

The Number Core: Later 2s/3s

Overview

Children can take a crucial step during this period: They can coordinate counting and seeing cardinality to understand that the last word they say when they count tells how many there are (tells the cardinality). They begin to see numbers hiding inside small numbers (e.g., “I see one and two in the three cups”). They can say the number word list to ten. Their counting becomes more accurate, with frequent correct counts of groups up to five or six. With experience, they can read the written number symbols 1 through 5.

Coordinating the components of the number core

The last word said in counting a group of things tells *how many* are in that group. This idea involves a count-to-cardinal shift in word meaning. When counting, one must at the end of the counting action make a mental shift—

- from thinking of the last counted word as referring to *the last* counted *thing* (when pointing to the last cup)
- to thinking of that word as referring to *all the things* (the number of things in the whole set, i.e., the *cardinality* of the group).

For example, when counting five plates on the table as one, two, three, four, five, the *five* refers to the single last plate pointed to when saying “five.” But then the counter must shift to thinking of all of the plates and think of the “five” as meaning all of them: “There are five plates on the table.” This is a major conceptual milestone for young children (see fig. 2.1). Adults and even older children make that shift so quickly and automatically that we do not even know we are doing it.

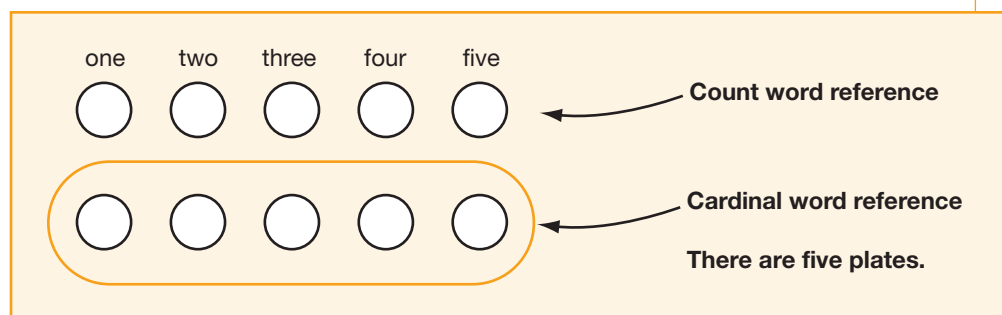


Fig. 2.1. Relating counting and cardinality

When children discover this relationship, they tend to apply it to all counts no matter the size of the set of objects. Therefore, children immediately generalize and apply this type of principle learning fairly consistently. It is relatively easy to teach children this relationship. For example, a statement of this principle followed by three demonstrations followed by another statement of the principle (see sidebar “Teaching the Cardinality Principle”)

Teaching the Cardinality Principle

The adult puts out three to seven objects in a row in front of the child each time.

“When you count, the *last* word you say tells you how many things there are. Watch me. One, two, three, four, five, six” (adult counts blocks pointing to each block). “Six. There are six blocks” (adult gestures in a narrow ellipse over the set of six blocks).

“Watch again.” Repeat for five small toy pigs. “When you count, the *last* word you say tells you how many things there are. Watch me. One, two, three, four, five” (adult counts toy pigs, pointing to each pig). “Five. There are five pigs” (adult gestures in a narrow ellipse over the set of five pigs).

“Watch again.” Repeat for seven pennies. “When you count, the *last* word you say tells you how many things there are. Watch me. One, two, three, four, five, six, seven” (adult counts pennies pointing to each penny). “Seven. There are seven pennies” (adult gestures in a narrow ellipse over the set of seven pennies).

“So the *last* word you say in counting tells you how many things you have.

Now you try it. How many trucks are here?”

was sufficient to move twenty of twenty-two children aged two years eight months to three years eleven months to using the principle (Fuson 1988).

Before children learn this relationship, they may count each time you ask “How many?” To them, “How many?” is a request to count, not a request for a cardinality. Or children may have a favorite cardinality that they say at the end of each count if asked how many, as this mother recorded in her notes about her daughter aged two years seven months:

Putting prunes back into a box, you correctly counted them up to nine. When asked how many prunes, you said *three* (your standard how many answer at this point: *three eyes*, etc.).

Even after being able to give the last counted word as the answer to “How many?” some children initially understand only that this last word answers the “How many?” question. They do not fully grasp the more abstract idea of cardinality as referring to all the objects. Thus, they may not point to all the objects when asked the cardinality question “Show me the four cups.” Instead, they may point at the last cup again. It is important to note that responding with the last word is progress. As children work more with this relationship, and with activities discussed in the relations and operations sections, they will work out the relationship between counting and cardinality more fully. Parents and teachers can show the meaning of “Show me the four cups” by gesturing around or across all four cups. But if the child does not get this cardinal idea after a couple of examples, drop it and try again at a later time when the child may be more able to take this conceptual step.

Extending cardinality

Numbers on fingers

Later 2s/3s begin to learn to recognize and to make groups of fingers that show small cardinalities. This ability often begins when their family shows them how to show their age on their fingers:

- “I am two” (showing two fingers), or
- “I am three years old” (showing three fingers).

This is an important process because these finger numbers will later on become tools for adding and subtracting. Therefore, it is important for later 2s/3s to work on showing fingers for one, two, three, four, and five without counting them out. Of course, initially they will need to raise a finger with each new count word to find out which fingers show a given number.

Cultures vary in how they show numbers on fingers. The three major ways are to—

- start with the thumb and move across the hand to the little finger;
- start with the little finger and move across the hand to the thumb;

- start with the pointing finger, move across to the little finger, and then raise the thumb.

Some cultures raise all the fingers and then lower them as they count (e.g., this is frequently done in Japan). But most cultures start with the hand closed and raise fingers with each count word. Some cultures show numbers one way for age and another for counting when adding, and so on. Teachers should find out how children in their class show numbers on their fingers and support that method. In classrooms where children use different methods, many children become fluent in showing numbers in different ways.

Partners hiding inside other numbers

Later 2s/3s begin to see and say the small numbers 1, 2, 3 inside other numbers if they have been given opportunities to hear and say such language. This example occurred at home after a child heard similar sentences on *Sesame Street* where pictures were shown with the sentence: The child asked for four olives, and her father gave them to her. She said, “Two and two make four.”

This process is often called *conceptual subitizing* (Clements 1999) because it involves seeing the small numbers rather than counting them. Calling these smaller numbers *partners* helps children relate these two smaller numbers within the total. For example, *one* and *two* are *partners* of *three*. With experience, children move rapidly from the partners to revisualizing them to see the total and can even express this relationship in words, as in the foregoing example. Children need opportunities to see and hear such partner sentences at home and at care or educational centers. Other examples are given in “Activities for Number Partners Hiding Inside Other Numbers” on the next page. These issues will be discussed more in the section on operations.

Extending the number-word list

If children have opportunities to hear the number-word list beyond their accurate range, they will extend this accurate portion of their list. With such experiences, later 2s/3s can count to ten and are working on the irregular teen patterns and the twenty to twenty-nine pattern.

Extending counting accuracy

Later 2s/3s continue to generalize what they can count and extend their accurate counting to larger sets. Children with little experience with books may have more difficulty counting pictures of objects rather than objects themselves. Therefore, it is important that all children practice counting pictures of objects as well as objects.

Very young children counting small rows with high effort make more errors in which their say-point *actions do not correspond in time* than errors in the

Activities for Number Partners Hiding Inside Other Numbers

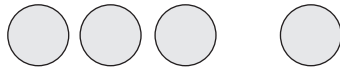
- 1) Adult: "How many balls?"



Child: "Two and 1 more is 3."



Child: "I saw 2 and 2, so I said 4."



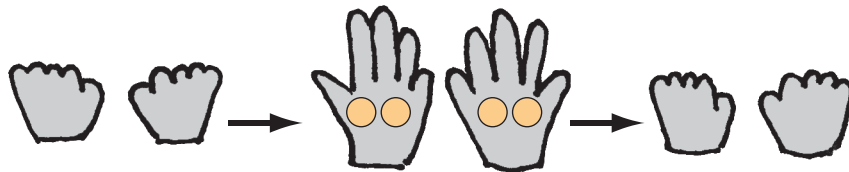
Child: "First I saw 3, then one more is 4."

- 2) Have the child make number partner displays (like in 1 above) for an adult or another child.

Matteo:  Becky: "3 and 2 more is 5." 

- 3) Have children look for number partners hiding inside other numbers in pictures in books.

- 4) For children who are already good at the previous activities, do a "snapshots" version of activity (1). Hide a small number of objects in your hands. Tell children they should take a snapshot picture in their mind when you open your hands. Open your hands briefly to show the objects. Then close your hands and ask how many objects there were in all. Allow more viewing time if needed.



spatial matching of the points and objects. Thus, they may need more practice coordinating their actions of saying one word and pointing at an object. Energetic collective practice in which children rhythmically say the number-word list and move their hand down with a finger pointed out as each word is said can be helpful. To vary the practice, the words can sometimes be said loudly and sometimes softly, but always with emphasis (a regular beat). The

points can involve a large motion of the whole arm or a smaller motion, but again in a regular beat with each word. Coordinating these actions of saying and pointing is the goal for overcoming this type of error. For variety, these activities can involve other movements, such as marching around the room with rhythmic arm motions or stamping a foot, saying a count word each time. See “Examples of Incomplete Understanding of Counting” on the next page.

Young children sometimes make multiple count errors on the last object. They either find it difficult to stop, or they think they need to say a certain number of words when counting and just keep on counting so they say that many. When they say the number-word list, more words *are* better. They need to learn that saying the number-word list when counting objects is controlled by the number of objects. Reminding them that even the last object gets only one word and one point can help. They also may need the physical support of holding their hand as they reach to point to the last object so that the hand can be stopped from extra points. When doing this, say the last word loudly and stretched out (e.g., “fi-i-i-ve”) to help the child inhibit saying the next word.

Regularity and rhythmicity are important aspects of counting. Activities that increase these aspects can be helpful to children making lots of correspondence errors. Children who are not discouraged about their counting competence generally enjoy counting all sorts of things and will do so if there are objects they can count at home or in a care or educational center. Counting in pairs of children to check each other so as to find and correct errors is often fun for the pairs. Counting within other activities, such as building towers with blocks, should also be encouraged.

Extending written number symbols

If children have opportunities to see a written number symbol and hear its number word, they will extend the number of symbols they know. With such experiences, later two- and three-year-olds can recognize and say the number words for 1, 2, 3, 4, 5 and perhaps other numerals.

Coordinating count words, quantities, and written numerals

Two- and three-year-olds are working on all six of the relationships shown by the three line segments in figure 2.2 on page 21 (each relationship goes both ways). Quantities as seen or as counted cardinalities, count words, and written numerals become connected and form the foundation for later work with these numbers in relations and operations.

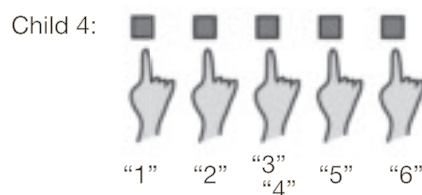
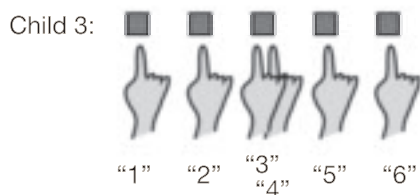
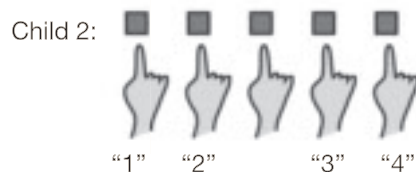
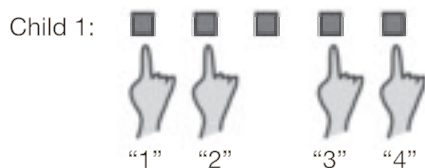
Monitoring and Correcting Children's Count Errors

Parents and teachers do not have to monitor or correct children's counting all the time. It is much more important for all children to get frequent counting practice, want to count and feel confident about it, and watch and help one another. Once children basically understand correct counting, they still will continue to make some errors, especially with larger sets. Children who do not show some correspondence between count words, points, and objects do need help until they can do so. Sometimes the adult can do part of the activity, such as the pointing, and can support the child by counting along. Some children need the adult to hold their pointing hand to get the motor feedback about what correct counting is. But for those who basically understand, frequent reminders to the whole class (or by a parent hearing or seeing a count error from afar) about the general process can be helpful:

- Remember not to skip over any.
- Remember to point one time to each thing.
- Try hard! You can do it!

Effort is a big part of correct counting. Children who make the same particular count errors do need support to overcome those errors through modeling by the adult or being helped physically with the pointing and counting. Establishing an exaggerated rhythm can help make correspondences; the child can back down to a more relaxed counting when the correspondences are going well.

Examples of Incomplete Understanding of Counting



Child 5: Keeps counting past 5.

Child 6: Doesn't know that the last number word said tells how many objects there are, so counts over again when asked "How many in all?"

Activities for Children to Practice

1) Rhythmic counting with hops (or claps, arm movements, and so on):



"We hopped _____ (or five) times!"

2) Counting wand: rhythmically count together while the teacher (or later, a child) points to children (or objects) one by one.

