Chapter 4

Eliciting, Supporting, and Guiding the Math

Three Key Functions of the Teacher's Role in Facilitating Meaningful Mathematical Discourse

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In this chapter, we examine the teaching-and-learning principle *facilitating meaningful mathematical discourse.* We begin by asking what is meant by *meaningful mathematical discourse* and what it means to *facilitate* such discourse. We then turn our attention to how teachers do this important work, including some of the challenges teachers face, and conclude with some suggestions for how teachers and districts might make progress toward having meaningful mathematical discourse become a daily event in their classrooms.

Facilitating Meaningful Mathematical Discourse: Getting Clear on Our Terms

What Is Meaningful Mathematical Discourse?

We focus on *meaningful mathematical discourse* as a discourse that is about *making mathematics reasonable in school* (Ball and Bass 2003). To clarify, discourse is not limited to spoken words but includes all mediums and methods that support communication and the expression and exchange of mathematical ideas, including diagrams, gestures, and other non-verbal signals that are part of how we convey and make meaning. That is to say, mathematical discourse is any form of communication that positions mathematics as a subject that makes sense, yielding insights when reasoning is used. It "sorts through" important ideas as students express ideas, clarify, and revise.

A meaningful mathematical discourse engages students with significant mathematics and targets a significant mathematical goal. The discussion can focus on concepts, procedures, problem-solving strategies, representations, or reasoning. The goal could be to compare definitions, develop a justification for why a strategy works, provide an argument to demonstrate whether a conjecture is true, determine how two different approaches can yield the same result, or identify when a particular strategy is more efficient than another (Chapin, O'Connor, and Anderson 2009; Kazemi and Hintz 2014). The meaningfulness of the discussion comes both from the significance of the mathematics and from the personal meaningfulness students ascribe as they generate ideas, are heard by others, consider others' viewpoints, and collectively develop new understandings.

Why Is Engaging Students in Meaningful Mathematical Discourse Important?

Engaging students in meaningful mathematical discourse promotes a range of desired learning outcomes (Ball and Bass 2003; Hiebert et al. 1997; NCTM 1989, 2000; NRC 2001). Mathematical discourse is a critical practice through which students develop mathematical communication and argumentation skills and the ability to critique the reasoning of others. The practice also supports students in developing a connected and strong understanding of mathematical concepts (Cross 2009; Kazemi and Stipek 2001; NRC 2001). Discourse is not an extension activity for students to engage in *after*

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they have learned content but should be part of the daily fabric of the mathematics classroom.

As students engage in meaningful mathematical discourse, they formulate ideas and present them to others, which creates opportunities for students to develop language to express ideas, represent evidence, and clarify their reasoning. They become increasingly proficient in articulating mathematical ideas and supporting them with mathematical arguments. These activities also offer opportunities for reflection and metacognition, two valuable practices that support learning and help students to solidify their thinking (NRC 2001).

As students experience sense-making and arrive at mathematical conclusions as a result of a classroom or group discussion, students begin to see themselves as thinkers, as people who can produce knowledge and who can do math. When engaged in meaningful mathematical discourse, the locus of authority in the classroom rests with the students and the discipline of mathematics, as validity and correctness are determined by the reasonableness of an argument or idea, and does not rest with the teacher or textbook (Hiebert et al. 1997). A shift in disciplinary authority has been connected with more productive beliefs about mathematics and its value (Boaler and Greeno 2000), high levels of student engagement (Boaler 1997; Engle and Conant 2002), and student perseverance (Boaler 1997). Furthermore, when students are involved with justification, reasoning, and expressing ideas, they learn to value the ideas and contributions of others as well as see the value of their own contributions (Boaler and Staples 2008; Nasir et al. 2014). These forces can disrupt status differences among students, which in turn promote more equitable participation (Cohen and Lotan 2014; Nasir et al. 2014) and may also support more equitable learning outcomes among students (Boaler and Staples 2008; Hiebert et al. 1997). As status differences shift and student participation increases, additional students have opportunities for "air time" and the classroom environment becomes increasingly responsive to students' thinking.

Finally, when students are engaged in mathematical discourse, teachers gain valuable insights into students' thinking. Discussions can provide valuable formative assessment data, revealing how students are making sense of information and reasoning about it in ways not afforded by students' written work. Student misconceptions are also frequently revealed and challenged during discussions, providing both teacher and student with useful information and opportunities to address and work through the misconception and enhance their learning (Hiebert et al. 1997; Hoffman, Breyfogle, and Dressler 2009).

What Is Facilitating?

The teaching practice highlighted in *Principles to Action: Ensuring Mathematical Success for All* is facilitating meaningful mathematical discourse. The term *facilitate* has many meanings and connotations. What does it mean for a teacher to facilitate meaningful mathematical discourse in the context of a mathematics classroom? The biggest challenge perhaps is to see what the teacher *is* doing (Chazan and Ball 1999), as facilitating can give the impression of a role that is "neutral" or unobtrusive, overseeing in a manner that does not impact the direction of the group or substance of the discussion. This type of facilitation is *not* what we mean when we describe a teacher as a facilitator of meaningful mathematical discourse. This hands-off vision of the role can lead to unproductive exchanges that fall short of being *mathematically meaningful* (Alfieri et al. 2011; Cross 2009; Nathan and Knuth 2003). Rather, when facilitating, the teacher plays a very active role—indeed multiple roles—toward the end of supporting meaningful mathematical discourse.

In our "active" definition, facilitating includes *guiding* as well as *supervising*. The nature of the teacher's guidance is critical for the productive organization of (or failure of) a discourse that engages students in meaning making, positions mathematics as reasonable, and builds the class's collective body of knowledge together. The supervisory role suggests that the teacher is responsible for coordinating the group, supporting productive interactions, and helping students do the work. A supervisor does not do the work for those under supervision but rather has the knowledge required to guide, troubleshoot, support, and bring out the best in a group. As Munter (2014) and others have pointed out, the teacher and students are mutually engaged, but the teacher is "a more knowledgeable partner who is responsible for ensuring that classroom mathematical practices come to resemble those of the discipline" (p. 590). Thus, although in some sense a co-participant, the teacher has unique responsibilities in supporting the group in attaining the desired goals.

The Teacher's Role in Facilitating Meaningful Mathematical Discourse

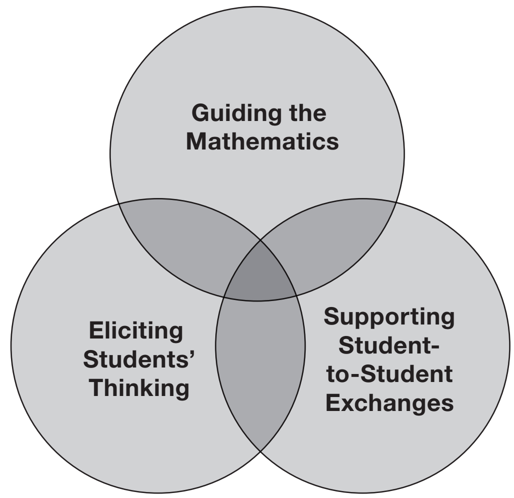
Having clarified what we mean by a meaningful mathematical discourse, we now consider the teacher's role in facilitating such discourse. What does it take to organize and engage students in meaningful mathematical discourse? The teacher's work to support meaningful mathematical discussions begins before the discussion takes place and includes identifying mathematical goals to pursue for the lesson and unit (chapter 1), selecting the task(s) for that lesson (chapter 2), establishing classroom norms, particularly those that support perseverance (chapter 7), and building relationships with students (Battey 2013), to identify a few. Necessarily, the exact work varies depending on the time of year, the newness of the content, the students' experience engaging in a math-talk community (Hufferd-Ackles, Fuson, and Sherin 2004), the instructional format (small group or whole class), and other factors. With this acknowledgment of the broader context, we focus on understanding the teacher's role *during* a meaningful mathematical discussion and address how teachers support meaningful mathematical discussions as they unfold.

Three Key Functions of the Teacher's Role

Drawing on frameworks and descriptions found in the research literature (e.g., Ball 1993; Bochicchio et al. 2009; Fraivillig, Murphy, and Fuson 1999; Hufferd-Ackles et al. 2004; Lampert 2001; Staples 2007), we describe the actions and decisions that a teacher makes to support meaningful mathematical discussions in terms of three key functions:

1. *Eliciting Student Thinking,* including providing opportunities for students to generate ideas and then share their ideas with the class;
2. *Supporting Student-to-Student Exchanges about Mathematical Ideas,* including establishing a common knowledge base from which to work; and
3. *Guiding and Extending the Math,* including pursuing common misconceptions and ensuring appropriate disciplinary norms to advance the learning of the class.

An overview of these three key functions, along with a brief description of each, is shown in figure 4.1. In this section, we clarify each function and describe "teacher moves" and strategies related to each function that have been shown to be useful in supporting meaningful mathematical discourse. Drawing from the extensive literature in this area, we give particular attention to teacher verbal discourse moves because what teachers say (and do not say) as discussions unfold significantly impacts the nature of the discussion and the degree to which it is mathematically meaningful to students.



Eliciting Student Thinking, including providing opportunities for students to generate ideas and then share their ideas with the class;

Supporting Student-to-Student Exchanges about Mathematical Ideas, including establishing a common knowledge base from which to work and helping students make sense of one another's ideas;

Guiding and Extending the Math, including guiding the development of mathematical ideas, pursuing common misconceptions and ensuring appropriate disciplinary norms to advance the learning of the class.

Fig 4.1. Three key functions of the teacher's role in facilitating meaningful mathematical discourse

Because of space limitations, we emphasize one format—whole-class discussion—over others (e.g., pair work, small group discussions). We also give less attention to questioning and types of questions, as these topics are addressed extensively in chapter 5.

Eliciting. Students' reasoning is central to a meaningful mathematical discourse. Consequently, one of the teacher's most important functions is eliciting student thinking. This process starts with the teacher providing students time and space to generate ideas. This "thinking time," or wait time (Chapin, O'Connor, and Anderson 2009; Rowe 1986), allows students to think through a problem, form a strategy, and work on how they might express their ideas. Teachers can create further opportunities for students by using structures like quick writes, turn-and-talk, think-pair-share, or the use of hand signals by students to let the teacher know they have an idea and are ready to share.

Another aspect of eliciting student thinking is inviting students to share their ideas with the class. Note that such requests can be very open ("Who would like to share their answer to number 2?") or very specific ("I'm looking for different strategies for number 2. Who can show a numerical approach?"). Chapter 8 on the teaching practice of questioning includes strategies for how to effectively question and elicit students' thinking. One point we emphasize here is that the teacher's request may be more productive in eliciting student ideas if the teacher uses language that focuses on students' ideas and not the "right" answer (Hiebert et al. 1997; Humphreys and Parker 2015; Truxaw and DeFranco 2008). This feature relates back to the meaningfulness of the conversation and the shift of authority. For example, students may respond differently to the request, "Who can tell me the answer to Number 3 and how to do it?," versus the request, "Who can tell me what they got for Number 3 and how they approached it?" Both aim to elicit student ideas, but the latter phrasing implies that students are sharing their *thinking* in ways that might open up a conversation, whereas the former implies that the student's response is expected to be correct and might be subject to immediate evaluation.

A third consideration when eliciting student thinking is encouraging or supporting clear communication of ideas. The goal is, not perfect use of vocabulary or formal sentences, but rather clear enough expression of ideas so that both the teacher and other students can consider the contribution. To support student communication and develop these skills over time, teachers can provide students with a word bank of specific vocabulary or sentence frames, such as "I think the answer is . . . because . . . ," which help them to express ideas and articulate their thinking in an organized structure. Teachers also need to incorporate the use of visuals or manipulatives and opportunities for students to reference these as they describe their ideas. This aspect of the teacher's role overlaps with the next role, supporting student-to-student exchanges, as that work cannot be done if the student who is sharing does not present his or her idea in a sufficiently clear manner.

Supporting. The function of supporting student-to-student exchanges about mathematical ideas has two key aspects. First, the teacher must ensure that the ideas being worked on are accessible to the students in an ongoing manner, particularly as students contribute new ideas, some questions are resolved, and new questions are put on the table. Second, the teacher must manage, as needed, the student-to-student exchanges and turn taking as the discussion unfolds.

We focus first on accessibility. In a meaningful mathematical discussion, not only are student ideas elicited, but those ideas become a focus of the discussion. Students need support in attending to, making sense of, and commenting on one another's ideas. They also must be able to track the conversation over time as its focus shifts, new questions arise, and others are resolved. Teachers must monitor the discussion and act as needed to help students stay in a position where they can make sense of and contribute to a collective conversation.

To support this work, teachers can use a variety of strategies. A guiding principle for teachers is to ensure that ideas are represented publicly and with multiple opportunities for students to hear and make sense of an idea. Teachers can encourage students to come to the board or document camera (or other public space) to share and record calculations, tables, graphs, or other diagrams that support their reasoning or solution methods. Color coding visuals and restating key ideas are also useful ways to direct students' attention toward critical information.

By publicizing these elements, other students in the class have a better chance of being able to make sense of the shared strategy and offer their own thinking about it. In addition, these records are then available to be referenced later in discussion. Indeed, Ball and Bass (2003) argue that to engage students in mathematics in ways that centralize sense making and reasoning requires building public records, as discussions can only be supported on a base of public knowledge, which must be visible and shared.

The accessibility of the conversation, however, goes beyond what is happening in the present moment, or even that class period. For productive exchanges to occur, students must have a common ground (Clark 1996) or common knowledge (Barnes 1976; Edwards and Mercer 1987) as the basis of their communication. The common ground comprises those reference points that a student (or teacher) in the class can assume others in the class understand or can access and use meaningfully as they explain or share their ideas. For example, suppose the class is considering whether one can have an obtuse triangle that is also a right triangle. One strategy to publicize a contributing student's idea for others to consider is to have a student come to the board to record and "show" her thinking and the examples she has in mind. The teacher might also ask for some labeling or the use of a new color for the angle under consideration as right. But for this conversation to be a meaningful one that advances the class's knowledge, shared definitions of right and obtuse triangles need to be in play; otherwise, students will not be able to make sense of one another's ideas. For example, a student might reference "John's triangle" in sharing his own thought, which will support the discussion only if that is a common reference point that students can then refer to (mentally or otherwise) as they continue to work through ideas.

These types of supports help students to make sense of each other's ideas and ultimately to come to agreement. In general, teachers can encourage students to use agreed upon vocabulary or refer to a common diagram or example when explaining. Teachers can ask students to remind the class of the question being discussed, summarize the discussion so far, or restate the different ideas being considered. By bringing attention back to established reference points, teachers help to ensure that students continue to be able to make sense of and contribute to the discussion (Staples 2007).

To further support student communication about mathematical ideas, teachers need discourse moves to help students hear, consider, and comment on one another's ideas. Chapin, O'Connor, and Anderson (2009) have developed a set of five talk moves, shown in figure 4.2, that are designed to support classroom discussions. These moves help publicize students' ideas and also help students attend to, make sense of, and build on one another's ideas.

1. Revoicing – Repeating what students have said and then asking for clarification
2. Repeating – Asking students to restate someone else's reasoning
3. Reasoning – Asking students to apply their own reasoning to someone else's reasoning
4. Adding on – Prompting students for further participation
5. Waiting – Using wait time

Fig. 4.2. Five productive talk moves. From Chapin, O'Connor, and Anderson (2009), pp. 12–17.

Revoicing and repeating are both important moves to publicize or establish an idea and help others access the idea, which increases the likelihood students will hear and make sense of an idea. Waiting after a student shares an idea is another form of wait time that supports the establishment of common ground, as students are provided the opportunity to consider and make sense of the contribution before being asked to respond to or use the idea.

Reasoning is a talk move that asks students to think about another student's idea or chain of reasoning and to consider whether or not they agree with the approach or idea and why. By doing so, students are prompted to make sense of another's idea and connect it with their own thinking. Adding on is another move that directs students to attend to another's idea, as it requests contributions that build on what has already been shared and potentially extends the mathematics. Each of these five talk moves positions the idea(s) already shared for further consideration by the class and offers opportunities for students to make sense of or comment on a peer's idea. Using the talk moves can help shift classroom discourse from sharing discussions, which can be supported by eliciting and publicizing student ideas, to collaborative discussions (Staples and Colonis 2007) where students work through ideas together, creating new understandings as a result of their conversations.

Guiding. A final function of the teacher's role in facilitating meaningful mathematical discussions is guiding and extending the math. That is, the teacher must guide the class's mathematical work during a discussion so that it advances and/or extends students' thinking and understanding. As a classroom discussion progresses, the teacher continually makes decisions regarding the direction of the lesson based on how it is unfolding with respect to her understanding of the discipline, her goals for the unit and lesson, her students' prior knowledge, and many other factors (Ball 1993; Chazan 2000; Lampert 2001). The teacher's listening skills are crucial here. She must hear students' contributions to monitor what they are wrestling with and where students are with their understanding. The students' contributions then inform her next moves as she decides how to productively support their continued engagement in a way that advances the class's mathematical work.

One challenge in facilitating a discussion is ensuring the mathematics is attended to with sufficient depth and rigor, which requires a teacher to decide when and how to step into the discussion (Rittenhouse 1998). These types of moves generally offer support for sustaining students' attention to an idea (particularly in the face of struggle) or advancing the discussion, by prompting students to consider new questions or content at a deeper level. While advancing the math, the moves also keep students "in the driver's seat."

More specifically, the teacher's work might include the following moves:

- Highlighting mathematically important aspects of student contributions for the class to further consider.
- Pressing students to offer mathematical reasoning and arguments and not just explanations of their procedural steps. (See Kazemi and Stipek 2001 for a discussion of high-press versus low-press classroom environments.)
- Regarding "errors [as] opportunities to reconceptualize a problem and explore contradictions and alternative strategies" (Kazemi 1998, p. 411; see also Hoffman, Breyfogle, and Dressler 2009).
- Offering "information that students need in order to test their ideas or generate a counterexample" (Lobato, Clarke, and Ellis 2005, p. 110).
- Asking students to compare two or more methods (Fraivillig et al. 1999; Rathouz 2011) and/or explicitly prompting the use of multiple representations.
- Introducing a new example for consideration, strategically chosen to draw students' attention to some aspect of the mathematics.

These moves support students' mathematical work and do not take over the work or reduce the cognitive demand of the task (Stein, Grove, and Henningsen 1996).

While it is the ultimate goal for student ideas to drive the discussion and for students' mathematical reasoning to be the "judge" about correctness, there are also times when it is appropriate for the teacher to intervene more directly by telling (Chazan and Ball 1999; Lobato, Clarke, and Ellis 2005). Specifically, Lobato, Clarke, and Ellis (2005) conceptualized some forms of telling as initiating, where the teacher may, for example, summarize student work in a way that inserts new information, supplies a definition, or describes a new concept. More important than what the teacher says is the intention of the teacher's insertion. Lobato and colleagues assert that the purpose should be "to prompt coherence and sense-making." They further note that initiating is often followed by eliciting—"an action intended to ascertain how students interpret the information introduced by the teacher" (p. 111).

To illustrate, Lobato, Clark, and Ellis (2005) offer an example where students are working on the idea of steepness. The teacher detects potential misconceptions and offers ideas intended to clarify, as well as attune students to, commonalities across examples:

> One thing I want to say is that when we're talking about steepness we're talking about this slantiness. We're not

> talking about whether it's harder to walk up it. They are definitely different ramps. You have to walk further on this one. . . . This one is higher. This one is longer. But there is something the same about them. (p. 128)

This kind of telling, or initiating, maintains students' authority and continues to position them as thinkers and doers of mathematics. The role of teachers at these moments—when students are satisfied with the depth and rigor (when the teacher is not) or are unsure of how to proceed further—is crucial. It is this type of guidance and structuring of students' attention to specific aspects of the mathematics that may make the key difference in whether the discussion is productive and meaningful and the degree to which it advances student learning (Alfieri et al. 2011; Cross 2009; Kazemi and Stipek 2001).

Table 4.1 offers a summary of the three functions of the teacher's role—eliciting student thinking, supporting student-to-student exchanges, and guiding and extending the math—with examples of teacher moves aligned to each. To help further illustrate these functions, we turn our attention to a classroom dialogue.

A Dialogue Is Worth a Thousand Words

A class of seventh-grade students is working on a hexagon patterns task as shown in figure 4.3. An excerpt from this lesson is shown in figure 4.4.

Table 4.1
Three sets of moves or strategies teachers can use as part of enacting each component of their facilitator role

Eliciting student ideas	Supporting student-to-student conversations and establishing common ground	Guiding and extending the mathematics
Providing students with time to think, generate, and work on expressing their ideas. (Quick Write, Think-Pair-Share, Turn-and-Talk, wait time) • "You have two minutes to talk, and at the end of that time, you need to be able to tell me who you agree with and why." Directly requesting students to share their ideas • "I'm looking for different strategies for number 2. Who can show a numerical approach?" Supporting clear communication • "I see you pointing and saying the corners. Let's make sure we know what you're referencing. Can you circle the corners you're talking about? And what's our mathematical word for those?" • "Lianne, as you explain, stand to the side so we can see your diagram and what you're referencing."	Ensuring a public space available to represent ideas • Small groups should have workspace in the middle of their table or desks • Teacher uses board or wall space carefully to record the main question, emerging ideas, and to focus the conversation Recording student ideas publically • "Come up and write your answer (thinking) on the board." • Use different colored markers to track different student ideas Encouraging students to engage and make sense of one another's ideas • "Vanessa, can you restate Felicia's idea in your own words?" • "Why do you think Albert chose to divide at this step?" Verbally recapping, or asking a student to recap, where the class is now with the ideas, and what they are still discussing	Prompting students to focus on particular aspects of the math, or to extend the mathematical thinking in a particular way: • "I'd like you two to compare your strategies." • "Hector just made a conjecture. Let's work on his conjecture as a class. That will get us into some other important mathematics." • "So we have the result. Now, why might that be true? Any thoughts?" Teacher inserts an idea for consideration to clarify or extend students' understandings • "From this discussion, I hear most agreeing it has to be an odd function, and some of you aren't sure yet. I'm going to draw a graph for you to consider. I want to know if this graph is odd as well, and why you think that." • "It sounds like we're not agreeing how to sort these objects because we're not agreeing on their definitions. Let's revisit the definitions and then come back to this."

Each figure in the pattern below is made of hexagons that measure 1 centimeter on each side.

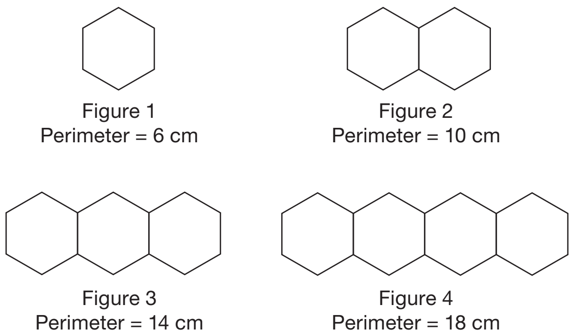


Fig. 4.3. Hexagon pattern task

In this excerpt, the class first considers the perimeter of a chain of four hexagons and then, using a particular method shared by a student, pursues the perimeter of a chain of ten hexagons. While the students drive much of this conversation, indicating a fairly mature math-talk community (Hufferd-Ackles et al. 2004), the teacher is still quite active, enacting each of the three role functions described above. Notice that the teacher does not do the mathematical work, however. In less mature communities the teacher enacts the same role functions but may need to do so in a different manner, such as in the example provided by Staples (2007, p. 16).

1. *Jackie* [*at board, pointing to a visual of four hexagons*]: Okay, well every time that you have one of these things, if it's *n* numbers, the middle ones will always have 4 and then these are going to be 5s. So I got 18.
2. *Ms. L*: So we all understand that? 18?
3. *Students*: Yeah.
4. *Ms. L*: No, do we all understand her strategy?
5. *Students*: Oh no. No.
6. *Ms. L*: No. Then, she is the presenter; you guys are the audience. Go ahead presenter, you're on.
7. *Sean*: How did you get the four?
8. *Jackie*: Well the 4 is the inside hexagons and then another 4 right here [*points to the second "interior" hexagon*] and then you have 5 on the ends [*points to the two "end" hexagons*].
9. *Cody*: So, what you are saying is all you did was 4 plus 4 plus 5 plus 5?
10. [*Jackie nods.*]
11. *Ms. L*: Okay [*addressing a student*], so go ahead, ask her how to find figure 10.
12. *Kate*: How did you find figure 10?
13. *Jackie*: Um, can I draw a picture?
14. *Ms. L*: Absolutely.
15. *Ms. L*: All right, while she is doing that up there and drawing it, why don't you try to use her strategy and find figure 10?
16. [*The teacher walks around the room, encouraging students to use Jackie's strategy. Jackie draws a chain of ten hexagons on the board and writes her calculations. After a minute, the teacher asks Jackie to explain.*]
17. *Jackie*: All right, well 8 right here [*pointing to the 8 "interior" hexagons of the chain that are not on the ends*], and then there is 4 on the [top and] bottom thing, so 8 times 4 equals 32, and then the ends there is 5 and then another 5, so plus 10 equals 42.
18. *Ms. L* [*to class*]: Does it work?
19. *Students*: Yeah.
20. *Ms. L*: Will it work for every single one?
21. *Students*: Yes.
22. *Ms. L* [*acknowledging a student*]: Do you have a question?
23. *Sean*: But for 100 it really wouldn't work, because you would have to draw 100 hexagons.
24. *Ms. L*: Okay, good question, would you have to draw them? What a good question. Would you have to draw it? Now Jackie, these guys over here used your method, too. Can they come up and support you and show how they don't need to draw it?
25. [*Jackie assents.*]
26. *Ms. L*: All right, let's do it.

Fig. 4.4. A classroom excerpt of a teacher facilitating meaningful mathematical discussions (edited for readability and anonymity)

We consider this an example of meaningful mathematical discourse, as the class is engaged in collaborative work, offering their ideas and attending to others' thinking, and making sense of significant mathematics. Looking more closely at the teacher's role in facilitating this meaningful mathematical discussion, let's begin with guiding the mathematics.

Line 4 is the teacher's first key move in guiding the class's mathematical work. She ascertains whether the class understands the presenter's strategy (beyond agreeing with the answer). Learning that the student's strategy is not well understood, in line 6 the teacher sets the agenda for the next part of the lesson–to understand Jackie's strategy for finding the perimeter of the hexagon chains.

In line 11, by encouraging a student to ask another student a question, the teacher moves the class to a consideration of a chain of 10 hexagons and, in line 15, asks the class to "use her strategy" to find the perimeter. This move further guides the class's mathematical work making sense of Jackie's approach, and we can see how this type of mathematical activity is setting the class up to consider the more general case. Line 23 is particularly interesting. The teacher hears a question that could lead to an important discussion about generalization: specifically, what kinds of strategies are general and can be used to determine perimeter for a hexagon chain of any length? She excitedly highlights it and invites another group to come up and share its work, which she indicates will help the class engage and ultimately answer the question. In this case, the teacher did not set the question herself to guide the math, but hearing a student question that was of import, gave weight to that and made it the focus of the conversation.

Some of the moves just discussed also support the other role components of eliciting students' ideas (e.g., lines 4) and supporting student exchanges (e.g., lines 6, 11, and 15). We also can see in this excerpt classroom norms as students seem comfortable and skilled at sharing and attending to each other's ideas as they question one another (lines 7 and 22).

Common Challenges and Productive First Steps: Supports for Making It Happen

Facilitating meaningful mathematical discourse is complex and challenging work. The research literature has documented the persistence of long-standing routines and discourse patterns of typical U.S. classrooms that run counter to engaging students in meaningful mathematical discourse and position mathematics as something to be directly transmitted and reproduced (Jacobs et al. 2006; Stigler and Hiebert 1999). Making meaningful mathematical discourse a regular part of mathematics classes requires deliberate effort by classroom teachers and targeted support by districts. In this section, we highlight productive steps to advance this agenda, first at the classroom level and then at the district level.

Using Routines

One potentially productive approach for shifting teaching practice to better engage students in meaningful mathematical discourse involves teachers enacting new planning and classroom routines, or activity structures. Such routines provide a sequence of steps for part or all of a lesson to help create a context for meaningful mathematical discussions.

For example, Number Talks (e.g., Humphreys and Parker 2015; Parrish 2010) is a routine originally designed to support developing students' number sense and understanding of mathematical operations through sharing and discussing multiple approaches to a computational problem (e.g., 16×25). After think time, where the students signal the teacher when they have an approach, the teacher asks for student ideas (eliciting) and records these on the board (publicizing). As the teacher elicits and records, the teacher supports students in attending to and making sense of others' approaches. She then guides the subsequent discussion about the ideas. A discussion might focus on comparing two methods, raise questions about how students decomposed numbers, or revise and extend a student's method.

Other routines, such as the Launch-Explore-Summarize structure of *Connected Mathematics Project* (Lappan et al. 2009), the Five Practices routine (Smith and Stein 2011), the Talk Frame (Casa 2013; Williams and Casa 2011/2012), and Kazemi and Hintz's (2014) "targeted discussion" formats similarly create a context where students' generated ideas are used as the basis of the subsequent conversation. Routines provide a structure in which to have a meaningful mathematical discussion but will fall short if they become a forum for presenting one best way or allow students to serially report their approaches with no further discussion or connection (see *strategy reporting* versus *inquiry and argument* as patterns of interaction in Wood and Turner-Vorbeck [2001]).

Teaching New Practices to Students

Engaging in meaningful mathematical discourse may be just as new to students as to teachers. As students come together at the beginning of the year, they bring with them various conceptions of what math is and what it means to do math based on prior experiences. It is critical that teachers appreciate the extent to which they are asking students to engage with math in new ways—ways that require students to take risks—as they share ideas and try to work through ideas in a more public way than they have been asked to do before (e.g., Chazan 2000; Lampert 2001). Perhaps surprisingly, students who have been successful with more traditional models of instruction may be some of the strongest "resisters" of these changes, as their success and identity as a "good math student" may seem threatened.

Lampert (2001) offers an informative example of this paradox from her work with a fifth-grade class. She wanted to introduce the practice of revising as a way to support student sense making and engagement in authentic mathematical practices.

> Introducing *revising* . . . would require a change in how . . . students would typically think about what one does to study mathematics. It would probably also require some changes in what they thought about the roles of "smart" and less smart classmates. . . . I did not expect that my students would come to fifth grade knowing how to evaluate their own assertions or those of their peers in order to decide whether or not such assertions needed revising. Nor did I expect that they would see such evaluation and revision as activities that would contribute to their leaning. (Lampert 2001, p. 65)

This excerpt highlights the magnitude of the changes being asked of students and points to similarly daunting changes for teachers as they learn to do mathematics differently. Through Lampert's careful work with her class introducing new practices, she was able to expand students' individual and collective capacity to participate in a meaningful mathematical discourse.

Research studies have documented how teachers support the development of the class's capacity to participate in meaningful mathematical discourse and have found that teachers must deliberately introduce their students to new ways of working together, be explicit about how their participation in these new practices and formats supports their learning, and provide opportunities to negotiate the meanings of these new practices (Chazan 2000; Goos 2004; Hufferd-Ackles et al. 2004; Lampert 2001; Staples 2007). Going hand-in-hand with developing these practices, teachers must carefully establish classroom norms to support productive exchanges among students, such as the expectation that everyone must listen to each other, all students have the right to ask questions and share their thinking, and discussions are about mathematical ideas, not people (e.g., I'm critiquing the idea, not you). For detailed discussion and strategies, see Chapin, O'Connor, and Anderson (2009, 2014).

District-level Support

A critical component to supporting teachers as they strive to organize meaningful mathematical discourse is aligning the system. Pacing guides, teacher evaluation protocols, and curricular materials need to support and affirm pedagogies that centralize mathematical discussion. Similarly, student assessments must reach well beyond procedural skill to assess reasoning. Without such alignment, efforts to advance meaningful mathematical discourse in classrooms will not have a consequential impact. To undertake the challenge of shifting classroom culture and developing new pedagogical techniques, teachers need time, model resources, non-evaluative support, and opportunities to step back and reflect on how the discourse is developing in their classrooms. Teachers also need high quality resources to support their learning, reflection, and collaboration (such as published material), representations of practice (including videos), and access to and time with their local community of teachers (e.g., grade-level team or professional learning community). Specifying times that are protected and can be used for such learning-focused work is critically important as well, as teachers' days are busy and crowded with many important demands.

To support districts in monitoring their progress, Munter (2014) developed a set of protocols to help ascertain teachers' visions of high-quality mathematics instruction during implementation of district-level, multiyear reform. Though labor intensive, these protocols document teachers' changing visions and gauge the degree to which the vision of quality math teaching put forth from the district is being adopted consistently by teachers in the district. These and similar tools can be useful in supporting districts in designing and monitoring their efforts.

Concluding Remarks

Facilitating meaningful mathematical discourse is an essential goal if we are to support students' participation in mathematics, advance a view of mathematics as a connected whole, and develop students' conceptual understanding and proficiencies with key practices such as problem solving, argumentation, and communicating mathematically. Teachers play a multifaceted role requiring deep knowledge, pedagogical skill, and judgment. This work is challenging. Nevertheless, teachers, when supported, can take concrete, continual steps toward organizing this type of valued interaction in their classrooms. Though not prescriptive in nature, we hope this chapter provides a rationale, the beginning ideas, examples, and resources to undertake this important work.

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