Chapter 1

Ratios and Proportional Relationships

The concepts of ratio and proportional reasoning are "big ideas" that permeate the middle school curriculum. The Common Core State Standards for Mathematics (CCSSM) consider these topics as one of five critical areas of the curriculum in both grade 6 and grade 7.

A ratio is a type of numerical comparison. Prior to middle school, students have been making *absolute* comparisons, using additive notions, to determine whether one quantity is more than, less than, or equal to, another. *Relative* comparisons, using multiplicative notions, allow quantities to be compared in a different way. For example, consider two different groups of students, each including both girls and boys. The first group has 20 girls, while the second group has 25. Clearly, the second group has more girls, using an absolute comparison. If we learn, however, that the first group has a total of 30 students while the second has a total of 150 students, we can make different types of comparisons for the number of girls, both to the total number of students in each group and to the number of boys in each group. Using a relative comparison, we see that the first group (20 girls out of 30 students) has more girls relative to the total number of students in it than the second (25 girls out of 150 students). If we consider a third group with 40 girls out of a total of 60 students, we see that the relative number of girls in the first group and the third is the same. The notions of *ratio* and *equal ratio* (or *proportion*) emerge naturally from such scenarios.

The type of thinking exemplified above is the conceptual underpinning for the big ideas of ratio and proportion, which are that ratio is a multiplicative comparison, and that proportional situations are based on multiplicative, and not additive, relationships (Van de Walle 1998). These notions are related to various ways of thinking about proportional concepts and their representations, including unit rate, equations and functions, graphing, and percent. As stated in *Principles and Standards for School Mathematics*, proportionality connects many of the mathematics topics studied in grades 6–8.

> Facility with proportionality involves much more than setting two ratios equal and solving for a missing term. It involves recognizing quantities that are related proportionally and using numbers, tables, graphs and equations to think about the quantities and their relationship. (NCTM 2000, p. 217)

It is essential to give students opportunities to develop a conceptual understanding of proportion so that solving problems involving proportion is not reduced to the procedure of cross multiplication only. It is worth noting that the Common Core State Standards require that students solve problems using proportional reasoning, not specifically using the cross product.

The CCSSM middle school standards include a Ratios and Proportional Relationships domain for grades 6 and 7 (though not one for grade 8). The single overarching standard in this domain for grade 6 is "Understand ratio concepts and use ratio reasoning to solve problems" (National Governors Association Center for Best Practices [NGA Center] and Council of Chief State School Officers [CCSSO] 2010, p. 42). These concepts include setting up ratios to represent real-life situations, unit rates, and using various representations to solve problems, including tables, graphs, and equations; percent concepts are also included in grade 6. The one overarching standard in the ratio and proportion domain for grade 7 is "Analyze proportional relationships and use them to solve real-world and mathematical problems" (NGA Center and CCSSO 2010, p. 48). In grade 7, students compute unit rates in more complex situations and examine proportional relationships in various situations, including the constant of proportionality and equations. More complex ratio and percent problems are also part of the CCSSM standards for grade 7.

Although ratio and proportion are not topics with separate domains in the standards for grade 8 and high school, proportional reasoning is present when working with similar figures, notions of trigonometry, and probabilistic concepts. The standards for grades 6 and 7 are structured so that students are ready for these more difficult concepts in grade 8 and high school.

What follows are six tasks to support ratio reasoning (tasks 1.1 and 1.2), percent (tasks 1.3 and 1.4), and proportion (tasks 1.5 and 1.6). The eight Standards for Mathematical Practice (MP), as listed on page vi, are woven throughout these domains. Depending on the problem, a subset of those standards is discussed. The problems are loosely grouped by grade level, although the extensions of some problems might allow students to meet a higher grade's standards. As in other chapters, we believe that all of these problems require "attention to precision," thus developing mathematically proficient students as required by the sixth Standard for Mathematical Practice. The tasks in this chapter are summarized in table 1.1.

Table 1.1
Content areas, grade levels, and standards met by the tasks in chapter 1

Content Areas	**Tasks**	**Grade 6 Standards**	**Grade 7 Standards**	**Grade 8 Standards**	**Standards for Mathematical Practice**
Ratio reasoning	1.1	6.RP.1, 6.RP.3		8.F.4*	MP.1, MP.2
Ratio reasoning	1.2	6.RP.1			MP.1, MP.2
Percent	1.3		7.RP.3	8.F.4*	MP.3, MP.4
Percent	1.4		7.RP.3		MP.3, MP.4
Proportion	1.5	6.RP.2, 6.RP.3a	7.RP.2a, 2b, 2c, 2d	8.F.2, 8.F.4*	MP.1, MP.4
Proportion	1.6		7.RP.1, 7.RP.2, 7.RP.3		MP.1, MP.4

*Extensions of the problem

Ratio Concepts

Grade 6

The first task in this section begins to lay the groundwork for ratio concepts and proportional thinking by giving students the opportunity to see the need for relative comparisons and how they are different from absolute (or additive) comparisons. Task 1.1 lends itself well to a problem-solving approach. Students need to understand the problem, and particularly what is different about its parts (*a*) and (*b*), in order to make sense of what is being asked in later parts of the problem, and to be able to devise a plan to approach the problem. In devising such a plan, students might, for example, draw a picture to visually compare the number of goals made and attempted in each part of the problem.

This task addresses the standard 6.RP.1, “Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities,” and 6.RP.3, “Use ratio and rate reasoning to solve real-world and mathematical problems” (NGA Center and CCSSO 2010, p. 42).

Task 1.1

Anna and Gerard both play basketball. In a recent game, Anna took 10 shots at the basket and made 3 of them, while Gerard took 20 shots and made 5 of them. (In this context, “making” a shot means the ball goes into the basket.)

(*a*) Who made more baskets in this game?

(*b*) Who performed better in this game?

(*c*) What is different about questions (*a*) and (*b*)? Why is “performing better” different from making more baskets? What arithmetic operations did you use to answer questions (*a*) and (*b*)?

(*d*) In a later game, Anna attempted 14 shots and Gerard attempted 20. Write your own two scenarios using those numbers:

- One in which Anna and Gerard made different numbers of baskets, but performed equally well
- One in which Anna made fewer baskets than Gerard, but Anna performed better

Parts (*a*) and (*b*) of task 1.1 each ask students a question that, on a first reading, might be misinterpreted to mean the same thing. The goal of these questions is to juxtapose the idea of absolute and relative comparison so that students can make a distinction between them. Teachers should introduce the language of ratio as a type of comparison when discussing this problem. In the second scenario of part (*d*), the idea of equal ratios is introduced.

Several changes and extensions may be made to this problem. Certainly, different contexts may be used. Continuing with the sports theme, students may consider turns at bat in baseball or the number of shots in a hockey game. The discussion of the number of boys and girls in a classroom may be used (as in the introduction earlier in this chapter). Teachers may consider different contexts as appropriate given the interests of their students. Students may find the idea of a test more obvious, since it might be easier to see that answering more questions correctly does not necessarily mean that one's test score will be higher if the number of questions is different.

Extensions to the mathematical aspects of the problem may be made as well. Students may be asked to find the percent of shots made in parts (*b*) and (*d*). Students may also be asked to answer further questions involving proportional reasoning. For example, the following question could be asked in part (*d*): *Michael attempted 12 shots but performed equally well as Anna and Gerard. How many baskets did he make?* A more open-ended extension to the same part of the problem might ask the following: *Michael performed as well as Anna and Gerard. Write two different possibilities for the number of shots he attempted and made.* Alternatively, the question could ask for a possibility of the number of shots attempted and made if he performed worse or better. In order to bring percent, as in standard 6.RP.3c (NGA Center and CCSSO 2010, p. 42) into this problem, students could be asked to find the percent of baskets made for each of the players. Finally, students could be given a particular unit rate for "shots made" as compared to attempted, and be asked to make tables of equivalent ratios. Students might be asked to plot points representing the equivalent ratios, considering one of the values as a function of the other, thus making this problem meet the grade 8 standard 8.F.4, "Construct a function to model a linear relationship between two quantities" (NGA Center and CCSSO 2010, p. 55). Predictions could then be made, such as "If Anna attempted 40 shots, predict how many she made," or other similar questions.

The next activity gives students the opportunity to formalize their use of the ideas and language of ratio. Although the task is conceptually simpler than task 1.1, the two tasks are ordered this way because task 1.1 creates a need for relative comparison first, while task 1.2 gives students the opportunity to respond to an open-ended question about ratio. This allows for informal conceptual development first, followed by formalization using proper mathematical language. This problem is open-ended and gives students the opportunity to practice writing their own ratios given a scenario involving books. The suggested extensions of the problem allow for a deeper discussion of the meaning of "smaller" and "larger" ratios. Task 1.2 is targeted for students to meet standard 6.RP.1, "Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities" (NGA Center and CCSSO 2010, p. 42).

Task 1.2

Tariq has several types of items on his J-Pad. He has 30 applications ("apps"), some songs, and 18 books. The books include 10 mysteries and 8 books focused on sports. He is not sure how many songs there are on the device.

(*a*) What is the ratio of the number of apps to the number of books?

(*b*) What is the ratio of the number of books to the number of apps?

(*c*) Write some other ratios that compare the number of items of one type to the number of items of another type in words and in symbols.

(*d*) Tariq realizes that the ratio of the number of mysteries to the number of apps is the same as the ratio of the number of books to the number of songs. How many songs must he have?

Students might be resistant to writing ratios involving the number of songs in part (*c*) because they need to use a variable to represent this number. They should, however, be encouraged to write ratios using variables. Therefore, teachers may want to require in part (*c*) that at least one ratio they write include the number of songs. This problem helps students develop flexibility in their thinking, as they need to make the decision about which comparisons to make and the order in which to make them.

The four parts of the task could be extended in a variety of ways. One possible extension of this problem would be to ask what percent of the total number of items is of a particular type. Students would need to have a specific value for the number of songs in order to do this, so this might be done following part (*d*).

Part (*d*) as written only has one answer, but teachers may also extend this part of the problem by introducing the concept of "larger" or "smaller" ratios. This not only makes the question open-ended, but it also introduces the difficult notion of comparing ratios that are not necessarily equal. A possible modification of part (*d*) is "Tariq knows that the ratio of the number of mysteries to the number of apps is smaller than the ratio of the number of books to the number of songs. What is a possible number of songs that he has?" Students must then realize that the ratio of the number of mysteries to the number of apps is $^{10}/_{30}$, or equivalently $^{1}/_{3}$, and determine a number of songs so that the ratio $18/s$ (where s is the number of songs) is less than $^{1}/_{3}$. Any value of s that is greater than 54 will meet the requirement. Students may also be asked what the smallest value for s would be that would meet the requirement. It may seem counterintuitive that having a larger number of books makes the ratio smaller, but this is a good opportunity to remind students about the relationships among fractions. Further, students should see that only integral values for s make sense for this problem.

Similarly, the questions may be modified by saying that the ratio of the number of mysteries to apps is larger than the ratio of books to songs. In this case the ratio $18/s$ would need to be greater than $^{1}/_{3}$, so any integral value for h that is less than 54 and greater than zero would be a solution. It would also be worthwhile for teachers to ask students why zero could not be a value for s.

DISCUSSION—*Tasks 1.1 and 1.2*

Tasks 1.1 and 1.2 require the use of various problem-solving strategies, thus allowing students to meet several of the CCSSM standards through the approach of making sure they understand what is being asked in the problem, making a plan for how to solve it, applying a strategy, and checking whether their answer makes sense. An appropriate strategy for task 1.1 might be to "draw a picture" or "use a model." Students may need to represent the comparison of shots made to shots attempted by visually representing the values, either on a number line or graph paper, or by using a discrete or concrete representation, such as dots or circles, or centimeter cubes, to represent the quantities. Such scale representations may help students more clearly see the meaning of relative comparison. An additional strategy that might be successful for task 1.2 would be to "use direct reasoning," working directly from the given information. Students may also benefit from using a model for this problem, such as drawing a picture or making a model using concrete or semi-concrete materials. For part (*d*) of task 1.2 and its extension, a productive approach might be "guess and check" or "work backwards," choosing values that make the ratio smaller (or larger) and determining how this fits the problem. For both problems, students should check their work and determine whether their solutions make sense. For example, if students write an inequality, such as $x > 54$, for their solution to the extension of the problem that asks for a smaller ratio, they need to examine whether non-integral values make sense in this problem and adjust the representation of their answer. Several of the CCSSM Standards for Mathematical Practice are targeted by these problems.

MP.1

Tasks 1.1 and 1.2 both require that students "make sense of problems and persevere in solving them" (NGA Center and CCSSO 2010, p. 6). Parts (*a*) and (*b*) of task 1.1 juxtapose two types of comparison, absolute and relative, that may not initially make sense to students. Students must think through and understand what the problem is asking them to do. In task 1.2, students must decide how to represent the unknown value of books using a variable, and then determine how to create equal, larger, or smaller ratios by examining different values of the variable.

MP.2

In developing the ideas of proportional reasoning through each of the above tasks, students "reason abstractly and quantitatively" (NGA Center and CCSSO 2010, p. 6). Students must make sense of the different types of reasoning used when making absolute comparisons, simply saying that one quantity is more (or less) than another, and making relative comparisons, saying that one quantity is more (or less) *relative* to another. Further, the extension of task 1.2 requires that students compare the actual ratios, determining what it means for a ratio to be smaller or larger than another ratio.

Percent

Grade 7

The tasks that follow target misconceptions that students often have about percent. Students often erroneously try to "operate" on percent values as though they were absolute quantities, and they forget that the percent is a relative quantity. Task 1.3 presents students with a context that allows them to consider whether this makes sense. Although it might seem reasonable to students initially, once they see that they eventually obtain a percent greater than 100 they should conclude that something is "wrong" and revisit their approach to the problem. Students should be given part (*a*) first, then after some thought and discussion they can be given part (*b*).

Task 1.3

Bright-O toothpaste advertises that 30 percent of tooth stains are removed after 1 week of use. As Gabriella was brushing her teeth, she thought, "Well, if I use this toothpaste for 2 weeks, then 60 percent of the stains will be gone, and if I use it for 3 weeks, then 90 percent of the stains will be gone."

(*a*) According to Gabriella's reasoning, what percent of stains will be gone after 4 weeks? Does this make sense? Why or why not?

(*b*) Now that you have seen the flaw in Gabriella's reasoning, what is a reasonable way to compute the percent of stains that have been removed after 2 weeks? 3 weeks? 5 weeks?

Although this type of problem might seem quite sophisticated for students in the middle grades, the concepts targeted are critical to students' understanding of percent. Of course, according to Gabriella's reasoning, 120 percent of the stains will be removed after 4 weeks, which is not sensible, and indicates fundamental misconceptions about percent. Teachers might first want to discuss with students what an advertiser might even mean by assigning a percent to such a statement, and how "stains" might be quantified and measured. As an aside, teachers might have a short discussion about misleading advertisements.

To initiate a discussion of the problem, teachers may want to assign a value to quantify the beginning "measure" of stains—say, 100. Of course, any value may be used, but using 100 helps illuminate the fact that if 30 percent of the stains are removed, then 70 percent remain. If we consider the number of stains remaining after one week, we can see that if we begin with 100, we subtract 30 percent of 100, that is $100 - 0.3(100) = 70$. At the start of the second week, the beginning value has changed, and is now 70. So, after two weeks, we subtract 30 percent of 70 from 70. That is, $70 - 0.3(70) = 49$. The same reasoning follows: At the beginning of the third week we begin with 49, and the logic continues. Students may notice that if they must subtract what is being removed, it makes more sense to consider what remains. They may also notice that the value at the end of the first week is 0.7(100), at the end of the second week is $0.7 \cdot 0.7(100)$, and at the end of the third week is $0.7 \cdot 0.7 \cdot 0.7(100)$, leading to the use of exponents.

The observation that we can work with the stains that remain can give students the opportunity to work with functions. This might be a bit sophisticated for most seventh graders, but can lend itself to informal work with exponential functions of the form $y = ab^x$, where a is the initial value, $b = 0.7$, and x is the number of weeks that the toothpaste has been used.

Other extensions of this problem would be to ask such questions as "How many weeks does it take to remove at least 75 percent of the stains?" The given problem may also be changed so that, instead of the advertisement stating that 30 percent of stains are removed, different rates of removal are used for different effects.

The next task also targets misconceptions about percent. Students do not always make the connection that for percents to be equal, they must be percents of the same quantity. The context of task 1.4, a sale at a store, is one with which many students are familiar. The scenario, with a "percent-off" sale, then an additional percent off of the sale price, is common in many stores.

Task 1.4

Kara is shopping at her favorite store, which is holding a sale that offers 20 percent off the price of every item in the store. She has a coupon that gives her an additional 50 percent off the sale price. She thinks this is great, because now she will get 70 percent off on the items she buys.

(*a*) Is Kara correct? Explain.

(*b*) If Kara wants to buy an item that originally costs $60, how much will the item cost given both discounts?

(*c*) If the tax rate is 7.5 percent, what will be the final cost of the item in part (*b*)?

(*d*) Which is the better deal: 20 percent off with an additional 50 percent off the sale price, or 50 percent off with an additional 20 percent off the sale price?

Part (*a*) targets a common misconception, which is that the percents can simply be added. Of course, this is not the case since the second percent is taken off of a smaller number. Teachers may decide to have students compute the sale price by first subtracting 20 percent of the original cost, then subtracting 50 percent of the sale price to determine final cost, and comparing this answer to subtracting 70 percent of the cost. Students will see that this is not the same result. Class discussion can help clarify that the discount is smaller because the percent is taken off of the sale price, which is lower than the original cost. This problem also opens up the opportunity to discuss the rather perplexing convention of computing a "percent off" and subtracting it from the original cost, instead of the simpler idea of computing what percent will be paid. That is, for an item whose cost is represented by C, it makes more sense to determine the cost at 20 percent off by considering that 80 percent of the price will be paid, and multiplying $0.8 \cdot C$, rather than by computing $C - 0.2 \cdot C$. Teachers may wish to discuss this with their students.

Part (*d*) brings up the interesting question of order, and students may be surprised to see that the order in which the discounts are taken does not matter. If the cost of an item is represented by C, we see that what we pay, which equals $C \cdot 0.8 \cdot 0.5$, is the same as $C \cdot 0.5 \cdot 0.8$ due to the commutativity of multiplication of real numbers. Further, when computing tax along with a discount, it does not matter whether the tax is computed first, then the discount taken (as long as the discount is also taken on the tax) or whether the discount is taken first, then the tax computed (as long as the tax is only computed on the discounted amount).

Finally, this task sets the stage for asking the somewhat bigger and more general question "Which is a bigger discount, 80 percent or 10 percent?" Of course, many students instinctively might answer that 80 percent is bigger, but the more important question is "Of what?" If the percents are of the same amount, then 80 percent of it is certainly bigger, but 80 percent of 5 is not bigger than 10 percent of 1000. This reconnects to the idea of relative comparison as opposed to absolute comparison as discussed earlier in this chapter.

DISCUSSION—*Tasks 1.3 and 1.4*

Employing problem-solving strategies will aid in students' success in solving the problems presented in tasks 1.3 and 1.4. In understanding what is being asked in each of the problems, students must understand that problems involving percent do not make sense until the answer to the question "Of what?" is determined. Students might make a chart or organize a list to answer each of the questions. When checking their answers, students need to determine whether their answers make sense in the context of the original problem.

MP.3

Tasks 1.3 and 1.4 both present students with the opportunity to examine and critique the approach of a fictional student, and to determine that the approach is flawed in some way. Students must then determine what the correct approach to each of the problems is and explain their answers, thus "construct[ing] viable arguments and critiqu[ing] the reasoning of others" (NGA Center and CCSSO 2010, p. 6). Specifically, task 1.3 asks students to determine the flaw in Gabriella's thinking when she multiplies a percent by an integer, not realizing that the percent is of a different amount. Task 1.4 asks students to consider that percents cannot simply be added when they are percents of different amounts.

MP.4

The real-life situations presented in tasks 1.3 and 1.4 require that students "model with mathematics"; specifically, they must "identify important quantities in a practical situation" and "interpret their mathematical results in the context of the situation and reflect on whether the results make sense" (NGA Center and CCSSO 2010, p. 7). Task 1.3 presents students with an advertising context that they are likely to see in their everyday life, but do not usually have the opportunity to analyze mathematically. Similarly, task 1.4 gives students the opportunity to analyze a situation involving a sale with which they might be familiar.

Proportion

Grades 6 and 7

Tasks 1.5 and 1.6 present students with real-life contexts in which they must use proportional reasoning to solve problems. The tasks synthesize several ideas, including percent, making tables, and plotting points, as well as examining unit rate and slope in the context of functions, if teachers wish to integrate functions into this topic. Task 1.5 gives students a price for 60 chocolate candies and a different monetary contribution from five different students, and it asks students to divide the chocolate in proportion to the amount paid by each student. In part (*a*) of the task, students are asked to determine how many chocolates each person should get in an unstructured way. Students may be encouraged to brainstorm in pairs, and they may decide to determine the unit rate per chocolate, or to make a table to compare the different values.

Later parts of the problem provide more structure for different approaches, and because of this, teachers might want to give part (*a*) first, and then, after eliciting different approaches, have students do the later parts of the problem. Task 1.5 addresses the grade 6 standards 6.RP.2, "Understand the concept of a unit rate," and 6.RP.3a, "Make tables of equivalent ratios relating quantities with whole-number measurements" (NGA Center and CCSSO 2010, p. 42). Grade 7 standard 7.RP.2, "Recognize and represent proportional relationships between quantities" (NGA Center and CCSSO 2010, p. 48), is also met.

Task 1.5

Five friends contribute money to buy 60 chocolate candies. In total, the chocolate candies cost $15. The friends contribute as follows, and they want to divide the chocolate candies fairly, based on what was paid:

Carmen	\$3
Luwen	\$1
Julia	\$4
Otis	\$5
Quinn	\$2

(*a*) How many chocolate candies should each friend get, based on what was paid? Explain how you approached this problem.

(*b*) Make a table that compares the amount paid by each friend to the number of chocolate candies they should receive. Plot the values in your table on a coordinate plane. What do you notice about all of the points you plotted?

(*c*) What is the unit rate for the cost of chocolate candies?

(*d*) A different friend, Rachel, went to a different store, and purchased 10 chocolate candies for \$2. Plot the coordinates associated with these values on the same graph as part (*b*). What do you notice about the point you just plotted in relation to the line? What is different about the price Rachel paid?

Teachers may elect to take several different approaches with students to this problem. The intention of the problem as written is to give students part (*a*) first, allowing them to think about it alone, and then brainstorm in pairs, or as a whole class. If students get stuck, the teacher might ask such questions as, "What fraction of the total payment was made by Carmen?" and "What fraction of the chocolate candies should Carmen get?" Students might decide to make a table or to determine the unit rate for the cost of chocolate candies.

Part (*b*) asks students to make a table (they might have already done this) and to plot the points on the coordinate plane. The problem does not, however, tell students which variable should be dependent and which independent. Therefore, students may end up with a line whose slope is 4, and the unit rate is "four chocolate candies for one dollar" or a line whose slope is $^1/_4$, and the unit rate is "one chocolate costs $^1/_4$ dollar." This idea is worth discussing with students. Students can also be asked to examine the point whose x-coordinate is 1, and to connect its y-coordinate to the unit rate. Teachers may explicitly connect slope to the unit rate to make a more direct connection to functions.

Part (*d*) requires that students plot a point with a different unit rate. Students should notice that the point does not lie on the line that they graphed in part (*b*), and teachers may ask questions to help students conclude that the unit rate for the chocolate candies in part (*d*) is different, helping them to see that different unit rates lead to different lines. As a further extension, teachers may ask students to construct a table of values based on the unit rate in part (*d*) and have them examine the different slopes and their meanings in context. Teachers may also have students write equations for either or both of the unit prices of chocolate candies.

Task 1.6 includes many of the same concepts as task 1.5, but it uses fractions, rendering the problem a bit more difficult. The problem has several parts, targeting different standards, so teachers may elect to parse out the parts of the problem over several days. This problem meets many of the same standards as task 1.5, with the addition of 7.RP.1, "Compute unit rates associated with ratios of fractions," and 7.RP.3, "Use proportional relationships to solve multistep ratio and percent problems" (NGA Center and CCSSO 2010, p. 48).

Task 1.6

A cupcake recipe with the following ingredients yields 24 cupcakes:

$4\,^1/_2$ cups flour
$^3/_4$ cup sugar
1 tablespoon baking soda
2 eggs

(*a*) Owen only needs 12 cupcakes. How much of each ingredient will he need?

(*b*) To cut down on sugar, Owen's mother prefers that he only use recipes in which the volume of sugar is not more than 25 percent of the dry ingredients (not including baking soda, since the amount is so small). Would Owen's mother find this recipe acceptable?

(*c*) Use the table to determine several different equivalent ratios of flour to sugar:

Flour	$4\,^1/_2$					
Sugar	$^3/_4$					

(*d*) Plot the points in part (*c*) on the coordinate plane.

(*e*) What is the ratio of flour to sugar in lowest terms? Call this value *r*.

(*f*) Plot the point (1, *r*) using the value for *r* you determined in part (*e*). What do you notice about this point in relation to the other points from part (*c*)?

(*g*) Write an equation for the proportional relationship between flour and sugar in this recipe. Use *F* to represent the number of cups of flour, and *S* to represent the number of cups of sugar.

(*h*) Eileen has a different cupcake recipe that calls for $^1/_2$ cup sugar and $2\,^1/_2$ cups of flour. She says that this recipe has a smaller ratio of sugar in relation to flour, and it is less sweet. Is Eileen correct?

The main point of this problem is to have students examine proportions that involve fractions, and to determine whether or not the ratio of sugar to flour is more or less than $^1/_4$, or 25 percent. The constant of proportionality can be interpreted to be 6 or $^{1}/_{6}$, depending on which variable students decide to plot on the horizontal and vertical axes. Part (*h*) asks students to compare ratios to determine which is smaller. Teachers may choose to extend this problem by asking students to create a table for Eileen's recipe and to graph the coordinate pairs on the same set of axes as the graph made in part (*d*).

DISCUSSION—*Tasks 1.5 and 1.6*

Several different problem-solving approaches are appropriate for solving the problems in tasks 1.5 and 1.6. Students may wish to use several methods when devising their plan to solve each problem. In part (*a*) of task 1.5, the question is intentionally nonprescriptive, so students may decide to make a table of values or find a unit rate, or examine different representations of the ratio in order to determine their solution. When working on task 1.6, students might revert to an easier problem (perhaps one not involving fractions) so that they can better understand and solve the problem. Of course, students must check their solutions in the context of the original problem so that they can determine whether or not their answers are reasonable.

MP.1

When approaching tasks 1.5 and 1.6, students must make sense of each of the problems and the constraints and relationships in the given information. In task 1.5, they must be able to determine what it means to have a ratio of chocolate candies that is equal to the ratio of money they spent. In task 1.6, they must understand what it means to have a ratio of ingredients smaller than 25 percent. In both problems, students must use different analyses to determine the answers to a variety of questions about the quantities in the recipe.

MP.4

The realistic contexts in each of the tasks require that students model with mathematics. In both cases they are making tables and examining a variety of ways of representing the important quantities with which they are presented. They also bring in additional constraints and examine the meaning of the graph, table, and, if necessary, slope in the context of the problems.