Introduction

This book is one of two designed to bring awareness to various international perspectives on mathematical topics taught throughout the world. Classroom activities herein showcase topics discussed at the International Congress on Mathematical Education (ICME–12) held in Seoul, South Korea, in July 2012.

ICME meets once every four years and is organized under the auspices of the International Commission on Mathematical Instruction (ICMI), an international body with representatives from around the world. The aim of ICME is to present current states and trends in mathematics education research and to examine current practices of mathematics teaching at all levels. Each ICME includes a wide range of participants, including mathematics education researchers, teacher trainers, practicing teachers, mathematicians, and others interested in mathematics education.

This book features activities for students ages thirteen through eighteen, while the companion book features activities for students ages five through thirteen. As we collected activities for the books, it became abundantly clear to us that different countries give different meanings to and have different ideas about what an "activity" is. These differences led to our massaging the submitted activities into a truly international model that has remained primarily true to the original while infusing a degree of commonality for the purposes of these books. Classroom-tested activities were sought to exemplify some standard or guideline in different countries. Where possible, those standards are noted along with the Common Core State Standards for Mathematics in the United States.

The general features of an activity include the following: mathematical content; materials needed; setting the scene for the activity, including both country and classroom; teacher notes; extensions; research notes with references; and activity sheets. These features vary, depending on the age of students. We have tried to keep the country flavor of the activities by including native languages in some instances and by using currency and measures of the country, or even symbolism and idiomatic language in others, if the language was important in the country of the writers.

Research notes in the activities place the mathematics topics in an international setting, with some sources in different languages. As editors, we have tried to make the research notes consistent throughout. What we found is that most activities can be centered in a wider setting than in a single country. What the user will see, however, is that some activities for one country would never be used in other countries at the age or grade level given but might be used either earlier or later.

The activities presented are ordered by age level in the country where they are used and by content if there is more than one activity in an age level. Activity sheets are available for download at NCTM's More4U website (www.nctm.org/more4u). Check the title page in this book for your access code.

The audience for the book is classroom teachers, teacher educators, math coaches, secondary school mathematics specialists, and those who provide professional development.

Lesson 1

Problem Solving with Mr. Splash

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MATH CONTENT

Graphs of authentic data

Linear functions

Linear measurement

MATERIALS NEEDED

Poster or digital pictures of Mr. Splash

Rulers (30 centimeter and 1 meter) and tape measures

Calculators, if available

Butcher paper or graph paper

Markers ("textas" in Australia)

A set of activity pages for each group of students

Setting the Scene

Country of Context

The Australian curriculum for mathematics (see www.australiancurriculum.edu.au) has four proficiency strands: understanding, fluency, reasoning, and problem solving. Within these strands, teachers can emphasize a breadth of mathematical actions. For example, the following description suggests what teachers might focus on in the problem-solving strand:

> Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify that their answers are reasonable. (Australian Curriculum Assessment and Reporting Authority [ACARA] 2012)

The problem-solving activity included here is a rich task suitable for students from grade 7 to grade 8 (ages twelve to thirteen).

Classroom Context

In Australian mathematics classrooms, tasks such as this one are regularly chosen and used, are open-ended, and are designed for a diverse range of learners. Such tasks have different entry points, allow all students to succeed in finding a solution or range of solutions, and provide learning experiences that extend all students' mathematical understanding.

Teacher Notes

Task A

To introduce the lesson, the teacher may show students the pictures of Mr. Splash that appear in figures 1.1 and 1.2 and read the following problem:

> "I have some photographs of Mr. Splash. I wonder if we can work out how tall he might be?"
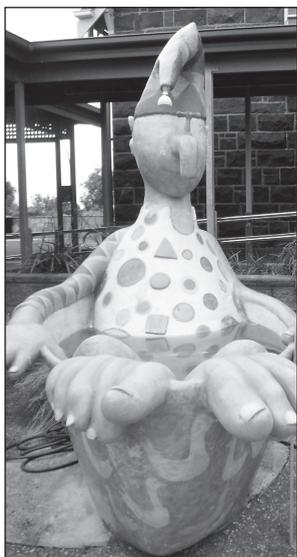
Fig. 1.1. Mr. Splash in bathtub
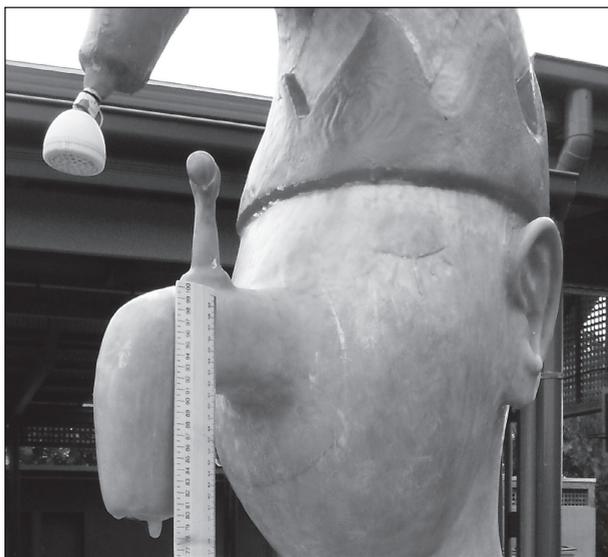
Fig. 1.2. Close-up of Mr. Splash

The teacher may then ask students to volunteer their predictions of the height of Mr. Splash and encourage them to explain how they decided on these predictions.

Additionally, the students may be asked to compare their predictions with their own heights and the heights of other students in the class. To help students determine class members' heights and measurements more accurately, the teacher should explain that in small groups, students are to measure their heights and some body dimensions, as shown in table 1.1.

Table 1.1. Data recording table

Name	Height (cm)	Head length (cm)	Arm span (cm)	Leg length (cm)	Foot length (cm)

Alternatively, if students have access to the data exploration software TinkerPlots (Konold and Miller 2005), they could record the data in the TinkerPlots program, as in figure 1.3.
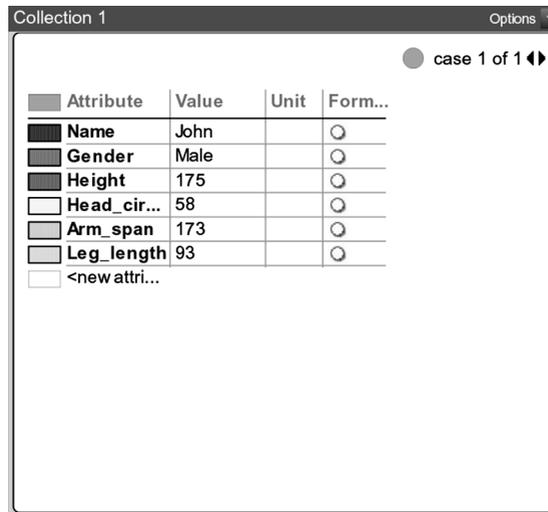

Fig. 1.3. TinkerPlots display of data

Before group work, the teacher should establish with the students how to measure each dimension and how to use measuring instruments correctly.

Depending on time, the teacher could allocate one relationship to each group to investigate and subsequently represent in a graph (e.g., height compared with head length, arm span, or leg length). Figure 1.4 shows a sample TinkerPlots graph displaying a relationship between height and foot length.
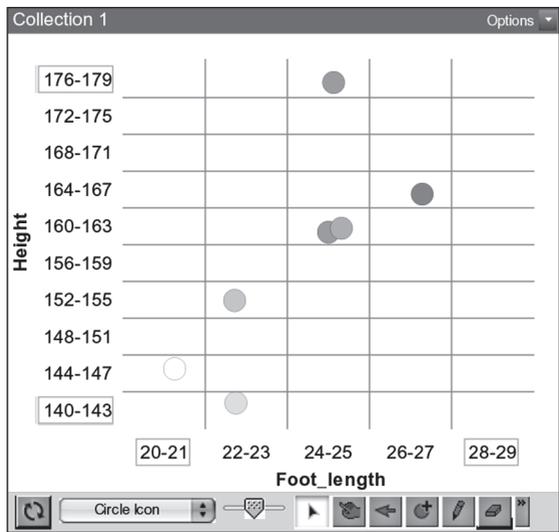
Fig. 1.4. TinkerPlots graph showing a relationship between height and foot length

After students have completed the graphing task, the teacher may ask them to compare their graphs and answer the following questions:

What conclusions can you make?

What is the relationship between height and each of the other measurements?

Is there a rule that describes the relationship?

The task may be enhanced by having groups that have plotted similar data collect all the information that they have in common and try to answer the same questions with those data. If the answers are not the same for the combined data and the individual sets, then the students might explain in writing why they think this difference occurred.

The teacher should take the opportunity to talk about sample spaces and how changing the sample space changes the data. This would also be an excellent opportunity to talk about the danger of trying to generalize beyond the sample data.

Task B

The teacher shows students the photo in figure 1.5 that displays a part of Mr. Splash's hand and gives them the activity page with the problem for task B:

"Below is a photograph of Mr. Splash's hand. What might be the measure of his hand span?"
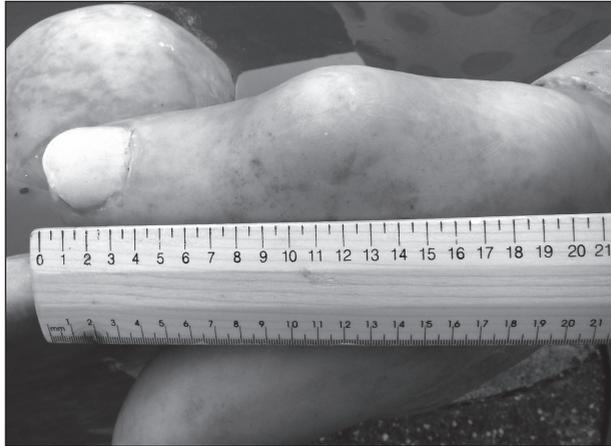
Fig. 1.5. Mr. Splash's hand

Working in groups of four, the students are asked to measure their thumb lengths and hand spans. The teacher may choose to establish guidelines for taking the measurements. The teacher may also suggest or discuss with the students how to record their data in the table on the activity page. If students have access to the TinkerPlots software, they might use it for recording and making a graph of the data. Otherwise students should construct a graph of hand span compared with thumb length manually.

Students should be asked to draw conclusions based on their sample, compare that sample with that of their classmates, describe any rule that could be inferred to describe the data, and make overall conclusions.

As an enhancement, the data in task B may be treated as in task A, where all groups combine their data into one set, draw graphs, and try to answer the questions. Again, the teacher has the opportunity to discuss overgeneralization from a limited sample.

Extensions

Extension activities include comparing the data collected from different classes, discussing or predicting what the differences would be, and using the Australian Bureau of Statistics website (www.cas.abs.gov.au) to collect data from the random sampler to make a comparison with a larger sample of students of similar age across Australia.

Research Notes

When group work begins, the teacher may observe and assist students who have difficulties. For example, a common measuring difficulty that students in Australia have is the following:

> "Zeroing" is a common difficulty that students experience with many measuring devices. For example, students may be unable to use a ruler to measure length to the nearest centimetre, because they do not align the 0 of the ruler with one end of the object to be measured. This problem is exacerbated by the fact that 0 is often not right at the end of the ruler. (State of Victoria Department of Education and Early Childhood Development 2012)

In the United States, the Common Core State Standards for Mathematics (CCSSM) for grade 8 state that students should "construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities" (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA Center and CCSSO] 2010, p. 56).

REFERENCES

Australian Curriculum Assessment and Reporting Authority (ACARA). *The Australian Curriculum: Mathematics V4.2.* Sydney, Australia: Australian Curriculum, Assessment and Reporting Authority, 2012. http://www.australiancurriculum.edu.au/Mathematics /Curriculum/F-10.

Konold, Clifford, and Craig D. Miller. TinkerPlots: Dynamic data exploration. [Computer software] Emeryville, CA: Key Curriculum Press, 2005.

National Governors Association Center for Best Practices and Council of Chief State School Officers (NGA Center and CCSSO). *Common Core State Standards for Mathematics. Common Core State Standards (College- and Career-Readiness Standards and K–12 Standards in English Language Arts and Math).* Washington, D.C.: NGA Center and CCSSO, 2010. http://www.corestandards.org.

State of Victoria Department of Education and Early Childhood Development. "Formal Units for Measuring: 2.25." East Melbourne, Victoria, Australia: State of Victoria Department of Education and Early Childhood Development, 2012. http://www.education.vic.gov.au/school/teachers/ teachingresources/discipline/maths/continuum/Pages/formalunits225.aspx.

Activity Page

Task A

Photographs of Mr. Splash are shown below. How tall might he be?

Mr. Splash in bathtub
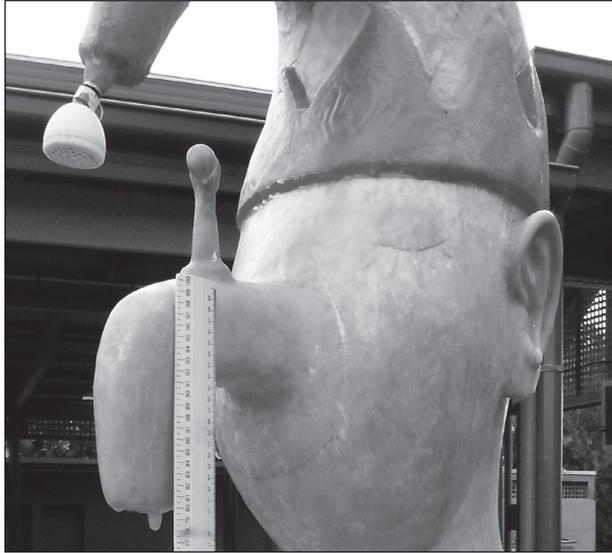
Close-up of Mr. Splash

- Explain how you made your predictions.
- Compare your predictions with those of your classmates.
- Working in your group, determine one another's heights and other requested measurements. Record your data in the table below.

Data recording table

Name	Height (cm)	Head length (cm)	Arm span (cm)	Leg length (cm)	Foot length (cm)

- Choose one set of measurements to investigate (for example, head length, arm span, or leg length), and then make a graph comparing your height data with those data.
- Based on your graphs, what tentative conclusions can you make?
- What is the relationship between height and the other measurements?
- What rule describes the relationship?
- Write a paragraph explaining whether you could apply your tentative conclusions, the relationship that you found, and the rule that you came up with to other individuals. (*Hint:* Think about the sample that you used to obtain this information.)

Task B

Below is a photograph of Mr. Splash's hand. What might be the measure of his hand span?
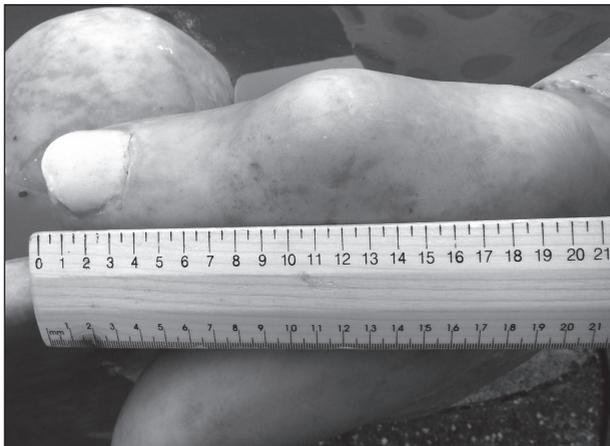
Mr. Splash's hand

- Working in your group, measure thumb lengths and hand spans, using the guidelines that the class agreed on for taking the measurements.
- Record your measures in the table below.

Data recording table for thumb lengths and hand spans

Name	Thumb length (cm)	Hand span (cm)

- Use the data in the table to make a graph of hand span compared with thumb length. (You may use technology if available.)

- What conclusions can you draw?

- What is the relationship between thumb length and hand span?

- What rule describes the relationship?

- Write a paragraph describing the limitations of making generalizations with small samples of data.