

figures are triangles because they look the same” (Fuys, Geddes, and Tischler 1988). In the Triangle task presented in figure 1.1, Alison, the third-grade student whose work is shown in figure 1.2, selected only shape E, whereas Patrick, whose work is shown in figure 1.3, selected only shape F—after extending the sides. Patrick may have ignored shapes A, D, and E because they did not “fit” his image of a triangle. Both Patrick and Alison demonstrated level 1 thinking in relation to triangles. Triangles apparently had to have a specific shape and orientation to “fit” their notions of triangle. Angelina, whose work appears in figure 1.4, selected not only shapes A, D, and E but also—after adding a couple of lines—shape G. In fact, 7 of the 39 third graders included G (the chevron) in their selection of triangles. However, Angelina’s case is noteworthy because she drew two additional lines before she decided it was a triangle. In such a case, posing follow-up questions would be helpful: “Why did you choose shape G as a triangle, Angelina?” “Why did you draw these additional lines?”

For students in grades 3–5, focusing on attributes is important. Noting that triangles have three straight sides (connected at their endpoints) helps students move away from a focus on general appearance to a focus on properties. Students have developed mental images of triangles and associated properties that match their experiences. For example, many students identify as triangles only those shapes that have a side parallel to the bottom of the page on which they appear, or those with a “point” at the “top,” or those with all sides the same length. Adults and children alike often refer to mental images and the properties that they associate with an object when thinking about, creating, or evaluating a representation of that object. Tall and Vinner (1981) refer to this as a “concept image.”

If you find that you have students who have developed a concept image of a triangle that requires that all triangles have a point on top, then you need to design tasks and instruction that will address that faulty concept image. You must place consistent emphasis on the definition of a triangle as having three (straight) sides and give your students opportunities to consider examples such as shape E in figure 1.1, which is a triangle that does not have a “pointy top.”

To gain further insight into the different ways that students may view basic shapes, we designed the Rectangle task also shown in figure 1.1, reproduced here as figure 1.5. We gave this task to 127 students who had nearly completed grade 3, 4, or 5.

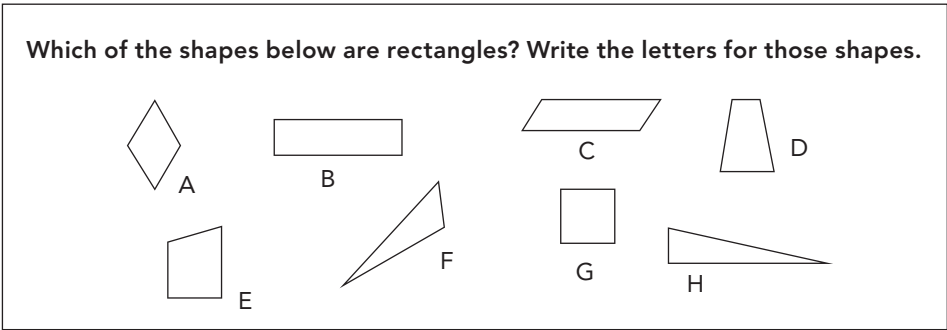


Fig. 1.5. Rectangle task, reproduced from figure 1.1

The Rectangle task elicited a wider variety of answers than the Triangle task; figure 1.6 shows the outcome. Examine these results, and respond to the questions in Reflect 1.3 to probe deeper into the students’ responses to this task.

Grade	Students choosing only shape B	Students choosing shapes B and C	Students choosing shapes B and G	Students choosing shapes B, C, and G	Other
3 (43 students)	16 (37%)	17 (40%)	4 (9%)	2 (5%)	4 (9%)
4 (35 students)	10 (29%)	5 (14%)	15 (43%)	2 (6%)	3 (9%)
5 (49 students)	16 (33%)	2 (4%)	31 (63%)	0	0
Total (127 students)	42 (33%)	24 (19%)	50 (39%)	4 (3%)	7 (6%)

Fig. 1.6. Students’ responses by grade level to the Rectangle task

Reflect 1.3

Examine the shapes presented in the Rectangle task in figure 1.5. How would you characterize the mathematical understandings and misunderstandings of the students who selected only shape B as a rectangle?

How would you characterize the mathematical understandings and misunderstandings of the students who selected shapes B and C?

**Reflect 1.3, *continued***

**How would you characterize the mathematical understandings and misunderstandings of the students who selected shapes B, C, and G?**

**What specific strategies would you use or questions would you pose to move these various students forward in their understanding?**

Providing your students with opportunities to examine examples and non-examples of specific shapes, followed by opportunities to participate in a facilitated discussion (or debate), will challenge their conceptions of the shapes. You can offer such opportunities through tasks that—

- involve images (such as the Rectangle task in fig. 1.5);
- require students to sort shapes; or
- require students to identify shapes in real-world settings (for example, the classroom) or pictures (from magazines, for instance).

Through these types of activities, you can determine which students are focusing only on the appearance and the mental or visual image of the shape (van Hiele level 1: Visual) and which students are analyzing figures in terms of their attributes and properties (van Hiele level 2: Descriptive/Analytic).

To help teachers distinguish student responses according to these two levels, Fuys, Geddes, and Tischler (1988, pp. 58–63) created tables that provide sample responses for each level. These examples can not only help you assess your students' responses in relation to the van Hiele levels of geometric development but also guide you as you select and design tasks and discussion questions. Figures 1.7 and 1.8 provide excerpts of Fuys, Geddes, and Tischler's tables for the first two levels. Compare and contrast the tables, and then respond to the questions in Reflect 1.4.