Preface

Until relatively recently, the study of algebra was reserved for college-intending high school students. Other high school students were relegated to the "general math" track or to courses that provided a review of grades 4–8 arithmetic. Different political and educational professional groups have pointed out that this practice of "tracking" students at the high school level serves to exacerbate inequities between ethnic and socioeconomic groups. One proposal to reduce these inequities was to have all students study algebra. From this proposal was born the "algebra for all" movement. School districts nationwide began to require first-year algebra of all their students in either grade 8 or 9, and the completion of that course became a condition for high school graduation.

Shortly thereafter, articles began to appear in the popular press describing the poor performance of students in algebra. Several districts claimed that failure in first-year algebra was the major contributor to school dropout. What could be causing this failure?

Some mathematics educators suggested that students' difficulties with algebraic ideas could be attributed to a lack of preparation for the study of algebra. They claimed that the elementary school curriculum, with its emphasis on arithmetic, did not treat aspects of algebra that would better prepare students for the formal study of algebra.

Recognizing the importance of early exposure to fundamental ideas of algebra and the need for the development of those ideas to be well articulated across grade levels, the National Council of Teachers of Mathematics (NCTM) dedicated its 1988 Yearbook, *The Ideas of Algebra K–12,* to the teaching of algebra. A year later, NCTM published *Curriculum and Evaluation Standards for School Mathematics,* in which algebra was identified as one of the core content strands for exploration by students in grades K–12. In 2000, NCTM published *Principles and Standards for School Mathematics,* which reaffirmed the need for well-articulated instruction in algebra across grades beginning at the prekindergarten level. Accompanying each of the *Standards* documents were books (the Addenda series for the 1989 *Standards* and the Navigations series for the 2000 *Standards*) designed to illustrate how the goals cited in the *Standards* documents could be achieved in school mathematics programs.

Unfortunately, many attempts to prepare students better for algebra have not resulted in greater achievement in first-year algebra. Students in grades 8 and 9 are still struggling with important algebraic concepts and skills, and many are discontinuing their study of higher-level mathematics because of their lack of success in the course. At the elementary school level, teachers are not clear about what algebraic ideas they should explore with their students and what approaches and

pedagogical methods they should employ. The need to attend to algebra in the elementary school curriculum is highlighted in the charge given to the National Mathematics Advisory Panel, appointed by the president of the United States in 2006. According to the document, the panel is to identify "the skills needed for students to learn algebra and to be ready for higher level mathematics" and to consider better ways of delivering instruction.

The Seventieth Yearbook is designed to bring us up to date on the status of algebra in our schools and, in particular, the history of changes in the algebra curriculum; the current situation in enrollment in, and success with, courses in algebra; the nature of algebra and algebraic thinking; research into the teaching and learning of algebra; classroom strategies for developing students' algebraic thinking abilities and their understanding of some of the big ideas of algebra; and methods for preparing or updating teachers to teach algebra. The Yearbook is separated into five parts.

Part 1 focuses on historical perspectives on algebra in the curriculum. Kilpatrick and Izsák trace the journey of algebra instruction from the years when it became a course option for students at the university level to its entrance into high school and then middle school mathematics programs. They describe changes in the algebra curriculum from a focus on equations to that of functions. Chazan considers the current status of algebra in our schools and, in particular, who studies algebra, when they study the subject, what they study, and how their learning is assessed.

Part 2 considers the nature of algebra and algebraic thinking. Arcavi leads the reader through a variety of tasks aimed at helping students see the purpose of algebra and of ways to use algebra to model, analyze, and "demystify" mathematical problems. Cuoco identifies "general purpose methods," or tools that students can use to investigate and gain deeper understanding of important algebraic ideas. Using vignettes illustrating difficulties students have with algebra, Saul advances the idea that algebra is not simply the study of variables and functions; rather, algebra is a tool that can be used to gain greater understanding of major mathematical ideas. Bezuszka and Kenney focus on recursive thinking as one aspect of algebraic thinking and on recursion as a core concept of algebra. They illustrate both topics with different types of problems. Banchoff considers the nature of algebraic thinking by demonstrating its connections with geometric thinking, and he stresses the value of using both types of reasoning to solve mathematical problems.

Part 3 addresses research on the learning of algebra and how topics from traditional mathematics programs can be modified or extended to develop students' algebraic thinking. Warren and Cooper describe tasks, problems, and sets of questions they used to help young children distinguish repeating from growing patterns, identify relationships between sets of data, and generalize relationships. Fujii and Stephens present vignettes that show that students in grades 2 through 8 naturally employ algebraic types of strategies to the solution of number sentences that contain variables. Baek describes elementary school students' multiplication techniques

and points out how those techniques involve the application of algebraic generalizations. Moss, Beatty, Barkin, and Shillolo describe their study of the effects of online conferencing on fourth-grade students' abilities to solve complex function problems. Cai and Moyer present results of international studies that compare how educators in the United States, Singapore, and China approach and treat the development of algebraic thinking of elementary school students.

Part 4 focuses on classroom practices that have proved successful in developing students' algebraic reasoning abilities and their understanding of important ideas of algebra. Watanabe gives a Japanese perspective on the development of algebraic thinking in the elementary school—a perspective in which algebraic ideas are explored in conjunction with the study of addition, multiplication, and proportions. Ferrucci, Kaur, Carter, and Yeap offer a detailed description of the "model" method used in the Singapore mathematics program and the value of this type of representation of mathematical relationships in the identification of problem structures and the solution of algebra problems. To engage their middle school students in the study of variables, Gay and Jones use the context of an amusement park to help their students understand what variables represent, how they can be used to construct equations, and how they can be manipulated. Using variations of a toothpick problem, Tabach and Friedlander describe how such a context-based approach to algebra can facilitate the understanding of variables in expressions, the difference between changing and constant quantities, and the equivalence of algebraic expressions. Anderson describes a sequence of problems and activities she used with her middle school students to develop their understanding of linear relationships and, in particular, the equation $y = mx + b$. This part of the Yearbook concludes with Zbiek and Heid's examination of ways to use technology (computer algebra systems, dynamical geometry construction environments, and graphing calculators) to facilitate students' understanding of important concepts and functions in intermediate algebra.

Part 5 presents strategies employed for preparing and updating teachers to develop their students' algebraic thinking abilities and their knowledge of algebra. Schifter, Bastable, Russell, Seyferth, and Riddle use case studies of elementary school students in the process of developing their algebraic thinking abilities as the centerpiece of a professional development program for teachers. Recognizing the importance of having teachers grow in their own mathematical abilities prior to—or while—working with their students, Billings focuses on four pictorial growth pattern tasks that she uses with preservice and in-service teachers to develop their algebraic thinking talents. Herbel-Eisenmann and Phillips describe a professional development program in which teachers' pedagogical content knowledge of algebra is developed in concert with analyses of their students' work.

On behalf of the Seventieth Yearbook Editorial Panel, I thank the authors for their outstanding contributions, for their willingness to respond to all requests for additional information "ASAP," and for their dedication to mathematics education.

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Carole E. Greenes
Seventieth Yearbook Editor