Chapter 2

Creating Mathematical Futures Through an Equitable Teaching Approach

The Case of Railside School

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Soon after moving to California from England, I (Boaler, 2002a) was interested in conducting a study to follow through on the one I had conducted in England—looking at the impact of different teaching approaches upon student learning.[1] I had heard about an interesting school where the math teachers collaborated greatly and used an unusual pedagogical approach called "Complex Instruction." Complex Instruction is an approach to teaching developed by Elizabeth Cohen and Rachel Lotan that focuses on disrupting status differences in the classroom and utilizing groupwork for optimal engagement and learning. I visited the school and immediately saw a very unusual teaching approach, so we were thrilled when the department agreed to be part of a comparative study that would allow us to learn from their work.

The low and inequitable mathematics performance of students in urban American high schools has been identified as a critical issue contributing to societal inequities (Moses & Cobb, 2001) and poor economic performance (Madison & Hart, 1990). Thousands of students in the United States and elsewhere struggle through mathematics classes, experiencing repeated failure. Students often disengage from mathematics, finding little intellectual challenge, as they are asked only to memorize and execute routine procedures (Boaler, 2002a). But the question of how best to teach mathematics remains controversial, and debates are dominated by ideology and advocacy (Rosen, 2001).

In this chapter, we report upon a 5-year longitudinal study of approximately 700 students as they progressed through three high schools. One of the findings of the study was the important success of one of the schools. At Railside School students learned more, enjoyed mathematics more, and progressed to higher mathematics levels. What made this result more important was the fact that Railside is an urban school on what locals refer to as the "wrong" side of the tracks. Students come from homes with few financial resources, and the population is culturally and linguistically diverse, with many English language learners. At the beginning of high school, at the start of 9th grade, the Railside students were achieving at significantly lower levels than the students at the other two, suburban schools in our study. Within 2 years the Railside students were significantly outperforming students at the other schools. The students were also more positive about mathematics, they took more mathematics courses, and many more of them planned to pursue mathematics in college. In addition, achievement differences among students of different ethnic groups were reduced in all cases and were eliminated in most. By their senior year, 41% of Railside students were taking advanced classes of precalculus and calculus, compared to approximately 27% of students in the other two schools. Mathematics classes at Railside had a high work rate and few behavioral problems, and the ethnic cliques that form in many schools were not evident. In interviews, the students told us that they learned to respect students from other cultures and circumstances through the approach used in their mathematics classes.

The mathematics teachers at Railside achieved something important that many other teachers could learn from—they organized an effective instructional program for students from traditionally marginalized backgrounds and they taught students to enjoy mathematics and to include it as part of their futures. In this chapter, we present evidence of these important achievements and report upon the ways that the teachers brought them about.

We conducted our study of student learning in different schools with the knowledge that a multitude of schooling variables—ranging from district support and departmental organization (Talbert & McLaughlin, 1996) to curricular examples and classroom interactions—could impact the learning of students and the promotion of equity. Our study centered upon the affordances of different curricula and the ensuing teaching and learning interactions in classrooms. It also considered the role of broader school factors and the contexts in which the different approaches were enacted.

UNDERSTANDING MATH AT RAILSIDE

The Stanford Mathematics Teaching and Learning Study was a 5-year longitudinal study of three high schools with the following pseudonyms: Greendale, Hilltop, and Railside. These three schools are reasonably similar in terms of their size and share the characteristic of employing committed and knowledgeable mathematics teachers. They differ in terms of their location and student demographics (see Table 2.1).[2]

Railside High School, the focus of this analysis, is situated in an urban setting. Lessons are frequently interrupted by the noise of trains passing just feet away from the classrooms. Railside has a diverse student population, with students coming from a variety of ethnic and cultural backgrounds. Hilltop High School is situated in a more rural setting, and approximately half of the students are Latino and half White. Greendale High School is situated in a coastal community with very little ethnic or cultural diversity (almost all students are White).

Railside School used a reform-oriented approach and did not offer families a choice of math programs in which to enroll. The teachers worked collaboratively, and they had designed the curriculum themselves, drawing from different reform curricula such as the College Preparatory Mathematics Curriculum (Sallee, Kysh, Kasimatis, & Hoey, 2000) and Interactive Mathematics Program (Alper, Fendel, Fraser, & Resek, 2003). In addition to a common curriculum, the teachers also shared teaching methods and ways of enacting the curriculum. As they emphasized to us, their curriculum could not be reduced to the worksheets and activities they gave students. Mathematics was organized into the traditional sequence of classes—algebra followed by geometry, then advanced algebra and so on—but the students worked in groups on longer, more conceptual problems.

Another important difference to highlight was the heterogeneous nature of Railside classes. Whereas incoming 9th-grade students in Greendale and Hilltop could enter geometry or could be placed in a remedial class, such as "Math A" or "Business Math," all students at Railside entered the same algebra class. The department was deeply committed to the practice of mixed-ability teaching and to giving all students equal opportunities for advancement.

We monitored three approaches in the study—"traditional" and "IMP" (as labeled by Hilltop and Greendale) and the "Railside approach." As the numbers in the IMP approach were insufficient for statistical analyses, the main comparison groups of students in the study were approximately 300 students who followed the traditional curriculum and teaching approaches in Greendale and Hilltop schools and approximately 300 students at Railside who were taught using reform-oriented curriculum and teaching methods. These two groups of students provide an interesting contrast, as they experienced the same content, taught in very different ways. Class sizes were similar across the schools: approximately 20 students in each math class in Year 1, in line with the class-size reduction policy that was in place in California at that time, and 25–35 students in Years 2 and 3.

WHAT WE LEARNED AT RAILSIDE

Given our goal of understanding the highly complex phenomena of teaching and learning mathematics, we gathered a wide array of data, both qualitative and

Table 2.1. Schools, Students, and Mathematics Approaches

	Railside	Hilltop	Greendale
Enrollment (approx.)	1,500	1,900	1,200
Study demographics	40% Latino/a 20% African American 20% White 20% Asian/ Pacific Islanders	60% White 40% Latino/a	90% White 10% Latino/a
ELL[a] students	30%	20%	0%
Free/reduced lunch	30%	20%	10%
Parent education, % college grads	20%	30%	40%
Mathematics curriculum approaches	Teacher – designed, reform-oriented curriculum, conceptual problems, group work	Choice between "traditional" (demonstration and practice, short problems) and IMP (groupwork, long, applied problems)	Choice between "traditional" (demonstration and practice, short problems) and IMP (groupwork, long, applied problems)

[a] ELL is English language learners.

quantitative, on the teaching approaches and classroom interactions, and student achievement, and students' views of mathematics. Although students' learning experiences ultimately happen on the classroom level, they are shaped and organized by factors in the broader context, such as curriculum design, course sequencing, and departmental collaboration. Consequently, we sought to work across multiple levels to understand the organization of instruction on the classroom level.

We report our results in two parts. The first set of results is more quantitative, describing broader trends and documenting differences. We offer more statistical detail than is perhaps necessary in this section, so that the interested reader is provided more information. The statistics, however, need not be read in detail to understand the overall results and the story of Railside. The second set of results is more qualitative in nature, with descriptions and quotes from the students, which helped to show *how* the documented differences came about.

The Teaching Approaches

Most of the students in Hilltop and Greendale high schools were taught mathematics using a traditional approach, as described by teachers and students—the students sat individually, the teachers presented new mathematical methods through lectures, and the students worked through short, closed problems. Our coding of lessons showed that approximately 21% of the time in algebra classes was spent with teachers lecturing, usually demonstrating methods. Approximately 15% of the time teachers questioned students in a whole-class format, 48% of the time students were practicing methods in their books, working individually, and students presented work for approximately 0.2% of the time. The average time spent on each mathematics problem was 2.5 minutes, or an average of 24 problems in one hour of class time. Our focused analysis of the types of questions teachers asked, which classified questions into seven categories, was conducted with two of the teachers of traditional classes (325 minutes of teaching). This analysis showed that 97% and 99% of the two teachers' questions in traditional algebra classes fell into the procedural category (Boaler & Brodie, 2004).

At Railside School the teachers posed longer, conceptual problems and combined student presentations with teacher questioning. Teachers rarely lectured (for approximately 4% of the time), and students were taught in heterogeneous groups. Approximately 9% of the time teachers questioned students in a whole-class format, 72% of the time students worked in groups while teachers circulated the room showing students methods, helping students and asking them questions of their work, and students presented work for approximately 9% of the time. The average time spent on each mathematics problem was 5.7 minutes, or an average of 16 problems in a 90-minute class period—less than half the number completed in the traditional classes. Our focused analysis of the types of questions teachers asked, conducted with two of the Railside teachers (352 minutes of teaching), showed that Railside math teachers asked many more varied questions than the teachers of traditional classes. Sixty-two percent of their questions were procedural, 17% conceptual, 15% probing, and 6% fell into other questioning categories (Boaler & Brodie, 2004). The broad range of questions they asked was typical of the teachers at Railside, who deliberately and carefully discussed their teaching approaches, a practice that included sharing good questions to ask students. We conducted our most detailed observations and analyses in the 1st-year classes when students were taking algebra, but our observations in later years as students progressed through high school showed that the teaching approaches described above continued in the different mathematics classes the students took.

Student Achievement and Attainment

At the beginning of high school we gave a test of middle school mathematics to all students starting algebra classes in the three schools.[3] At Railside, all incoming students were placed in algebra, as the school employed heterogeneous grouping. Comparisons of means indicated that at the beginning of Year 1, the students at Railside were achieving at significantly lower levels than students at the two other schools using the traditional approach ($t = -9.141$, $p < 0.001$, $n = 658$), as can be seen in Table 2.2. At the end of Year 1 we gave all students a test of algebra to measure what students had learned over the year. The difference in means (1.8) showed that the scores of students in the two approaches were similar (traditional = 23.9, Railside = 22.1), a difference that was significant at the 0.04 level ($t = -2.04$, $p = 0.04$, $n = 637$). Thus the Railside students' scores were approaching comparable levels after a year of algebra teaching. At the end of Year 2 we gave students a test of algebra and geometry, reflecting the content the students had been taught over the first 2 years of school. By the end of Year 2 Railside students were significantly outperforming the students in the traditional approach ($t = -8.309$, $p < 0.001$, $n = 512$).

There were fewer students in the geometry classes in Railside due to the flexibility of Railside's timetable, which allowed students to choose when they took geometry classes (as will be described in the next section). The students in geometry classes at Railside did not represent a selective group; they were of the same range as the students entering Year 1.

Before proceeding, we describe in more detail the type of information presented in Table 2.2. Table 2.2 reports the mean score, or the average of the students' scores on the assessment, for each group of students (Traditional, Railside).

Table 2.2. Assessment Results

	Traditional			**Railside**			
	Mean score	*Std Deviation*	*n*	*Mean score*	*Std Deviation*	*n*	***t*** **(level of significance)**
Y1 Pretest	**22.23**	8.857	311	**16.00**	8.615	347	–9.141 ($p < 0.001$)
Y1 Posttest	**23.90**	10.327	293	**22.06**	12.474	344	–2.040 ($p = 0.04$)
Y2 Posttest	**18.34**	10.610	313	**26.47**	11.085	199	–8.309 ($p < 0.001$)
Y3 Posttest	**19.55**	8.863	290	**21.439**	10.813	130	–1.75 ($p = 0.082$)

Along with the mean scores, Table 2.2 reports Std Deviation, which is the standard deviation of those scores. This value indicates how "spread out" the scores were. The higher the standard deviation, the wider the range of the scores. A standard deviation of 8.8 indicates that about 70% of the students' scores were between 8.8 points below and 8.8 points above the mean score. (Note that one uses a standard deviation on data that are assumed to be normally distributed.) The "*n*" represents how many students, or the sample size. Finally, the "*p*" value in the last column is the "level of significance," and it indicates how likely the observed differences are to be the result of random chance. In the case of $p < .0001$, it means that these observed differences between mean scores, given the spread of the data, are likely to happen by chance less than 1 in 10,000 times. A *p* value of .05 means that the observed differences from the given populations are likely to happen by chance 1 in 20 times. By convention, we set the bar at this .05 level (1 in 20), and when observed differences are less likely than this to have occurred by chance, we say that the difference is statistically significant, represented by *t*.

The scores in Table 2.2 were for all assessments taken by students. In order to determine whether student attrition impacted the mean scores, we also compared the scores for students in both approaches who took all three tests. We wanted to know if scores in later years were artificially inflated by students who transferred or dropped out of high school. These results show that the Railside students taking all three assessments started at significantly lower levels and ended Year 2 at significantly higher levels (see Table 2.3). (These analyses include only those students who went straight from algebra to geometry in each school [a smaller number] and do not capture students who repeated a course or took time off from math.)

Interestingly, the students most advantaged by the teaching approach at Railside, compared to those in traditional, tracked classes, appeared to be those who started at the highest levels. These students showed the greatest achievement advantage in Year 2 when compared with students in tracked classes, a finding that should alleviate concerns that high-attaining students are held back by working in heterogeneous groups. Interview data, reported in the next sections, suggest that the high-attaining students developed deeper understanding from the act of explaining work to others.

Table 2.3. Scores of Students who took Y1 Pretest, Y1 Posttest, and Y2 Posttest

		Y1 Pretest		Y1 Posttest		Y2 Posttest	
	n[a]	*Mean*	*SD*	*Mean*	*SD*	*Mean*	*SD*
Railside	90	20.58	8.948	29.19	11.804	24.96	10.681
Traditional	163	23.44	8.802	25.86	10.087	16.58	8.712
t (level of significance)		2.463 ($p = 0.014$)		2.364 ($p = 0.019$)		6.364 ($p = 0.000$)	

[a] *n* is number of students taking all three tests.

In Year 3 the students at Railside continued to outperform the other students, although the differences were not significant ($t = -1.75$, $p = 0.082$, $n = 420$). The Railside students' achievement in Year 3 classes may not have been as high in relation to the traditional classes, as the Year 3 Railside curriculum had not been developed as much by the department, and the classes were taught by teachers in their first 2 years of teaching. In Year 4 we did not administer achievement tests. However, more students at Railside continued to take higher-level math courses. By their senior year, 41% of Railside students were taking advanced classes of precalculus and calculus, compared to about 27% of students in the other two schools.[4]

The Railside mathematics teachers were also extremely successful at reducing the achievement gap among groups of students belonging to different ethnic groups at the school. Table 2.4 shows significant differences among groups at the beginning of the 9th-grade year, with Asian, Filipino, and White students each outperforming Latino and Black students ($p < .001$).

At the end of Year 1, there were no longer statistically significant differences between the achievement of White and Latino students, nor the Filipino students and Latino and Black students. The significant differences that remained at that time were between White and Black students and between Asian students and Black and Latino students (ANOVA $F = 5.208$; $df = 280$; $p = 0.000$). Table 2.5 shows these results.

In subsequent years the only consistent difference that remained was the high performance of Asian students, who continued to significantly outperform Black and Latino students. Differences among White, Black, and Latino students' scores on our tests were not present. At the other schools, achievement differences between students of different ethnicities remained. At Railside there were also no gender differences in performance on any of the tests, and young women were well represented in higher mathematics classes.

Student Perceptions and Relationships with Mathematics

In addition to high achievement, the students at Railside also enjoyed mathematics more than the students in the other approach. In questionnaires given to the students each year, the Railside students were always significantly more positive about their experiences with mathematics. For example, 71% of Railside students in Year 2 classes ($n = 198$) reported "enjoying math class," compared with 46% of students in traditional classes ($n = 318$) ($t = -4.934$; $df = 444.62$; $p < 0.001$). In the Year 3 questionnaire students were asked to finish the statement "I enjoy math in school" with one of four time options: all of the time, most of the time, some of the time, or none of the time. Fifty-four percent of students from Railside ($n = 198$) said that they enjoyed mathematics all or most of the time, compared with 29% of students in traditional classes ($n = 318$), which is

Table 2.4. Railside Year 1 Pretest Results by Ethnicity

Ethnicity	*n*	Mean	Median	Std Dev.
Asian	27	22.41	22	8.509
Black	68	12.28	12	6.286
Hispanic/Latino	103	14.28	12	7.309
Filipino	23	21.61	22	8.289
White	51	21.20	21	9.362

Table 2.5. Railside Year 1 Posttest Results by Ethnicity

Ethnicity	*n*	Mean	Median	Std Dev.
Asian	27	29.44	30	12.148
Black	68	18.21	16.50	10.925
Hispanic/Latino	103	21.31	21	11.64
Filipino	23	26.65	26	10.504
White	51	26.69	28	13.626

a significant difference ($t = 4.758$; $df = 286$; $p < 0.001$). In addition, significantly more Railside students agreed or strongly agreed with the statement "I like math," with 74% of Railside students responding positively, compared with 54% of students in traditional classes ($t = -4.414$; $df = 220.77$; $p < 0.001$).

In Year 4 we conducted interviews with 105 students in the three different schools. Most of the students were seniors, and they were chosen to represent the breadth of attainment displayed by the whole school cohort. These interviews were coded, and students were given scores on the categories of *interest*, *authority*, *agency*, and *future plans for mathematics*. The categories of *authority* and *agency* (Holland, Lachiotte, Skinner, & Cain, 1998) emerged as important, as students in the different approaches varied in the extent to which they believed they had authority (the capacity to validate mathematical methods and ideas using their own knowledge rather than the teacher or textbook) or that they could work with agency (having the opportunity to inquire and use their own ideas; see Boaler, 2009). Significant differences were found in all of these categories, with the students at Railside being significantly more interested in mathematics ($x^2 = 12.806$, $df = 2$, $p = 0.002$, $n = 67$) and believing they had significantly more authority ($x^2 = 29.035$, $df = 2$, $p = 0.000$, $n = 67$) and agency ($x^2 = 22.650$, $df = 2$, $p = 0.000$, $n = 63$). In terms of future plans, *all* of the students interviewed at Railside intended to pursue more mathematics courses, compared with 67% of students from the traditional classes, and 39% of Railside students planned a future in mathematics, compared with 5% of students from traditional classes ($x^2 = 18.234$, $df = 2$, $p = 0.000$, $n = 65$).

Because of the challenges of accessing individual student data and confidentiality issues, we are unable to report anything beyond school scores for the students on state-administered tests. Despite this limitation, these school-level data are interesting to examine and raise some important issues with respect to testing and equity, as Railside students performed higher on our tests, district tests, and the California Standards Test of algebra but did not fare as well on the CAT 6, a standardized test, nor on indicators of Adequate Yearly Progress (AYP), which are determined primarily by standardized tests.

In contrast, the California Standards Test, a curriculum-aligned test taken by students who had completed algebra, showed the Railside students scoring at higher levels than the other two schools (see Table 2.6). Fifty percent of Railside students scored at or above the basic level, compared to 30% at Greendale[5] and 40% at Hilltop. Students at Hilltop and Greendale scored at higher levels on the CAT 6, and these schools had higher AYP numbers, as seen in Tables 2.7 and 2.8.

The relatively low performance of the Railside students on the state's standardized tests is interesting and may be caused by the cultural and linguistic barriers provided by the state tests. The correlation between students' scores on the language arts and mathematics sections of the AYP tests, across the whole state of California, was a staggering 0.932 for 2004. This data point provides strong indication that the mathematics tests were testing language as much as mathematics. This argument could not be made in reverse, as the language tests do not contain mathematics. Indeed, the students at Railside reported in open-ended interviews that the standardized tests used unfamiliar terms and culturally biased contexts that our tests did not use (see also Boaler, 2003). Tables 2.7 and 2.8 also show interesting relationships between mathematics and language, as the Greendale and Hilltop students were more successful on tests of reading and language arts, a trend that held across the state, but the Railside students were as or more successful on mathematics. Another interesting result to note is that 40% more White students scored at or above the 50th percentile than Latino students at Hilltop

Table 2.6. California Standards Test, Algebra, 2003: Percentage of Students Attaining Given Levels of Proficiency

	Greendale	Hilltop	Railside
Advanced	0	0	0
Proficient	10	10	20
Basic	30	30	30
Below basic	60	40	40
Far below basic	10	20	20

Table 2.7. CAT 6, 2003, STAR, Grade 11 (Year 3): Percentage of Students at or Above 50th Percentile

	Railside	Hilltop	Greendale
Reading	40	60	70
Language	30	50	70
Mathematics	40	50	70

Table 2.8. AYP (Adequate Yearly Progress), 2003: Difference Between Percentage of Students Scoring at "'Proficient" Level in Language Arts and Mathematics (Data Rounded to Nearest Whole Number)

	Difference (% proficient in language arts—% proficient in mathematics)	"Similar schools" average difference
Railside	1	13
Hilltop	9	11
Greendale	15	12

(the only other sizeable group of ethnic minority students in the study) on the CAT 6. At Railside the difference between the same two groups was only 10%. The data in Tables 2.6–2.8 may indicate the inability of the state standardized tests to capture the mathematical understanding of the Railside students that was demonstrated in many other formats.

Summary Comments

The students at Railside enjoyed mathematics more than students taught more traditionally, they achieved at higher levels on curriculum-aligned tests, and the achievement gap between students of different ethnic and cultural groups was lower than those at the other schools. In addition, the teachers and students achieved something that Boaler (2006b, 2008) has termed *relational equity*. In studying equity most researchers look for reductions in achievement differences for students of different ethnic and cultural groups and genders when tests are taken. But Boaler has argued that a goal for equity should also be the creation of classrooms in which students learn to treat each other equitably, showing respect for students of different cultures, genders, and social classes. Schools are places where students learn ways of acting and being that they are likely to replicate in society, making respect for students from different circumstances an important goal. It is not commonly thought that mathematics classrooms are places where students should learn about cultural respect, but students at Railside reported that they learned to value students who came from very different backgrounds to themselves because of the approach of their mathematics classes (for more detail, see Boaler, 2006b, 2008).

ANALYZING THE SOURCES OF SUCCESS

I. The Department, Curriculum, and Timetable

Railside had an unusual mathematics department. During the years of our study, 12 of the 13 teachers worked collaboratively, spending vast amounts of time designing curricula, discussing teaching decisions and actions, and generally improving their practice through the sharing of ideas. Unusually for the United States, the mathematics department strongly influenced the recruitment and hiring of teachers, enabling the department to maintain a core of teachers with common philosophies and goals. The teachers shared a strong commitment to the advancement of equity, and the department had spent many years working out a coherent curriculum and teaching approach that teachers believed enhanced the success of all students. The mathematics department had focused their efforts in particular upon the introductory algebra curriculum that all students take when they start at the school. The algebra course was designed around key concepts, with questions drawn from various published curricula such as College Prepatory Mathematics (CPM), IMP, and a textbook of activities that use Algebra Lab Gear™ (Picciotto, 1995). A theme of the algebra and subsequent courses was multiple representations, and students were frequently asked to represent their ideas in different ways, using math tools such as words, graphs, tables and symbols. In addition, connections between algebra and geometry were emphasized even though the two areas were taught in separate courses.

Railside followed a practice of block scheduling, and lessons were 90 minutes long, with courses taking place over half a school year, rather than a full academic year.[6] In addition, the introductory algebra curriculum, generally taught in one course in U.S. high schools, was taught in the equivalent of two courses at Railside. The teachers spread the introductory content over a longer period of time partly to ensure that the foundational mathematical ideas were taught carefully with depth and partly to ensure that particular norms—both social and sociomathematical (Yackel & Cobb, 1996)—were carefully established. The fact that mathematics courses were only half a year long at Railside may appear unimportant, but this organizational decision had a profound impact upon the students' opportunities to take higher-level mathematics courses. At Greendale and Hilltop (as in most U.S. high schools), mathematics classes were 1 year long and a typical student began with algebra. Consequently, students couldn't take calculus unless they were advanced, as the standard sequence of courses was algebra, geometry, advanced algebra, then precalculus. Furthermore, if a student failed a course, the level of content he or she would reach is limited. In contrast, a Railside student could take two mathematics classes each year. Consequently, students could fail classes, start at lower levels, and/or choose not to take mathematics in a particular semester and still reach calculus. This relatively simple scheduling decision was part of the reason why significantly more students at Railside took advanced-level classes than students in the other two schools.

Because the teachers at Railside were deeply committed to equity and to heterogeneous teaching, they had worked together over the previous decade to develop and implement a curriculum that afforded multiple points of access to mathematics and comprised a variety of cognitively demanding tasks. The curriculum was organized around units that each had a unifying theme such as "What is a linear function?" The department placed a strong emphasis on problems that satisfied the criterion of being *groupworthy*. Groupworthy problems are those that "illustrate important mathematical concepts, allow for multiple representations, include tasks that draw effectively on the collective resources of a group, and have several possible solution paths" (Horn, 2005, p. 219). The Appendix to this chapter includes an example of a problem that the department deemed groupworthy.

An important feature of the Railside approach we studied that cannot be seen in the curriculum materials was the act of asking follow-up questions. For example, when students found the perimeter of a figure (see the Appendix) with side lengths represented algebraically, as $10x + 10$, the teacher asked a student in each group, "Where's the 10?," requiring that students relate the algebraic expression to the figure. Although the tasks provided a set of constraints and affordances (Greeno & Middle School Mathematics Through Applications Project, 1997), it was in the implementation of the tasks that the learning opportunities were realized (Stein, Smith, Henningsen, & Silver, 2000). The question of "Where's the 10?," for example, was not written on the students' worksheets, but was part of the curriculum, as teachers agreed upon the follow-up questions they would ask of students.

Research studies in recent years have pointed to the importance of school and district contexts in the support of teaching reforms (McLaughlin & Talbert, 2001; Siskin, 1994; Talbert & McLaughlin, 1996). Railside, however, is not a case of a district or school that initiated or mandated reforms. The reforms put in place by the mathematics department were supported by the school and were in line with other school reforms, but they were driven by the passion and commitment of the mathematics teachers in the department. The school, in many ways, was a demanding context for the reforms, not least because it had been managed by five different principals in 6 years and had been labeled an "'underperforming school" by the state because of low state test scores. The department, under the leadership of two strong and politically astute co-chairs, fought to maintain their practices at various times and worked hard to garner the support of the district and school. While the teachers felt well supported at the end of our study, Railside does not represent a case of a reforming district encouraging a department to engage in new practices. Rather, Railside is a case of an unusual, committed, and hardworking department that continues to grow in strength through its teacher collaborations and work.

II: Groupwork and "Complex Instruction"

Many teachers use groupwork, but groups do not always function well, with some students doing more of the work than others, and some students being excluded or choosing to opt out. At Railside the teachers employed strategies to make groupwork successful. They adopted an approach called *Complex Instruction*, designed by Cohen and Lotan (Cohen, 1994a; Cohen & Lotan, 1997) for use in all subject areas. The system is designed to counter social and academic status differences in classrooms, starting from the premise that status differences do not emerge because of particular students but because of group interactions. The approach includes a number of recommended practices that the school employed that we highlight below.

Multidimensional Classrooms. In many mathematics classrooms one practice is valued above all others—that of executing procedures correctly and quickly. The narrowness by which success is judged means that some students rise to the top of classes, gaining good grades and teacher praise, while others sink to the bottom. In addition, most students know where they are in the hierarchy created. Such classrooms are unidimensional—the dimensions along which success is presented are singular. In contrast, a central tenet of the Complex Instruction approach is what the authors refer to as *multiple ability treatment*. This treatment is based upon the idea that expectations of success and failure can be modified by the provision of a more open set of task requirements that value many different abilities. Teachers should explain to students that "no one student will be 'good on all these abilities' and that each student will be 'good on at least one'" (Cohen & Lotan, 1997, p. 78). Cohen and Lotan provide theoretical backing for their multiple ability treatment using the notion of multidimensionality (Rosenholtz & Wilson, 1980; Simpson, 1981).

At Railside, the teachers created multidimensional classes by valuing many dimensions of mathematical work. This was achieved, in part, by implementing open problems that students could solve in different ways. The teachers valued different methods and solution paths, and this enabled more students to contribute ideas and feel valued. When we interviewed the students and asked them, "What does it take to be successful in mathematics class?," they offered many different practices, such as asking good questions, rephrasing problems, explaining well, being logical, justifying work, considering answers, and using manipulatives. When we asked students in the traditional classes what they needed to do in order to be successful, they talked in much more narrow ways, usually saying that they needed to concentrate and pay careful attention. Railside students regarded mathematical success much more broadly than students in the traditional classes, and instead of viewing mathematics as a set of methods that they needed to observe and remember, they regarded mathematics as a way of working with many different dimensions.

The multidimensional nature of the classes at Railside was an extremely important part of the increased success of students. Put simply, when there are many ways to be successful, many more students are successful. Railside students were aware of the different practices that were valued, and they felt successful because they were able to excel at some of them. Given the current high-stakes testing climate, teachers may shy away from promoting the development of practices outside of procedure execution because they are not needed on state tests, but the fact that teachers at Railside valued a range of practices and more students could be successful in class appears to have made students feel more confident and positive about mathematics.[7] This may have enhanced their success in class and their persistence with high-level mathematics classes.

The following comments given by students in interviews provide a clear indication of the multidimensionality of classes:

> Back in middle school the only thing you worked on was your math skills. But here you work socially and you also try to learn to help people and get help. Like you improve on your social skills, math skills and logic skills. (Janet, Y1)

> *J*: With math you have to interact with everybody and talk to them and answer their questions. You can't be just like, "Oh, here's the book, look at the numbers and figure it out."
> *Int*: Why is that different for math?
> *J*: It's not just one way to do it. . . . It's more interpretive. It's not just one answer. There's more than one way to get it. And then it's like: "Why does it work"? (Jasmine, Y1)

It is not common for students to report that mathematics is more "interpretive" than other subjects. The students at Railside recognized that helping, interpreting, and justifying were critically valued practices in mathematics classes.

One of the practices that we found to be particularly important in the promotion of equity was justification. At Railside students were required to justify their answers at almost all times. There are many good reasons for this—justification is an intrinsically mathematical practice (Martino & Maher, 1999; RAND, 2002), but this practice also serves an interesting and particular role in the promotion of equity. The practice of justification made space for mathematical discussions that might not otherwise be afforded, particularly given the broad range of students' prior knowledge in the Railside mathematics classes. Students had both the right to receive a justification that satisfied them, and the obligation to provide a justification in response to another's question. Justifications then were adapted to the needs of individuals, and mathematics that might not otherwise be addressed was brought to the surface.

The following excerpt gives an indication of how two students viewed the role that justification played in helping different students:

> *Int*: What happens when someone says an answer?
> A: We'll ask how they got it.
> *L*: Yeah, because we do that a lot in class. . . . Some of the students—it'll be the students that don't do their work, that'd be the ones, they'll be the ones to ask step by step. But a lot of people would probably ask how to approach it. And then if they did something else they would show how they did it. And then you just have a little session! (Ana & Latisha, Y3)

The following boy was achieving at lower levels than other students, and it is interesting to hear him talk about the ways he was supported by the practices of explanation and justification:

> Most of them, they just like know what to do and everything. First you're like "why you put this?" and then like if I do my work and compare it to theirs. Theirs is like super different 'cause they know, like what to do. I will be like—let me copy, I will be like, "Why you did this?" And then I'd be like: "I don't get it why you got that." And then like, sometimes the answer's just like, they be like, "Yeah, he's right and you're wrong" But like—why? (Juan, Y2)

Juan also differentiated between high and low achievers without referring to such adjectives as "smart" or "fast," instead saying that some students "know what to do." He also made it very clear that he was helped by the practice of justification and that he felt comfortable pushing other students to go beyond answers and explain why their answers are given. At Railside the teachers prioritized the message that students had two important responsibilities—both to help someone who asked for help, but also to ask if they needed help. Both are important in the pursuit of equity, and justification emerged as an important practice in the students' learning.

The Importance of Student Roles. A large part of the success of the teaching at Railside came from the complex, interconnected system in each classroom, in which students were taught to take responsibility for one another and were encouraged to contribute equally to tasks. When in groups, students were given a particular role to play, such as *facilitator*, *team captain*, *recorder/reporter*, or *resource manager* (Cohen & Lotan, 1997). The premise behind this approach is that all students have important work to do in groups, without which the group cannot function. At Railside the teachers emphasized the different roles at frequent intervals, stopping, for example, at the start of class to remind facilitators to help people check answers or show their work. Students changed roles at the end of each unit. The teachers reinforced the status of the different roles and the important part they played in the mathematical work that was undertaken. These roles contributed to a classroom environment in which everyone had something important to do and all students learned to rely upon one another.

Assigning Competence. An interesting and subtle approach that is recommended within the Complex Instruction literature is that of assigning competence. This practice involves teachers raising the status of students who may be of a lower status in a group, for example, by praising something they have said or done that has intellectual value, and bringing it to the group's attention; asking a student to present an idea; or publicly praising a student's work in a whole-class setting. For example, during a classroom observation at Railside a quiet Eastern European boy muttered something in a group that was dominated by two outgoing Latina girls. The teacher who was visiting the table immediately picked up on what Ivan said, noting, "Good, Ivan, that is important." Later, when the girls offered a response to one of the teacher's questions, the teacher said, "Oh, that is like Ivan's idea; you're building on that." The teacher raised the status of Ivan's contribution, which would almost certainly have been lost without such an intervention. Ivan visibly straightened up and leaned forward as the teacher reminded the girls of his idea. Cohen (1994a) recommends that if student feedback is to address status issues, it must be public, intellectual, specific, and relevant to the group task (p. 132). The public dimension is important, as other students learn about the broad dimensions that are valued; the intellectual dimension ensures that the feedback is an aspect of mathematical work; and the specific dimension means that students know exactly what the teacher is praising. This practice is linked to the multidimensionality of the classroom, which values a broad range of work and forms of participation. The practice of assigning competence demonstrated the teachers' commitment to equity and to the principle of showing what different students could do in a multifaceted mathematical context.

Teaching Students to Be Responsible for One Another's Learning. A major part of the equitable results attained at Railside came from the serious way in which teachers taught students to be responsible for one another's learning. Groupwork, by its nature, brings an element of responsibility, but Railside teachers went beyond this to encourage the students to take the responsibility very seriously. In previous research on approaches that employ groupwork, students generally report that they prefer to work in groups and they list different benefits, but the advantages usually relate to their own learning (see Boaler, 2000, 2002a, 2009). At Railside, students' descriptions of the value of groupwork were distinctly reciprocal, as they also voiced a clear concern for the learning of their classmates. For example:

> *Int*: Do you prefer to work alone or in groups?
> A: I think it'd be in groups, 'cause I want, like, people that doesn't know how to understand it, I want to help them. And I want to, I want them to be good at it. And I want them to understand how to do the math that we do. (Amado, Y1)

Students talked about their enjoyment of helping others and the value in helping one another:

> It's good working in groups because everybody else in the group can learn with you, so if someone doesn't understand—like if I don't understand but the other person does understand they can explain it to me, or vice versa, and I think it's cool. (Latisha, Y3)

One unfortunate but common side effect of some classroom approaches is that students develop beliefs about the inferiority or superiority of different students. At Railside the students did not talk in these ways. This did not mean that they thought all students were the same, but they came to appreciate the diversity of classes and the different attributes that different students offered:

> Everybody in there is at a different level. But what makes the class good is that everybody's at different levels so everybody's constantly teaching each other and helping each other out. (Zane, Y2)

The students at Railside not only learned to value the contributions of others, they also developed a responsibility to help one another.

One way in which teachers nurtured a feeling of responsibility was through the assessment system. Teachers graded the work of a group by, for example, rating the quality of the conversations groups had. The teachers also gave both individual and group tests. A third way in which responsibility was encouraged was through a practice of asking one student in a group to answer a follow-up question after a group had worked on something. If the student could not answer, the teacher would leave the group and return to ask the same student again. In the intervening time it was the group's responsibility to help the student learn the mathematics he or she needed to answer the question. This move of asking one member of a group to give an answer and an explanation, without help from his or her group mates, was a subtle practice that had major implications for the classroom environment. In the following interview extract the students talk about this particular practice and the implications it holds:

> *Int*: Is learning math an individual or a social thing?
>
> *G*: It's like both, because if you get it, then you have to explain it to everyone else. And then sometimes you just might have a group problem and we all have to get it. So I guess both.
>
> *B*: I think both—because individually you have to know the stuff yourself so that you can help others in your groupwork and stuff like that. You have to know it so you can explain it to them. Because you never know which one of the four people she's going to pick. And it depends on that one person that she picks to get the right answer. (Gisella & Bianca, Y2)

These students made the explicit link between teachers asking any group member to answer a question and being responsible for their group members. They also communicated an interesting social orientation that became instantiated through the mathematics approach, saying that the purpose in knowing individually was not to be better than others but so "you can help others in your group." There was an important interplay between individual and group accountability in the Railside classrooms.

The four practices described—multidimensionality, group roles, assigning competence, and encouraging responsibility—are all part of the Complex Instruction approach. We now review three other practices in which the teachers engaged that are also critical to the promotion of equity. These relate to the challenge and expectations provided by the teachers.

III. Challenge and Expectations

High Cognitive Demand. The Railside teachers held high expectations for students and presented all students with a common, rigorous curriculum to support their learning. The cognitive demand that was expected of all students was higher than other schools', partly because the classes were heterogeneous and no students were precluded from meeting high-level content. Even when students arrived at school with weak content knowledge well below their grade level, they were placed into algebra classes and supported in learning the material and moving on to higher content. Teachers also enacted a high level of challenge in their interactions with groups and through their questioning, for instance, in the earlier example where students found the perimeter of a set of algebra lab tiles to be $10x + 10$ and the teachers asked students to explain where the +10 came from. Importantly, the support that teachers gave to students did not serve to reduce the cognitive demand of the work, even when students were showing signs of frustration. The reduction of cognitive demand is a common occurrence in mathematics classes when teachers help students (Stein et al., 2000). At Railside the teachers were highly effective in interacting with students in ways that supported their continued thinking and engagement with the core mathematics of the problems.

When we interviewed students and asked them what it took to be a good teacher, students demonstrated an appreciation of the high demand the teachers placed upon them, for example:

> She has a different way of doing things. I don't know, like she won't even really tell you how to do it. She'll be like, "Think of it this way." There's a lot of times when she's just like—"Well, think about it"—and then she'll walk off and that kills me. That really kills me. But it's cool. I mean it's like, it's alright, you know. I'll solve it myself. I'll get some help from somebody else. It's cool. (Ana, Y3)

The following students, in talking about the support teachers provided, also referred to their teachers' push for understanding:

> *Int*: What makes a good teacher?
>
> *J*: Patience. Because sometimes teachers they just zoom right through things. And other times they take the time to actually make sure you understand it, and make sure that you actually pay attention. Because there's some teachers out there who say: "You understand this?" and you'll be like, "Yes," but you really don't mean yes you mean no. And they'll be like, "OK." And they move on. And there's some teachers that be like—they *know* that you don't understand it. And they know that you're just saying yes so that you can move on. And so they actually take the time out to go over it again and make sure that you actually got it, that you actually understand this time. (John, Y2)

The students' appreciation of the teachers' demand was also demonstrated in our questionnaires. One of the questions started with the stem: "When I get stuck on a math problem, it is most helpful when my teacher . . . " This was followed by answers such as "tells me the answer," "leads me through the problem step by step," and "helps me without giving away the answer." On a four-point scale (SA, A, D, SD), almost half of the Railside students (47%) *strongly* agreed with the response: "Helps me *without* giving away the answer," compared with 27% of students in the traditional classes at the other two schools ($n = 450$, $t = -4.257$; $df = 221.418$; $p < 0.001$).

Effort Over Ability. In addition to challenging students through difficult questions that maintained a high cognitive demand, the teachers also gave frequent messages to students about the nature of high achievement in mathematics, continually emphasizing that it was a product of hard work and not of innate ability. The teachers reassured students that they could achieve anything if they put in the effort. This message was heard by students and communicated to us in interviews:

> To be successful in math you really have to just like, put your mind to it and keep on trying—because math is all about trying. It's kind of a hard subject because it involves many things. . . . but as long as you keep on trying and don't give up then you know that you can do it. (Sara, Y1)

In the Year 3 questionnaires we offered the statement "Anyone can be really good at math if they try." At Railside, 84% of the students agreed with this, compared with 52% of students in the traditional classes ($n = 473$, $t = -8.272$; $df = 451$; $p < 0.001$). But the Railside students did not only come to believe that they could be successful. They developed an important practice that supported them in that—the act of persistence. It could be argued that persistence is one of the most important practices to learn in school—one that is strongly tied to success in school as well as in work and life. We have many indications that the Railside students developed considerably more persistence than the other students. For example, as part of our assessment of students we gave them long, difficult problems to work on for 90 minutes in class, which we videotaped. The Railside students were more successful on these problems, partly because they would not give up on them and continued to try to find methods and approaches even when they had exhausted many. When we asked in questionnaires: "How long (in minutes) will you typically work on one math problem before giving up and deciding you can't do it?," the Railside students gave responses that averaged 19.4 minutes, compared with the 9.9 minutes averaged by students in traditional classes ($n = 438$, $t = -5.641$; $df = 142.110$; $p < 0.001$). This response is not unexpected, given that the Railside students worked on longer problems in classes, but it also gives some indication of the persistence students were learning through the longer problems they experienced.

In the following interview extract, the students link this persistence to the question-asking and justification highlighted earlier:

> *A*: Because I know if someone does something and I don't get it I'll ask questions. I'm not just going to keep going and not know how to do something.
>
> *L*: And then if somebody challenges what I do then I'll ask back and I'll try to solve it. And then I'll ask them: "Well how d'you do it?" (Ana & Latisha, Y3)

CONCLUSION

Railside is not a perfect place—the teachers would like to achieve more in terms of student achievement and the elimination of inequities, and they rarely feel satisfied with the achievements they have made to date, despite the vast amounts of time they spend planning and working. But research on urban schools (Haberman, 1991) and the experiences of mathematics students in particular tells us that the achievements at Railside are extremely unusual. There were many features of the approach at Railside that combined to produce important results. Not only did the students achieve at significantly higher levels, but the differences in attainment among students of different ethnic groups were reduced in all cases and disappeared in some.

We have attempted to convey the work of the teachers in bringing about the reduction in inequalities as well as general high achievement among students. In doing so we also hope to have given a sense of the complexity of the relational and equitable system that the teachers implemented. People who have heard about the achievements of Railside have asked for the curriculum so that they may use it, but while the curriculum plays a part in what is achieved at the school, it is only one part of a complex, interconnected system. At the heart of this system is the work of the teachers, and the numerous different equitable practices in which they engaged. The Railside students learned through their mathematical work that alternate and multidimensional solutions were important, which led them to value the contributions of the people offering such ideas. This was particularly important at Railside, as the classrooms were multicultural and multilingual. It is commonly believed that students will learn respect for different people and cultures if they have discussions about such issues or read diverse forms of literature in English or social studies classes. We propose that all subjects have something to contribute to the promotion of equity, and that mathematics, often regarded as the most abstract subject, removed from responsibilities of cultural or social awareness, has an important contribution to make. The discussions at Railside were often abstract mathematical discussions, and the students did not learn mathematics through special materials that were sensitive to issues of gender, culture, or class. But through their mathematical work, the Railside students learned to appreciate the different ways that students saw mathematics problems and learned to value the contribution of different methods, perspectives, representations, partial ideas, and even incorrect ideas as they worked to solve problems. As the classrooms became more multidimensional, students learned to appreciate and value the insights of a wider group of students from different cultures and circumstances.

The role of multidimensionality in the promotion of equity is not one that has reached the attention of many researchers in the United States. Multidimensionality is encouraged by open curriculum materials that allow students to work in different ways and bring different strengths to their work. The use of open materials in mixed-ability classrooms is something Boaler (2009) also found to promote equity in her study of English schools. Freedman, Delp, and Crawford (2005) also noted many aspects of a teacher's work that promoted equity and that are consistent with our findings, including learners being taught to be responsible for their own learning, a learning community that appreciates diverse contributions, opportunities for different ways of learning, and high challenges for all students. In Freedman et al.'s study they also found that equitable teaching did not rely on culturally sensitive materials, nor on the groupwork that the teachers in our study used, reminding us that there are many different routes to equity. In our study we found that mathematical materials and associated teaching practices that encouraged students to work in many different ways, supporting the contributions of all students, not only resulted in high and equitable attainment, but promoted respect and sensitivity among students.

The mathematical success shared by many students at Railside gave them access to mathematical careers, higher-level jobs, and more secure financial futures. The fact that the teachers were able to achieve this through a multidimensional, reform-oriented approach at a time in California when unidimensional mathematics work and narrow test performance were all that was valued (Becker & Jacob, 2000) may give other teachers hope that working for equity and mathematical understanding against the constraints of the system is both possible and worthwhile.

APPENDIX: GROUPWORTHY TASK

Figure 2.1. Graphic from Groupworthy Task

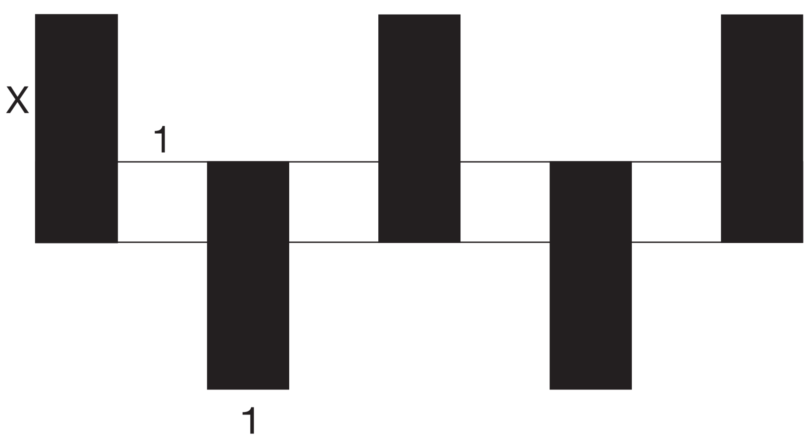

Figure 2.1. Graphic from Groupworthy Task (continued)

Explanation of figure:

There are two types of tiles used to create the above configuration. The dark tiles are x by 1 in dimension. The light square tiles are 1 by 1 in dimension.

Task prompt:

Build the arrangement of Lab Gear™ blocks (shown in the diagram given to students), and find the perimeter of the arrangement.

Result (which students derive in groups):

The perimeter is $10x + 10$.

Teacher follow-up question as she moves from group to group:

Where's the 10 in the $10x + 10$?

Students must discuss "where" the 10 is, and all students must be able to explain this to the teacher.