

FOREWORD

THE COMMON CORE STATE STANDARDS INITIATIVE (CCSSI) may be the most profound and widely distributed educational reform activity in recent history. One of the results of its efforts, the *Common Core State Standards for Mathematics* (CCSSM 2010), will affect nearly every K–12 student and the majority of the nation’s teachers over the next decade. CCSSI focuses on a foundational element of K–12 education—curriculum—as a means to improve student learning opportunities. Although CCSSI was formed using a top-down approach at the prompting of a group of governors and chief state school officers, the primary audience for and ultimate users of the standards are classroom teachers.

The question of whether CCSSM represents the best thinking in the field or whether the existence of common standards, in general, will advance or inhibit innovative curriculum development and evaluation is unresolved. However, what is clear is the need to support teachers in their work helping students achieve a high-quality mathematics education. Given the centrality of curriculum in the work of teachers (and students), supporting teachers as they prepare, plan, and implement instruction that attends to shared educational goals is critical. The adoption of CCSSM by 45 states and the District of Columbia represents a watershed moment of consensus and an opportunity to support teachers in a common goal.

The primary focus of this volume is on elaborating the Standards for Mathematical Practice outlined in CCSSM. These standards are given front-page attention in the CCSSM (that is, they are introduced early in the document and intended to permeate the entire K–12 curriculum). However, the brevity with which they are described and the lack of integration into the standards for mathematical content make them easy to overlook or ignore. Doing so will substantially dilute the impact of and potential for CCSSM to improve student learning of mathematics.

The ideas in the Standards for Mathematical Practice are not new but rather linked to mathematical goals articulated in previous documents and by other groups. In fact, problem solving and reasoning are at the core of all the practices outlined in CCSSM, as they have been at the core of the work of the National Council of Teachers of Mathematics (NCTM) since the publication of *An Agenda for Action* in 1980. In fact, the first recommendation of the *Agenda* was that “Problem solving must be the focus of school mathematics” (p. 2). The document went on to say, “Performance in problem solving will measure the effectiveness of our personal and national possession of mathematical competence” (p. 2).

In subsequent NCTM curriculum recommendations—*Curriculum and Evaluation Standards for School Mathematics* (1989) and *Principles and Standards for School Mathematics* (2000)—the role and place of mathematical processes and practices are underscored and further elaborated. There are other instantiations where these ideas have been eloquently presented. For example, in 1996, Cuoco, Goldenberg, and Mark wrote about the importance of mathematical habits of mind as a central focus of mathematics instruction:

Although it is necessary to infuse courses and curricula with modern content, what is even more important is to give students the tools they will need in order to use, understand, and even make mathematics that does not yet exist. A curriculum organized around habits of mind tries to close the gap between what the users and makers of mathematics do and what they say. (p. 376)

Likewise, the authors of *Adding It Up: Helping Children Learn Mathematics* (NRC 2001) highlighted the need to think more broadly about what it means to learn mathematics:

Recognizing that no term captures completely all aspects of expertise, competence, knowledge, and facility in mathematics, we have chosen mathematical proficiency to capture what we believe is necessary for anyone to learn mathematics successfully. . . . Mathematical proficiency, as we see it, has five components, or strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, productive disposition. (p. 116)

The Standards for Mathematical Practice outlined in CCSSM, and discussed more fully in this volume, are a reaffirmation of the significance of and a call for renewed emphasis on these same themes (habits of mind, mathematical processes, mathematical proficiency) as a significant aspect of learning mathematics. While some terms may be new to teachers, the central ideas have been around a long time and remain the same. The authors of this volume recognize and make explicit connections between these related ideas and the CCSSM Standards for Mathematical Practice.

A major contribution of this volume is the attention to making explicit, through classroom vignettes, images of the mathematical practices. These vignettes come from classrooms at different grade levels (elementary, middle, and secondary) and within the context of different mathematical content. This type of elaboration is essential in helping users of CCSSM (e.g., teachers, curriculum and assessment developers, teacher educators) apply and connect the practices to content and classroom instruction.

In the 1980 *An Agenda for Action*, the NCTM Board of Directors acknowledged its professional obligation:

We recognize as valid and legitimate the role of public opinion in the determination of educational goals. But this philosophy is predicated on a well-informed public. Thus, the National Council of Teachers of Mathematics, as an organization of professional educators, has a special obligation to present its responsible and knowledgeable viewpoint of the directions mathematics programs should be taking in the 1980s. (p. i)

With this NCTM volume, the authors continue a tradition of supporting teachers in their enactment of a core curriculum that is based on this vision.

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INTRODUCTION

Working at teaching turns out to be something like navigating in a multidimensional terrain, getting safely across the street while also crossing the city, sighting one island from another while catching the wind that makes it possible to get around the corner and across the ocean. (Lampert 2001, p. 50)

TEACHING MATHEMATICS IS AN INCREDIBLY COMPLEX TASK. Teachers must help students learn mathematical content while also guiding them in what it means to *do mathematics*. The eight Standards for Mathematical Practice in the new *Common Core Standards for School Mathematics* (CCSSI 2010) provide one vision of doing mathematics, and this book is intended as a roadmap to help teachers navigate these practices.

The *Common Core State Standards for Mathematics* (CCSSM) were developed as part of a project sponsored by the National Governors Association and the Council of Chief State School Officers in an effort to “define what students should understand and be able to do in their study of mathematics” (CCSSI 2010, p. 1). Although the authors of this document assert that the CCSSM were not intended to be national standards, 45 states and the District of Columbia have formally adopted or endorsed the standards. Only 5 states have not adopted the standards at this time.

While grade-specific content standards make up the body of CCSSM, it begins with the eight Standards for Mathematical Practice. Although the description of each practice is brief in the CCSSM document, the mathematical practices describe what it means to do mathematics and should therefore permeate mathematics instruction across grade levels and content areas. Careful integration of the mathematical practices into classroom instruction is of particular importance because mathematics content is often foregrounded (and assessed) over mathematical practice and process.

The mathematical practices are written as different skills, dispositions, and understandings of mathematics that students should have; however, it is teachers who must provide meaningful experiences in which students may develop these various forms of expertise. This book is designed to help in that endeavor by elaborating on each of the eight practices: describing each practice in more detail by discussing connections to the National Council of Teachers of Mathematics’ (NCTM) five Process Standards (NCTM 2000), providing elementary and secondary classroom examples of the practices, and giving readers additional resources to seek out for each practice. Similar to the mathematical practices themselves, each chapter is slightly different in terms of structure, scope, and size. While they are presented in the CCSSM as individual practices, we believe it is important for teachers to integrate the practices in their pedagogy. Therefore, the final chapter of this book highlights some of the important connections among practices.

Note: While the authors are listed in order as they joined the project, all four authors contributed equally in the development and writing of this book. The authors wish to thank Dr. Jack Smith and Dr. Joel Amidon for reviewing our work and providing feedback and Janet M. Liston for contributing to the high school vignette section of practice 1.

NCTM Resources

While each chapter lists additional resources, most from NCTM, that are relevant to the specific mathematical practice, there are two important sets of NCTM resources that span all the practices, and we strongly encourage you to read them to deepen your understanding of the mathematical practices.

- NCTM's Navigations series is an excellent resource to support teachers in infusing their teaching with the *Principles and Standards for School Mathematics* (NCTM 2000). Each volume is geared toward a specific grade level or grade band and focuses on one or more aspects of the *Principles and Standards*. The series is full of excellent tasks that teachers can use with their students to support their learning with understanding. Each book also comes with a CD-ROM containing activities to use with students, printable activity pages, articles from NCTM's journals, and interactive applets. See the NCTM catalog online at <http://www.nctm.org>.
- NCTM's Essential Understandings series is another excellent resource. Each book in the series provides a comprehensive overview of the big ideas and essential understandings of mathematical topics in the K–12 curriculum. Currently eleven titles are available, with titles including *Developing Essential Understanding of Expressions, Equations, and Functions for Teaching Mathematics in Grades 6–8* and *Developing Essential Understanding of Multiplication and Division for Teaching Grades 3–5*, with six more titles forthcoming. These books are a helpful tool in learning about the conceptual structures of the ideas in each topic and are useful in planning lessons that assist students in recognizing important mathematical concepts and structures.

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