Introduction

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At the heart of your work as teachers of mathematics in high school is the development of student self-efficacy. *Student self-efficacy* references students' *belief* in their capability to learn the mathematics we *need them to know* by the end of each grade.

But what exactly *does* a high school mathematics student need to know by the end of each unit of study throughout each course? And, more important, how does the teacher develop his or her personal teacher self-efficacy to adequately plan for and then deliver on the promise of the mathematics standards for those mathematics units of study to students?

I have been trying to answer this question my entire professional life.

In 1987, I coauthored my first mathematics textbook: a geometry book for high school students who found mathematics a difficult subject. It was my first real experience in taking a wide body of content for the complete school year and breaking the standards down into reasonable chunks for every teacher and student to learn.

As I eventually expanded my textbook writing to include K–12 mathematics students and teachers, I realized the time spent teaching these manageable chunks of content could vary in length from twenty to thirty-five days, and they often had names like *units* or *chapters* or *modules*. As you know, mathematics is a vertically connected curriculum, and the units of study at each grade level cannot be taught in a random order; they need to happen in the right place and the right time in the story arc for each grade level. There is an order to the flow of the high school mathematics content story. And, as high school teachers, we need to fully understand the *how* and the *why* of the content trajectories across these grades.

I have spent over half of my life trying to get these story arcs correct—trying to create textbooks that would make contextual sense to both the student and the teacher. In a way, I wanted to help students and teachers develop their self-efficacy to learn mathematics.

Now for more than a decade, and with the help of our incredible team of mathematics thought leaders and professional developers, lead author of this *Mathematics Unit Planning in a PLC at Work®* series Sarah Schuhl and I realize every teacher and teacher team of high school mathematics needs to work collectively with their textbook and other resources to *own* the planning process for each unit of study.

Developing *collective teacher efficacy* is at the heart of the Professional Learning Community (PLC) at Work process, defined as "Social interactions firmly anchored in instructional practice [that] can move teachers beyond contrived collegiality to a culture that can in turn influence a teacher's sense of efficacy" (Neugebauer, Hopkins, & Spillane, 2019, p. 13). However, Sabina Neugebauer, Megan Hopkins, and James Spillane (2019) point out that social team interactions must be "anchored in actual teaching and assessing episodes" (p. 13). Teachers then place those episodes into manageable chunks of content for their teams' discussion and work.

As we enter the 2020s, it is interesting to observe the integrated nature of how the high school mathematics *content* standards have shifted and changed, and also how the pedagogical *process* standards have become more integrated. To some extent, these changes are best viewed through the lens of *essential learning standards* for every high school student, and from the vantage point of each state's expectations for college and career readiness.

Going back to 2000, the National Council of Teachers of Mathematics (NCTM) describes *integrated mathematics* as a term used in the United States to connect topics or strands of mathematics throughout *each year* of secondary school. So what has changed in the past twenty years since NCTM's 2001 observation? Quite a bit, it turns out. The story arc for each year of high school mathematics is more integrated today than ever before, regardless of how we choose to name high school mathematics courses.

In making sense of the high school mathematics standards, we have tended to get a bit too entangled in the details. We think at the "139 standards level" about what we must teach students during the three-year college-readiness sequence. In most states, students reach proficiency of these standards through defined courses like algebra 1, geometry, and algebra 2. In some cases, there might be other names for these courses, such as math I, math II, and math III, or perhaps year 1, year 2, and year 3.

No matter what we call these high school courses, it is best to look at the approximately 139 learning targets through the lens of an ongoing set of *integrated standards* built around the primary essential learning standards developed throughout *every* unit in each high school mathematics course. These essential learning standards are built around five domains, which also each include modeling: number, algebra, functions, geometry and measurement, and statistics and probability.

Notice that each of these six essential categories are not courses per se; rather, they represent an integrated set of standards that teacher teams package into a progression of topics to create a scope and sequence story arc that makes sense as students pass through each high school course. For example, whether we call the courses *year 1*, *math I*, or *algebra 1*, a first-year high school mathematics course will integrate authentic mathematical modeling standards and mathematical tasks, statistics that integrate bivariate data sets to linear function connections, and algebraic equations as an equality of two functions.

Thus, teams integrate high school students' learning experiences through a flow of mathematical tasks that are both higher level and lower level in cognitive demand, for almost every daily learning target we teach. There is a significant difference between asking high school mathematics year 1 students to determine $f(x) = 500(1.015)^x$ at $x = 0$ versus asking students to compare and contrast the graphs of $f(x) = 500(1.015)^x$ and $g(x) = 500(1.021)^x$. Both of these mathematical tasks are important skills, yet they require varying demonstration levels of understanding.

The verbs most states' standards use, such as *create*, *understand*, *build*, *compare*, *describe*, and *justify*, indicate we are to present our high school students with an integrated *learning experience* as well. Thus, there is a pedagogical learning expectation that sometimes students learn by observing teacher thinking, taking notes, and answering teacher questions, also known as *whole-group discourse*, during the mathematics lesson. Yet sometimes students are to experience learning a particular standard through small-group discourse tasks, working with peers as the teacher provides formative feedback, and following prompts for perseverance on carefully chosen mathematical tasks. The expectation is that both whole-group and small-group discourse are integrated into every unit of mathematics.

For students to fully learn and demonstrate understanding of mathematics in each of the high school courses, a balanced use of technology is expected. It will be very difficult in the 2020s not to integrate technology into the high school mathematics curriculum. As students learn to explore mathematical models and apply various statistics and functions, including transformations, into this integrated curriculum, they will need proficiency in the use of appropriate graphing, exploration, and statistical tools for the primary college- and career-readiness mathematics courses in your department.

We have chosen in this book to reference the high school mathematics curriculum within the more widely used and understood algebra 1, geometry, and algebra 2 course names. These course names do not sound like they promote an integrated mathematics program. However, in this era, the high school mathematics curriculum is integrated, no matter the name you choose to give each year of study as the standards of functions and statistics permeate all years of study.

My coauthors of this Mathematics at Work unit planning guide for high school—Sarah Schuhl, Bill Barnes, Darshan Jain, Matt Larson, and Brittany Mozingo—and I serve or have served in many mathematics teaching and leading roles. One such role is to serve on our Mathematics at Work team of national thought leaders. As we travel around the United States helping high school teachers improve student learning in mathematics, those teachers often ask us, "How do we collectively plan for a unit of study in mathematics at our grade level?"

Answering that question is the purpose of this book.

The Purpose of This Book

We want to help your grade-level team learn *how to* work together to perform the following seven collaborative tasks for each unit of mathematics study throughout the year.

Generate Essential Learning Standards for Each Unit

Unwrap standards into daily learning targets and write those standards in student-friendly language for essential learning standards. Then use those essential learning standards to drive feedback on common mathematics assessments, classwork, independent practice, and intervention as a collaborative team.

Create a Team Unit Calendar

Decide the number of days needed to teach each essential learning standard, and the start and end dates of the unit. Decide the dates to administer any common mid-unit or end-of-unit assessments. Establish each date the team will analyze data from any common mid-unit and end-of-unit assessments to plan a team response to student learning.

Identify Prior Knowledge

Determine and identify the recent prerequisite content knowledge students need to access the grade-level learning in each unit of study. Decide which mathematical activities (tasks or prompts) to use for students to connect the prior knowledge at the start of each lesson throughout the unit. Use these activities to discern student readiness and entry points into each lesson.

Determine Vocabulary and Notations

Identify the academic vocabulary students will be reading and using during discourse throughout the unit. Identify any mathematical notation students will need to read, write, and speak during the unit.

Identify Resources and Activities

Determine which lessons in the team's current basal curriculum materials align to the essential learning standards in the unit. Determine examples of higher-level and lower-level tasks students must engage in to fully learn each essential learning standard.

Agree on Tools and Technology

Determine any manipulatives or technology needed to help students master the essential learning standards of the unit. Identify whether the tools or technology needed in the unit will support student learning of the essential learning standards with a focus on conceptual understanding, application, or procedural fluency. Identify which tools and technology, if any, will be part of instruction or available as a resource for common assessments.

Record Reflection and Notes

When planning the unit, record notes of things to remember when teaching. Do so by answering, for example, these questions: When should students use technology to foster learning? How will students develop and express their understanding of transformations on a coordinate plane, functions, proofs, statistics, and equations? What are the expectations for quality student work? Which mathematical strategies should teachers use throughout the unit? After the unit, reflect on instruction and assessments to keep or change for next year, and record ideas to use when planning the unit for next year.

The Parts of This Book

Part 1 provides detailed insight into how your mathematics team can effectively enact these seven planning tasks for the essential standards you expect students to learn in grades 9, 10, and 11.

Part 2 provides three detailed model units and describes a function-transformation story arc for a unit in algebra 1, geometry, and algebra 2. The units connect foundational elements of transformation with quadratic functions, geometric functions, and trigonometric functions, as students move through three years of a college-readiness mathematics sequence. We hope part 2 provides an inspiring model for your high school mathematics course-based teams.

The epilogue shares an example for how to organize your course-based team's work on a unit-by-unit basis so your mathematics department can grow and learn from its work in future years. If your collaborative team does not already have a mathematics unit of study yearlong plan with standards, appendix A provides a proficiency map protocol as a way to organize your standards and to determine when students should be proficient with each standard. Finally, appendix B contains a team checklist and questions for your team to answer as you plan each mathematics unit. Appendix B summarizes the elements of unit planning shared in parts 1 and 2 of this book and is intended to be a quick reference to guide the work of your team in your unit planning.

A Final Thought

You might wonder, "Why is this book titled *Mathematics Unit Planning in a PLC at Work, High School*?" In 1980, my second mathematics teaching job landed me on the doorstep of an educational leader who would later start an education movement in the United States that would spread throughout North America and even worldwide. He, along with Robert Eaker, was the architect of the Professional Learning Communities at Work movement and my principal for many years. Dr. Richard DuFour expected every grade-level or course-based team in our school district to answer four critical questions for each unit of study in mathematics (DuFour, DuFour, Eaker, Many, & Mattos, 2016).

1. What do we want all students to know and be able to do? (essential learning standards)
2. How will we know if they know it? (lesson design elements, assessments, and tasks used)
3. How will we respond if they don't know it? (formative assessment processes)
4. How will we respond if they do know it? (formative assessment processes)

As your collaborative team pursues this deep work, remember it all begins with a robust and well-planned response to PLC critical question one (What do we want all students to know and be able to do?). That is the focus of our high school unit planning book.

We want to help you plan for and answer the first question for each high school mathematics unit, for each course (regardless of the names you choose for these courses), and for every student. We wish you the best in your mathematics teaching and learning journey, *together*.