Introduction

In our article "Practicing Algebraic Skills: A Conceptual Approach," which appeared in the April 2012 issue of *Mathematics Teacher* (Friedlander and Arcavi 2012), we proposed to promote students' learning and practicing of algebraic procedures by employing a conceptual approach, which, in our view, lies at the core of becoming competent in algebra.

This book is the result of an invitation from the National Council of Teachers of Mathematics (NCTM) to expand the ideas and examples in that article regarding the three main themes of school algebra: expressions, equations, and functions. For that purpose, we adapted our framework of skills and competencies required for an integrated procedural and conceptual learning of algebra, in alignment with the goals and recommendations of the Common Core State Standards (CCSS).

Through a collection of tasks, we aim to describe the main ideas of algebra, its central topics, its learning objectives, and the expected algebraic competencies to be enacted and consolidated. Thus, the main purposes of this book are—

- to present opportunities for learning algebra using an integrated conceptual and procedural approach;
- to increase the cognitive demand of tasks by encouraging students to employ a variety of thinking processes, strategies, and reflective practices;
- to promote qualitative and divergent thinking and creativity in school algebra;
- to create opportunities for meaningful discussions of algebraic concepts, procedures, and reasoning;
- to provide examples of algebraic activities based on both pure mathematical contexts and contextualized situations, and to require students to construct and interpret mathematical models; and
- to provide examples of algebraic activities that encourage students to employ multiple representations and alternative solution methods.

In other words, the aim of this book is to contribute to current efforts to provide alternatives to the traditional emphasis on a rote and decontextualized approach to algebra.

We based the design and analysis of our collection of tasks in this book on the following student competencies.

***Understanding and applying concepts*—**Besides procedural proficiency, students who are competent in algebra should understand the meaning of concepts and should be able to apply them in different contexts. Students' understanding of algebra can be supported by discussing alternative solution methods, by using multiple representations, and by requesting students to think qualitatively by predicting, monitoring, and interpreting results.

***Global comprehension*—**A global view in algebra implies seeing expressions, equations, and functions as entities that can be identified and operated upon as units in themselves. Thus, students should learn to view algebraic expressions, equations, and functions not only as a collection of "atomic" components such as numbers, letters, signs (e.g., equals, greater than) and operations, but also as "whole" objects.

***Representing, modeling, and interpreting*—**Algebraic expressions, equations, and functions represent quantities, patterns of, and relationships between quantities. As such, they can be used as mathematical models of real or mathematical situations, relationships, or properties. Students should be able to analyze and operate on these models, interpret the results, and on that basis, derive new knowledge about the situations they represent.

***Reverse thinking*—**Some tasks require students to reverse the "direction" of an activity by resorting to backward thinking, or by reconstructing a procedure already performed but missing. Thus, students should be able to reconstruct expressions or equations based on partial information or on the given final result.

***Generating examples*—**Students should be encouraged to explore, try, create, and review their understandings by generating examples or counterexamples to a given assertion. Providing a variety of examples or counterexamples and comparing them with those provided by others also stimulates divergent thinking.

***Monitoring one's own or others' work*—**Students should be able to understand, interpret, and evaluate algebraic solutions produced by themselves or by others, and to compare results obtained by working with multiple representations of a concept. By developing this competency, students will learn to identify, understand, analyze, justify, and explain to themselves and to others sources of errors and misconceptions, and they will be able to consolidate their mathematical knowledge.

***Generalizing*—**Students should experience a main activity in algebra consisting of identifying patterns, relations, properties, and processes of variation, and expressing them in symbolical, graphical, numerical, or verbal representations. In addition, students should be able to reach a higher level of generalization by using parameters to handle families of expressions, equations, or functions.

***Justifying and proving*—**Students should also experience algebra (and not only geometry) as a domain that allows us to engage in proofs of statements/properties either provided by the teacher or conjectured by themselves.

Each task in this book was intended to allow students to resort to some of these competencies and, thus, to consolidate and integrate their procedural and conceptual knowledge of algebra.

Structure of the Book

This book consists of four chapters. Each of the first three chapters is based on a central algebraic concept: *algebraic expressions, equations,* and *functions.* The tasks of the last chapter are intended to make connections between these three concepts and to occasionally integrate other mathematical topics (such as geometry or probability) as well.

Each chapter contains twelve activities and is preceded by an introductory section consisting of a brief description of the mathematical background of the chapter's main concept, some of the principles mentioned by the Common Core State Standards with regard to the chapter's main concept, a brief discussion of the connection between the CCSS principles and the book's framework of student competencies, a summary of the chapter's twelve tasks, and a table showing the main student competencies required by each task.

Each task can be planned for one or two lesson periods, depending on its scope, the teacher's goals, and the characteristics of the particular class. The tasks are presented in a classroom-use format, and each of them is followed by a task analysis consisting of the following three sections:

- **About the Task—**stating the task's main ideas and objectives
- **Comments and Solutions—**providing guidance for each task item, suggesting solution methods, describing possible student reactions, and sometimes providing suggestions on how to expand the task
- **Competencies—**describing the cognitive competencies required in students' work on the task

The items within each task are arranged in an increasing order of complexity and sophistication.

We strongly believe in the potential of digital technologies to enrich existing learning scenarios and also to create new and promising ones. Moreover, these technologies may provide alternative learning trajectories as well as rich experiences for learning and teaching algebra. Thus, we have occasionally interspersed some suggestions about how to implement and/or expand certain tasks by means of technology. We have decided, however, in the present book to focus on paper-and-pencil tasks that have a strong potential for integrating meanings, concepts, and relevant procedures in algebra.

We hope that teachers, teacher educators, and curriculum designers will find the conceptual framework and the tasks presented in this book useful for teaching and learning the topic of algebra.