."

Reflect 1.3

- **How could questions, comments, or additional activities extend students' understanding of shapes and their properties while facilitating the use of precise language?**
- **How would classification activities relate specifically to the standards?**

Four topics or instructional strategies stand out when the understanding that the students demonstrated in response to task 1 is considered in relation to the recommendations in *Curriculum Focal Points* (NCTM 2006) and CCSSM—as well as suggestions offered by Goldenberg and Clements (2014):

1. Exploring examples and non-examples of shapes
2. Using right angles as a defining attribute of some shapes
3. Working with properties of three-dimensional shapes, including their two-dimensional faces
4. Classifying and reclassifying shapes on the basis of different properties

These four topics or strategies are important to incorporate into work with students and lead naturally to the development of tasks that use them to build on task 1.

Tasks 2–5: Using four fundamental sorting ideas to build on task 1

Tasks 2–5 are examples of tasks that offer experiences with each of the four strategies listed above. Many other such tasks are possible.