# INTRODUCTION

We have all had the unsettling experience of discovering flaws in our carefully planned approaches to teaching particular topics in mathematics. One of this book's authors, Jeff Barrett, offers a personal account of one such experience related to teaching measurement:

> *Early in September I was co-teaching a series of lessons with an experienced classroom teacher (Barrett et al. 2003). We had taught many measurement lessons together the previous year with another group of second graders. Now, as we addressed a new group of students, we came to our first day of learning measurement. We pulled out our rulers and some classroom objects to measure. Before we started, I reminded students of five important steps in measurement, which I recorded on the board:*
>
> 1. *Lay your ruler down in front of you with the inches marked on the top.*
> 2. *Place the object to be measured along the top of the ruler.*
> 3. *Align the object to be measured with the left end of the ruler.*
> 4. *Read the number that is under the right end of the object from the ruler.*
> 5. *Remember to include your units!*
>
> *After we reviewed the steps, I encouraged students to measure the length of several preselected objects—a pen, a chalkboard*

*eraser, a box of chalk, and a tape dispenser—and to record their measurements in a simple chart.*

*Our students quickly got to work aligning objects with rulers and reporting measures. As we circled the room, we had to remind some students of step 5 by asking "What is your unit?" Aside from needing these few reminders, students successfully completed the measuring tasks, and I was satisfied that they had met the goals of our lesson. That is, they could correctly measure the lengths of objects. I felt confident as I reflected on the way students handled those tasks.*

*Soon after that, while reading* Principles and Standards for School Mathematics *(National Council of Teachers of Mathematics [NCTM] 2000), I came across a classroom episode: "The teacher observed that Mari's hand slipped as she was aligning her ruler with the pencil. Mari made no comment but recorded this measurement as 12 inches also" (p. 106). I stopped to think about my own experiences with those second graders. Surely they would not have made the mistake of disregarding step 3—"Align the object to be measured with the left end of the ruler"—right? Mari reported, "The book is longer, but they are both 12 inches" (p. 106). Why was she willing to say that the book was longer than the pencil but that they had the same measure? How would other second graders react in this situation? What does Mari's analysis mean about what she understands, or doesn't understand, about linear measurement? What did my students really learn when I taught them to follow those five measurement steps?*

*Later in my career, I set out to try to get answers to some of my questions. Rather than wait for the right measurement situation to arise naturally, I posed some related tasks to make it happen. I created a worksheet with images of measuring tools paired with line segments to be measured (fig. 0.1). Drawing the line segments directly on the page prevented the students from aligning them with the left end of the ruler (the essential third step in measurement).*


**Fig. 0.1. Tasks 2–4 on this worksheet were designed to prevent students from aligning the left end of the ruler with the object to be measured.**

Before reading ahead, take some time to think about how your students might respond to the tasks in figure 0.1. We have observed that this kind of exercise often reveals a variety of thinking strategies. Table 0.1 shows some measurement approaches that students used to complete the tasks. For a more complete discussion of this task and other related tasks, see chapter 3.

**Table 0.1.** Second graders' strategies for completing measurement tasks

| Strategy | Likely responses | | | |
|---|---|---|---|---|
| | Task 1 | Task 2 | Task 3 | Task 4 |
| Right-hand endpoint | 5 inches | 12 inches | 4 inches | 6 inches<br>6.5 inches<br>7 inches |
| Left-hand endpoint | 0 inches* | 8 inches | 1 inch* | 4 inches |
| Counting tick marks | 5 inches<br>6 inches | 5 inches | 4 inches | 3 inches<br>4 inches |
| Correct (counting intervals) | 5 inches | 4 inches | 3 inches | 2.5 inches** |

* These responses are included in the table for completeness, but we do not anticipate that students will use this strategy for this task.

** We may also consider 2 or 3 inches to be correct, depending on the student's ability to deal with partial units.

## Standards for Teaching and Learning Measurement

The primary purpose of this book is to improve the teaching and learning of measurement in kindergarten through grade 5. We draw on two main sources for measurement topics in this grade band: the measurement content standards from *Principles and Standards for School Mathematics* (NCTM 2000) and the Common Core State Standards for Mathematics (CCSSM; National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA Center and CCSSO] 2010). These publications not only provide specific content standards but also detail the ways in which we engage students in learning. These are described as Process Standards by NCTM and as Mathematical Practices in CCSSM. We recognize that coordinating these demands from two sources is challenging; but it is also worthwhile, especially in preparation for the high-stakes tests that will be used to assess and document student learning.

## Learning Trajectories: A Framework for Measurement Activities

We hope to help teachers improve student learning by offering activities that help reveal student thinking. Because activities in isolation are of limited value, we also provide frameworks called *learning trajectories* (Barrett, Clements, and Sarama 2017) to support teaching and learning with these tasks. Each learning trajectory (LT) consists of three main parts: (1) a description of successively more complex strategies, (2) researchers' interpretations of students' thinking and reasoning, and (3) a collection of tasks designed as both formative assessments and catalysts for students' growth as they move from using less complex strategies to more complex strategies. In this book, we discuss three different but related LTs for measurement: length, area, and volume.

## Applying Standards and Learning Trajectories in the Classroom

In the following example, we show how the length LT can be used to facilitate interpreting student thinking, making instructional decisions, and preparing classroom activities. Similar examples using the area and volume LTs appear in subsequent chapters. Here, we address a measurement and data standard for grade 1 from CCSSM:

> Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is

> the number of same-size length units that span it with no gaps or overlaps. (1.MD.A.2)

Below is a possible assessment task related to the standard:

> Give students three 1-inch strips of paper and a 5-inch object to be measured. Ask the students to use the 1-inch strips to measure the length of the object.

We anticipate that although there will be some variety in your students' responses, most will fit into two or three categories. We expect that some grade 1 students—let's call them group A—will say that they cannot complete the measurement task because they do not have enough 1-inch strips to span the length of the object. Others—we'll call them group B—may take the three 1-inch strips and space them out so that they start and end at the left and right ends of the object (fig. 0.2) and report a measure of "3" or "3 inches." Still other students—group C—are likely to take the three 1-inch strips, lay them end to end starting from the left end, and then move two of the strips to the end of the collection (fig. 0.3) and report an answer of 5 inches.

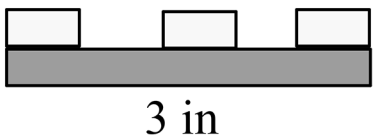


**Fig. 0.2. An incorrect measurement technique: spacing out the strips provided**

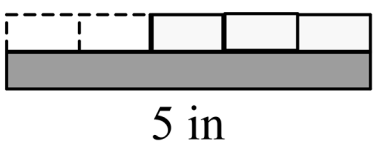


**Fig. 0.3. A correct measurement technique: iterating strips**

At this point, teachers will have to decide how to move forward in the lesson. How should we make this decision? Should all students be treated the same? Do we need to treat every student individually? An LT can be helpful in answering these questions. See table 0.2 for a brief excerpt from the length LT that can help us analyze the three hypothetical student responses from the task above. A summary of all levels for the length, area, and volume LTs is available at http://www.childrensmeasurement.org/learning-trajectories.html.

**Table 0.2.** Excerpt from the length learning trajectory (Barrett, Clements, and Sarama 2017)

| LT level | Thinking and strategies | Instructional tasks to motivate growth to next LT level |
|---|---|---|
| ***End-to-End Length Measurer***<br>Lays units end to end to create measures for comparison.<br>Can produce a meaningful measure when given all the objects but may struggle to do so when given fewer objects than needed to span the object to be measured. | Develops an initial understanding that an object can be measured by repetitions of shorter-length units.<br>Begins to understand the need to use equal-size units and to avoid gaps between units.<br>When provided with fewer unit pieces than needed to span a length, may assign a numerical value to the length by moving the pieces along the length while counting. | Provide more than enough unit pieces than needed to span the length of an object. Pause the activity before students align the object with all the unit pieces needed. Ask them to predict how many units would be needed to span the object. Allow them to check, if needed.<br>Use relatively large objects as units to create a measuring tool (e.g., pen lengths).<br>Challenge students by providing fewer than the total number of unit objects needed to lay end to end along the object to be measured. |
| ***Length Unit Relater and Repeater***<br>Iterates a single unit to measure.<br>Relates size and number of units, at least intuitively: "If you measure with centimeters, not inches, you'll need more because each one is smaller."<br>Can add two lengths to obtain the length of the whole. | Can maintain an image of each placement while moving the physical unit to the next iterative position and counting it.<br>Begins to anticipate that fewer numbers of larger units would be needed to span an object.<br>Iterates a single unit, sometimes with mistakes, to find how many would stretch end to end. | Lay a ribbon above a long measuring tool without aligning the ribbon with either endpoint. Cover a portion of the long measuring tool with dark construction paper or fabric. Ask students to find the length of the ribbon.<br>Ask students to draw and measure sequences of segments that are decreasing by length using only one unit object or a ruler. |
| ***Consistent Length Measurer***<br>Measures straight paths consistently. Aware of the need to use identical units when measuring, relationship between different units, partitions of unit, zero point on rulers, and accumulation of distance. Can resolve broken (or covered) ruler tasks effectively.<br>Iterates a single length unit without gaps or overlaps along a straight path. | Able to use rulers well because he or she sees the ruler as a collection of iterated units.<br>Sees that a length is a ratio comparison between the unit object and the measured object and begins to work with partitions of units, such as halves and quarters.<br>May make errors when considering the length of a bent path as the sum of its parts. | Ask students to predict lengths of objects in a room and then allow them to check with a measuring tool.<br>Provide students with bent paths. Ask them to determine the total length of the paths and to extend the path to a given target length. |

We can see that the students in the three groups described in the example fit into one of two levels: End-to-End Length Measurer or Length Unit Relater and Repeater.

In groups A and B, students were not able to resolve the task correctly when they did not have enough 1-inch strips to span the length of the object. Given the age of these students and their difficulties with this task, we anticipate that they are operating at the End-to-End Length Measurer level. To verify this, we suggest trying a similar task in which students are given more than enough 1-inch strips to span the object. If the students report the correct answer in this case, we would conclude that they are in fact operating at the End-to-End Length Measurer level because they demonstrated an ability to *lay units end to end to create measures for comparison* but were unable to *iterate a single unit to measure*. We would then select an instructional task from the third column of the table in the End-to-End Length Measurer row to prompt students' growth to the next level of the LT, Length Unit Relater and Repeater.

In group C, students demonstrated their ability to *iterate a single unit to measure* when they placed the 1-inch strip in the first position and again in the third, thus counting an empty space as a 1-inch strip. Before we conclude that these students are operating at the Length Unit Relater and Repeater level, we need to verify that they are not operating at the next higher level, Consistent Length Measurer. This can be done by repeating the task above with a longer object to be measured and fewer 1-inch strips, or a single 1-inch strip. If students successfully complete these modified tasks, we could check their ability to resolve a broken (or covered) ruler task, as described in the Length Unit Relater and Repeater row of the table. Assuming that students are unable to resolve this task correctly, we would conclude they are in fact operating at the Length Unit Relater and Repeater level because they have demonstrated an ability to *iterate a single unit to measure* but cannot yet *resolve a broken ruler task*. Finally, we would select an instructional task from the third column of the table in the Length Unit Relater and Repeater row to prompt reflection and reorganization of their thinking, supporting a shift to the next level of the trajectory, Consistent Length Measurer.

Although we believe the LTs are helpful tools for the teaching and learning of measurement, we also recognize the potential complexities that come with adding another resource. Keeping this in mind, we provide a crosswalk linking the CCSSM content standards for measurement, the NCTM content standards for measurement, and the length, area, and volume learning trajectories (see table 0.3). We anticipate that this crosswalk will help you connect the content standards to the LTs, which provide insight into student thinking and strategies along with instructional tasks.

**Table 0.3.** Measurement content standards and related learning trajectories

| CCSSM | NCTM Measurement Standards | Length learning trajectory | Area learning trajectory | Volume learning trajectory |
|---|---|---|---|---|
| K.MD.A.1: Describe measurable attributes of objects. | *Understand measurable attributes* of objects and the units, systems, and processes of measurement<br>**Expectations:** In prekindergarten through grade 2 all students should–<br>• recognize the attributes of length, volume, weight, area, and time;<br>• compare and order objects according to these attributes;<br>• understand how to measure using nonstandard and standard units; select an appropriate unit and tool for the attribute being measured.<br>*Apply appropriate techniques, tools, and formulas to determine measurements*<br>**Expectations:** In prekindergarten through grade 2 all students should– | (2) Length Quantity Recognizer | (2) Area Simple Comparer | (1) Volume Quantity Recognizer |
| K.MD.A.2: Directly compare objects with a measurable attribute. | | (3a) Length Direct Comparer | | |
| K.MD.B.3: Classify objects into categories; count and sort. | | (2) Length Quantity Recognizer | | |
| K.G.B.4: Analyze and compare two- and three-dimensional shapes. | | (3a) Length Direct Comparer–Mental | | |
| 1.MD.A.1: Order three objects by length; compare the lengths of two objects indirectly by using a third object. | | (3b) Indirect Length Comparer<br>(3c) Serial Orderer to 6+ | | |
| 1.MD.A.2: Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. | | (4) End-to-End Length Measurer | | |
| 2.NBT.A.1, 2.NBT.A.1a, 2.NBT.A.1b: Understand that digits of a three-digit number represent amounts of hundreds, tens, and ones. | | (6) Consistent Length Measurer | | |
| 2.NBT.A.2: Count within 1,000; skip-count by 5s, 10s, and 100s. | | (6) Consistent Length Measurer | | |
| 2.NBT.A.3: Read and write numbers to 1,000 using base-ten numerals, number names, and expanded form. | | (6) Consistent Length Measurer | | |

**Table 0.3.** continued

| CCSSM | NCTM Measurement Standards | Length learning trajectory | Area learning trajectory | Volume learning trajectory |
|---|---|---|---|---|
| 2.NBT.B.5: Add and subtract within 100 using strategies based on place value, properties of operations, and/or relationships. | • measure with multiple copies of units of the same size, such as paper clips laid end to end;<br>• use repetition of a single unit to measure something larger than the unit, for instance, measuring the length of a room with a single meterstick;<br>• develop common referents for measures to make comparisons and estimates. | (6) Consistent Length Measurer | | |
| 2.NBT.B.6: Add up to four two-digit numbers using strategies based on place value and properties of operations. | | (6) Consistent Length Measurer | | |
| 2.NBT.B.7: Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and (de)composes tens or hundreds. | | (6) Consistent Length Measurer | | |
| 2.MD.A.1: Measure the length of an object by selecting and using tools such as rulers, yardsticks, meter sticks, and measuring tapes. | | (5) Length Unit Relater and Repeater<br>(6) Consistent Length Measurer | | |
| 2.MD.A.2: Measure the length of an object twice, using length units of different lengths; relate to the size of the unit chosen. | | (5) Length Unit Relater and Repeater | | |
| 2.MD.A.3: Estimate lengths using inches, feet, and (centi)meters. | | (7) Conceptual Ruler Measurer | | |
| 2.MD.A.4: Measure to determine how much longer one object is than another; express the difference in terms of a standard unit. | | (4) End-to-End Length Measurer | | |
| 2.MD.B.5; 2.OA.A.1: Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers); use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns. | | (5) Length Unit Relater and Repeater<br>(6) Consistent Length Measurer<br>(8) Integrated Conceptual Path Measurer | | |

**Table 0.3.** continued

| CCSSM | NCTM Measurement Standards | Length learning trajectory | Area learning trajectory | Volume learning trajectory |
|---|---|---|---|---|
| 2.MD.B.6: Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points. | | (4) End-to-End Length Measurer<br>(5) Length Unit Relater and Repeater | | |
| 2.MD.D.9: Measure lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot. | | (5) Length Unit Relater and Repeater | | |
| 2.OA.C.4: Find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns. | | | (7) Area Row and Column Structurer | |
| 2.G.A.2: Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. | | | (5) Area Unit Relater and Repeater<br>(6) Partial Row Structurer<br>(7) Area Row and Column Structurer | |
| 3.NF.A.1: Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts. | *Understand measurable attributes* of objects and the units, systems, and processes of measurement. | (6) Consistent Length Measurer | | |
| 3.NF.A.2, 3.NF.A.2a, 3.NF.A.2b: Understand a fraction as a number on the number line; represent fractions on a number line diagram. | | (6) Consistent Length Measurer | | |
| 3.NF.A.3, 3.NF.A.3a, 3.NF.A.3b, 3.NF.A.3c, 3.NF.A.3d: Explain equivalence of fractions, and compare fractions. | | (6) Consistent Length Measurer | | |

**Table 0.3.** continued

| CCSSM | NCTM Measurement Standards | Length learning trajectory | Area learning trajectory | Volume learning trajectory |
|---|---|---|---|---|
| 3.MD.A.2: Measure and estimate liquid volumes and masses of objects by using standard units of grams (g), kilograms (kg), and liters (l). Use drawings (e.g., beaker with a measurement scale). | **Expectations:** In grades 3–5 all students should–<br>• understand such attributes as length, area, weight, volume, and size of angle and select the appropriate type of unit for measuring each attribute;<br>• understand the need for measuring with standard units and become familiar with standard units in the customary and metric systems;<br>• carry out simple unit conversions, such as from centimeters to meters, within a system of measurement;<br>• understand that measurements are approximations and how differences in units affect precision; | | | (5) Capacity Relater and Repeater |
| 3.MD.B.4: Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data using units–whole numbers, halves, or quarters. | | (5) Length Unit Relater and Repeater<br><br>(6) Consistent Length Measurer | | |
| 3.MD.C.5, 3.MD.C.5a, 3.MD.C.5b: Recognize area as an attribute of plane figures and understand concepts of area measurement. | | | (4) Primitive Coverer | |
| 3.MD.C.6: Measure areas by counting unit squares (square cm, square m, square in., square ft., and improvised units). | | | (5) Area Unit Relater and Repeater | |
| 3.MD.C.7: Relate area to the operations of multiplication and addition. | | | (7) Area Row and Column Structurer<br>(9) Array Structurer<br>(8) Area Conserver | |
| 3.MD.D.8: Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. | | (7) Conceptual Ruler Measurer<br>(8) Integrated Conceptual Path Measurer<br>(9) Coordinated and Integrated Abstract Measurer with Derived Units | | |

**Table 0.3.** continued

| CCSSM | NCTM Measurement Standards | Length learning trajectory | Area learning trajectory | Volume learning trajectory |
|---|---|---|---|---|
| 3.OA.A.1: Interpret products of whole numbers, e.g., interpret $5 \times 7$ as the total number of objects in 5 groups of 7 objects each. | • explore what happens to measurements of a two-dimensional shape such as its perimeter and area when the shape is changed in some way;<br>• understand the need for measuring with standard units and become familiar with standard units in the customary and metric systems.<br><br>*Apply appropriate techniques, tools, and formulas* to determine measurements<br>**Expectations:** In grades 3–5 all students should–<br>• develop strategies for estimating the perimeters, areas, and volumes of irregular shapes; | | (9) Array Structurer | |
| 3.OA.A.3: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings. | | | (7) Area Row and Column Structurer<br>(9) Array Structurer | |
| 3.OA.A.4: Determine the unknown whole number in a multiplication or division equation relating three whole numbers. | | | | |
| 3.G.A.2: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. | | | | |
| 4.OA.A.2: Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings; distinguish multiplicative comparison from additive comparison. | | (7) Conceptual Ruler Measurer | | |
| 4.NBT.B.5: Illustrate and explain multiplication by using rectangular arrays, and/or area models. | | | | |
| 4.NF.A.1: Explain why a fraction $a/b$ is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, noticing that the number and size of the parts differ but the fractions are the same. | | (8) Integrated Conceptual Path Measurer | | |
| 4.NF.B.4a, 4.NF.B.4b: Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. | | | | |

**Table 0.3.** continued

| CCSSM | NCTM Measurement Standards | Length learning trajectory | Area learning trajectory | Volume learning trajectory |
|---|---|---|---|---|
| 4.MD.A.2: Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems that require expressing measurements given in a larger unit in terms of a smaller unit. | • select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles;<br>• select and use benchmarks to estimate measurements;<br>• develop, understand, and use formulas to find the area of rectangles and related triangles and parallelograms;<br>• develop strategies to determine the surface areas and volumes of rectangular solids. | (5) Length Unit Relater and Repeater<br>(6) Consistent Length Measurer | | |
| 4.MD.A.3: Apply the area and perimeter formulas for rectangles. | | (6) Consistent Length Measurer | (9) Area: Array Structurer | |
| 4.MD.C.5, 4.MD.C.5a, 4.MD.C.5b: Recognize angles as two rays sharing a common endpoint; understand angle measurement. | | (8) Integrated Conceptual Path Measurer | | |
| 5.NBT.B.5: Fluently multiply multidigit whole numbers using the standard algorithm. | | | | |
| 5.NBT.B.6: Illustrate and explain division by using equations, rectangular arrays, and/or area models. | | | | |
| 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings. | | | | |
| 5.NF.A.1: Add and subtract fractions with unlike denominators by replacing given fractions with equivalent fractions. | | (8) Integrated Conceptual Path Measurer | | |
| 5.NF.A.2: Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models. | | (8) Integrated Conceptual Path Measurer | | |

**Table 0.3.** continued

| CCSSM | NCTM Measurement Standards | Length learning trajectory | Area learning trajectory | Volume learning trajectory |
|---|---|---|---|---|
| 5.NF.B.4, 5.NF.B.4a, 5.NF.B.4b: Multiply a fraction or whole number by a fraction. | | | | |
| 5.MD.A.1: Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multistep, real-world problems. | | (6) Consistent Length Measurer | | |
| 5.MD.C.3, 5.MD.C.3a, 5.MD.C.3b: Recognize volume as an attribute of solid figures and understand concepts of volume measurement. | | | | (6) Volume/Spatial Structuring: Partial 3-D Structurer |
| 5.MD.C.4: Measure volumes by counting unit cubes, using cubic cm, cubic in., cubic ft., and improvised units. | | | | (6) Volume/Spatial Structuring: Partial 3-D Structurer |
| 5.MD.C.5, 5.MD.C.5a, 5.MD.C.5b: Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. | | | | (9) Volume/Spatial Structuring: 3-D Array Structurer |

## How This Book Is Organized

The following chapters offer numerous examples of activities connected to grade-specific content standards. Each activity can be linked to NCTM's *Principles and Standards for School Mathematics* (2000), CCSSM, and the relevant length, area, and volume LTs, as shown in table 0.3 and summary tables within each chapter. (NCTM has produced supporting resources for *Principles and Standards for School Mathematics*, including a set of frequently asked questions, a Quick Reference Guide that outlines standards and expectations by grade band, and a CD that assists those interested in exploring the ten standards and learning more about them. In addition, the *Principles and Standards for School Mathematics* Navigations Series translates NCTM's Principles and Standards into action in the classroom and highlights major mathematics content areas in grade-band-specific volumes.)

Within each grade-level chapter of this book, we present activities to address key ideas as identified in CCSSM for that grade. We explain how to prepare for each activity, detail necessary materials and specific tasks within the activities, discuss anticipated student responses, and offer suggestions for extensions. The activities are structured as follows:

- **Essentials:** Gather necessary materials and prepare for the activity.
- **Engage:** Pose a problem or a situation based on a work of literature or art or using technology tools.
- **Explore:** Invite students to begin, and involve them in the measuring tasks.
- **Expect:** Anticipate, assess, and interpret students' thinking. Look for successes and struggles with the activity.
- **Extend:** Do more to help students build on the key measurement ideas presented in the activity.
- **Enrich:** Connect this concept to literature, relevant websites, or real-life situations; help students relate measurement ideas to people and their careers, such as Olympic athletes, chefs, or musicians.

Reproducible materials for a number of activities from throughout the book are available as classroom-ready PDF files at NCTM's More4U site. For access to these resources, readers should go to http://www.nctm.org/more4U and enter the access code on the title page of this book.

Although we have included references to children's literature in many of the sections labeled **Essentials** and **Enrich**, we expect that most of our activities can be adapted by replacing the suggested story or context with a different introduction to set the challenge or topic. Telling a story about your own experiences or posing a problem directly will work just as well.

We hope you and your students find it a pleasure to measure!