

Mathematics Curriculum Issues, Trends, and Future Directions

Seventy-second Yearbook

Barbara J. Reys

*Seventy-second Yearbook Editor
University of Missouri—Columbia
Columbia, Missouri*

Robert E. Reys

*Seventy-second Yearbook Editor
University of Missouri—Columbia
Columbia, Missouri*

Rheta Rubenstein

*General Yearbook Editor
University of Michigan—Dearborn
Dearborn, Michigan*



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Copyright © 2010 by
THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, INC.
1906 Association Drive, Reston, VA 20191-1502
(703) 620-9840; (800) 235-7566; www.nctm.org
All rights reserved

ISSN 0077-4103
ISBN 978-0-87353-643-1

The National Council of Teachers of Mathematics is a public voice of mathematics education, supporting teachers to ensure equitable mathematics learning of the highest quality for all students through vision, leadership, professional development, and research.

For permission to photocopy or use material electronically from *Mathematics Curriculum: Issues, Trends, and Future Directions*, please access www.copyright.com or contact the Copyright Clearance Center, Inc. (CCC), 222 Rosewood Drive, Danvers, MA 01923, 978-750-8400. CCC is a not-for-profit organization that provides licenses and registration for a variety of users. Permission does not automatically extend to any items identified as reprinted by permission of other publishers and copyright holders. Such items must be excluded unless separate permissions are obtained. It will be the responsibility of the user to identify such materials and obtain the permissions.

The publications of the National Council of Teachers of Mathematics present a variety of viewpoints. The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of the Council.

Printed in the United States of America

Contents

Preface	ix
----------------------	-----------

Introduction to the CD Collection: Classic Publications on the Mathematics Curriculum	1
--------------------------------------------------------------------------------------------------------	----------

Thomas A. Romberg

University of Wisconsin (Retired), Madison, Wisconsin

I. Curriculum Matters: Looking Back, Looking Forward	23
-------------------------------------------------------------------	-----------

1. The Current State of the School Mathematics Curriculum	25
-----------------------------------------------------------------	----

Zalman Usiskin

University of Chicago, Chicago, Illinois

2. Technology and the Mathematics Curriculum	41
----------------------------------------------------	----

James T. Fey

University of Maryland, College Park, Maryland

Richard M. Hollenbeck

University of Maryland, College Park, Maryland

Jonathan A. Wray

Howard County Public School System, Ellicott City, Maryland

3. National Standards: Lessons from the Past, Directions for the Future.....	51
---------------------------------------------------------------------------------	----

Margaret E. Goertz

University of Pennsylvania, Philadelphia, Pennsylvania

4. Recommendations for Statistics and Probability in School Mathematics over the Past Century	65
--------------------------------------------------------------------------------------------------------	----

Dustin Jones

Sam Houston State University, Huntsville, Texas

James E. Tarr

University of Missouri, Columbia, Missouri

5. Reflections on Five Decades of Curriculum Controversies	77
------------------------------------------------------------------	----

Stephen Willoughby

University of Arizona (Retired), Tucson, Arizona

II. The Intended Curriculum87

6. What We Teach Is What Students Learn: Evidence from
National Assessment89
Peter Kloosterman
Indiana University, Bloomington, Indiana
Crystal Walcott
*Indiana University Purdue University—Columbus,
Columbus, Indiana*
7. Curriculum Alignment in an Era of Standards and
High-Stakes Testing103
Shannon W. Dingman
University of Arkansas, Fayetteville, Arkansas
8. Preschool Mathematics Curricula115
Julie Sarama
*University of Buffalo—State University of New York, Buffalo,
New York*
Douglas H. Clements
*University of Buffalo—State University of New York, Buffalo,
New York*

III. The Written Curriculum127

Curriculum Development129

9. Supporting Focused and Cohesive Curricula through Visual
Representations: An Example from Japanese Textbooks131
Tad Watanabe
Kennesaw State University, Kennesaw, Georgia
Akihiko Takahashi
DePaul University, Chicago, Illinois
Makoto Yoshida
William Paterson University, Wayne, New Jersey
10. Cross-National Curriculum Collaboration: Examples Based
on Realistic Mathematics Education145
Margaret R. Meyer
University of Wisconsin—Madison, Madison, Wisconsin
Truus Dekker
Freudenthal Institute (Retired), Maarssen, Netherlands
Frank Eade
Manchester Metropolitan University, Manchester, England

11. Three Perspectives on the Central Objects of Study for Grades Pre-K–8 Statistics	157
Randall E. Groth	
<i>Salisbury University, Salisbury, Maryland</i>	
12. Designing Curricula to Expand and Extend Mathematical Knowledge	171
Debra I. Johanning	
<i>University of Toledo, Toledo, Ohio</i>	
13. Mathematics Applied to Curriculum Development: Lessons Learned on the Job	181
Al Cuoco	
<i>Education Development Center, Newton, Massachusetts</i>	
Jean Benson	
<i>Education Development Center, Newton, Massachusetts</i>	
Bowen Kerins	
<i>Education Development Center, Newton, Massachusetts</i>	
Sarah Sword	
<i>Education Development Center, Newton, Massachusetts</i>	
Kevin Waterman	
<i>Education Development Center, Newton, Massachusetts</i>	
Selection of Textbooks	197
14. How Do Districts Choose Mathematics Textbooks?	199
June Mark	
<i>Education Development Center, Newton, Massachusetts</i>	
Deborah Spencer	
<i>Education Development Center, Newton, Massachusetts</i>	
Julie Koehler Zeringue	
<i>Education Development Center, Newton, Massachusetts</i>	
Katherine Schwinden	
<i>Education Development Center, Newton, Massachusetts</i>	
15. Considerations in the Review and Adoption of Mathematics Textbooks	213
Rick A. Hudson	
<i>Indiana University, Bloomington, Indiana</i>	
Paula Elmer Lahann	
<i>Indiana University, Bloomington, Indiana</i>	
Jean S. Lee	
<i>Indiana University, Bloomington, Indiana</i>	

16. Curriculum as a Change Agent: High Schools That Rise to the Challenge and What They Stand to Gain	231
Kasi Allen-Fuller <i>Lewis and Clark College, Portland, Oregon</i>	
Margaret Robinson <i>Ithaca College, Ithaca, New York</i>	
Eric Robinson <i>Ithaca College, Ithaca, New York</i>	
IV. The Implemented Curriculum	247
17. Myths about Curriculum Implementation	249
Denisse R. Thompson <i>University of South Florida, Tampa, Florida</i>	
Sharon L. Senk <i>Michigan State University, East Lansing, Michigan</i>	
18. Technology and the Teaching of Mathematics	265
Richard M. Hollenbeck <i>University of Maryland, College Park, Maryland</i>	
Jonathan A. Wray <i>Howard County Public School System, Ellicott City, Maryland</i>	
James T. Fey <i>University of Maryland, College Park, Maryland</i>	
19. Understanding Teachers' Strategies for Supplementing Textbooks	277
Corey Drake <i>Iowa State University, Ames, Iowa</i>	
20. Teachers' Perspectives on Fidelity of Implementation to Textbooks	289
Mary Ann Huntley <i>Cornell University, Ithaca, New York</i>	
Kathryn Chval <i>University of Missouri—Columbia, Columbia, Missouri</i>	
V. Impact of Curriculum Materials on Students' and Teachers' Learning	305
21. Developing Curricular Reasoning for Grades PreK–12 Mathematics Instruction	307
M. Lynn Breyfogle <i>Bucknell University, Lewisburg, Pennsylvania</i>	

	Amy Roth McDuffie	
	<i>Washington State University—Tri-Cities, Richland, Washington</i>	
	Kay A. Wohlhuter	
	<i>University of Minnesota, Duluth, Minnesota</i>	
22. Secondary School Mathematics Curriculum Materials as Tools for Teachers' Learning		321
	Gwendolyn M. Lloyd	
	<i>Pennsylvania State University, University Park, Pennsylvania</i>	
	Vanessa R. Pitts Bannister	
	<i>Virginia Tech, Blacksburg, Virginia</i>	
23. Conducting Mathematics Curriculum Research: Challenges and Insights		337
	Paul Kehle	
	<i>Hobart and William Smith Colleges, Geneva, New York</i>	
	Kelly K. McCormick	
	<i>University of Southern Maine, Gorham, Maine</i>	
24. The Influence of Curriculum on Students' Learning.....		351
	Mary Kay Stein	
	<i>University of Pittsburgh, Oakmont, Pennsylvania</i>	
	Margaret S. Smith	
	<i>University of Pittsburgh, Oakmont, Pennsylvania</i>	

Preface

Mathematics Curriculum: Issues, Trends, and Future Directions

Why was *mathematics curriculum* chosen as a theme for the Seventy-second NCTM Yearbook? One explanation might be that mathematics curriculum—

... was chosen as the central theme because of the present interest in curriculum revision. Since it should be understood that such revision ought to be a continuous process, the discussions herein presented are not final. However, they furnish a basis that will help us to find better ways of determining how the proper content should be selected, arranged, and presented. (Reeve 1927, p. vii)

This is a timely response today, just as it was when it was originally stated by William D. Reeve in 1927 in the preface to the Second NCTM Yearbook, *Curriculum Problems in Teaching Mathematics*. It is a reminder that mathematics curriculum has long been a topic of keen interest in mathematics education. Some things don't change!

One thing is certain today, just as it has been for many decades. Mathematics curriculum remains a central issue in efforts to improve mathematics learning opportunities for students. Although times change, society changes, and people change, for many it is difficult to accept change in the mathematics curriculum. Terms such as *basic*, *old*, *new*, *modern*, *antiquated*, *traditional*, *conservative*, *liberal*, *contemporary*, and *reform* are commonly used in society. In fact, these terms are frequently mentioned in the context of mathematics curriculum. Some things don't change!

Returning to history, we find that the developers of the Second Yearbook tried to provide a balanced view of mathematics curriculum, as reflected in their statement—

The Committee tried to obtain contributors holding many different points of view and representing as widely separated sections of the country as possible. The result can be labeled neither liberal nor conservative. (Reeve 1927, p. vii)

Nearly a century ago, the Yearbook Editorial Committee believed that it needed to provide a balance between whatever was considered conservative and liberal. Some things don't change!

Most controversy about mathematics curriculum centers on either the need for change or the lack of change. The need for change, along with visions for change, has long been reflected by NCTM in such publications as *An Agenda for Action* (1980), *Curriculum and Evaluation Standards for School Mathematics* (1989), *Principles and Standards for School Mathematics* (2000), *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (2006), and *Focus in High School Mathematics: Reasoning and Sense Making* (2009). These publications have called for curricular change, and several have been met with mixed reactions. Although many strongly support the vision of school mathematics outlined by NCTM, some think the recommendations have gone too far in outlining changes, whereas others think they have not gone far enough.

Change in any period of time is difficult and creates challenges. In schools, mathematics curriculum change affects teachers, students, administrators, and parents. Change, particularly for parents, teachers, and administrators, has always been difficult. This concern is documented in one of the papers in the Fourth NCTM Yearbook (Reeve 1929, p. 132):

Tradition has been a hard factor to overcome in modernizing the curriculum in mathematics. The difficulty being largely a matter of clinging to the hazy and invalid objectives used in teaching the mathematics of many generations ago.

Tradition continues to maintain inertia that is difficult to overcome in changing mathematics curriculum. As a result, debates about the direction of mathematics curriculum in schools and cities throughout the world continue to this day. Some things don't change!

This Yearbook continues in a long line of NCTM Yearbooks, dating back to 1927, that have addressed various facets of the changing mathematics curriculum. Although some factors such as tradition can inhibit significant change, other factors such as policy (e.g., federal No Child Left Behind legislation), societal needs (mathematically literate graduates), and technological advances (computer software, calculators) foster and accelerate the need for change.

This Yearbook, *Mathematics Curriculum: Issues, Trends, and Future Directions*, was developed during a period of major curriculum change. The past two decades have seen an era of unprecedented mathematics curriculum development across grades K–12. In the past year alone, a major state-initiated process for developing “common core standards” is underway (NGA and CCSSO 2009). With forty-eight states (all but Texas and Alaska) and several territories participating in the articulation of “college and career-ready” high school graduation expectations and common grades K–12 standards in mathematics and language arts,

2010 promises to be a landmark year of discussion and dialogue about mathematics curriculum.

This Yearbook reflects some of the many issues that the field is currently discussing, so it serves as both a record of current advances and a summary of challenges regarding curriculum. We hope that it will both guide and stimulate thinking about where we have been, where we are, and where we need to go.

The Yearbook is organized to acknowledge the various forms of curriculum that shape the grades K–12 mathematics program, including the following:

The Intended Curriculum—Curriculum authorities at the local, state, and national level specify particular learning expectations, often delineated by grade, for school mathematics instruction. Often called “curriculum standards,” these learning expectations furnish guidance regarding what should be taught and when various mathematical content and processes should receive emphasis in the school program. They also guide the development of textbooks and assessments designed to monitor school programs.

The Written (Textbook) Curriculum—Publishers use curriculum standards to design textbooks and other instructional materials to implement the intended curriculum. These materials include textbooks typically developed to support the day-to-day teaching of mathematics over a school semester or academic year of study. They also include modules focusing on smaller amounts of mathematical content, workbooks, and computer software.

The Implemented Curriculum—Individual teachers make decisions every day regarding if and how they will use district-adopted curriculum materials. Therefore students, for example, using the same textbook may, in fact, have differing opportunities to learn mathematics. The implemented curriculum refers to the mathematics that students have an opportunity to learn, which is often a function of the district-adopted textbook and the individual teacher’s preferences.

Together, these different forms of curriculum have a direct impact on teachers’ decisions and students’ opportunities to learn. Each has been a focus of intense work over the past three decades (since the publication of *An Agenda for Action* in 1980) as each has served as a lever for school improvement.

So here we are today, one-tenth of the way through the twenty-first century, wrestling to improve students’ learning opportunities. This Yearbook includes articles focusing on the full range of curriculum issues, as well as articles that offer insight into the impact of curriculum on students’ and teachers’ learning. Specifically, the articles are organized into the following sections:

Curriculum Matters: Looking Back, Looking Forward

The Intended Curriculum

The Written Curriculum

Curriculum Development

Textbook Selection

The Implemented Curriculum

The Impact of Curriculum Materials on Students' and Teachers'
Learning

Access to Historical Documents on Mathematics Curriculum

In addition to sharing information and provoking discussion with the printed articles in this Yearbook, the Editorial Panel, under the direction of Tom Romberg, has assembled a set of sixty-eight articles, chapters, and relevant publications provided in an accompanying CD. These reflect issues and trends in the mathematics curriculum in the United States and Canada for the period 1843–1993. The articles are organized by time period: nineteenth and early twentieth century, mid-twentieth century, modern math era, post-modern math era, and standards-based era. (No articles more recent than 1993 were considered for inclusion in the CD.) For each era, the features of schools are briefly described. Then, as warranted, each era's documents have been organized by issues or topics of concern at that time, such as arithmetic, college-preparatory mathematics, and mathematics for all.

The Editorial Panel strove to set forth a broad, meaningful set of resources on the CD to offer significant perspectives on mathematics education curriculum philosophy and history. For the past century and a half, a constant dilemma faced by those choosing the mathematical content for the school mathematics curriculum has involved how one caters to the needs of the college-bound students who will study mathematics at universities (particularly those who will become professional mathematicians) and at the same time teaches other students the mathematical skills they will need to be productive citizens in a changing society. The selected articles reflect how each era dealt with this dilemma on the basis of the structure of schools and the social issues of that era.

Summary

For more than a century, mathematics curriculum has been changing, and these changes have generated much discussion. In fact, the past thirty years have wit-

nessed an unprecedented focus on school mathematics in the United States. In 2009–2010, the momentum is building, with the “common core standards” initiative (NGA and CCSSO 2009) well underway. Prompted by national reports and international assessments, attention has focused on the need to raise the quality of school mathematics programs for grades K–12 students to be more successful so as to compete in the global economy. Curriculum has been central to many of the recent school mathematics improvement efforts. As a result, grades K–12 students are studying more mathematics, often at an early grade, with a focus on conceptual understanding as well as skill development and problem solving. We hope the discussion stimulated by this Yearbook will advance our thinking and support continued reflection and productive work on the mathematics curriculum.

Acknowledgments

This Yearbook resulted from an open call for papers. On behalf of the Editorial Panel, we thank the authors for their contributions and working with us throughout the review process. Their willingness to accept suggestions, respond to queries, and meet deadlines was both exemplary and greatly appreciated. Behind the scenes and shaping this Yearbook have been authors, reviewers, members of the Editorial Panel, the NCTM staff, the General Editor, and the Yearbook coeditors all working together to write, review, edit, revise, lay out, and produce a set of papers addressing a wide range of issues related to mathematics curriculum.

This is the last in a series of NCTM Yearbooks (2008 to 2010) being shepherded by Rheta Rubenstein as General Editor. Rheta has done a stellar job advising, counseling, editing, and working to help in every way possible throughout the development of these yearbooks. On behalf of previous editors and the coeditors of this book, we thank Rheta for a job well done.

The members of the Editorial Panel reflect a deep knowledge about mathematics curriculum along with experience addressing a wide range of curricular issues. Their rich backgrounds served the yearbook development process well. Additionally, they worked tirelessly throughout all stages of the yearbook development process, and their contributions are greatly appreciated. The Seventy-second Yearbook Editorial Panel consisted of the following:

Randall Charles, Carmel, California
Kathleen Cramer, University of Minnesota—Twin Cities
Diana V. Lambdin, Indiana University Bloomington
Thomas A. Romberg, University of Wisconsin—Madison
James W. Wilson, University of Georgia

Yearbook Coeditors

Barbara J. Reys, University of Missouri—Columbia
Robert E. Reys, University of Missouri—Columbia

General Editor

Rheta Rubenstein, University of Michigan—Dearborn

REFERENCES

- National Council of Teachers of Mathematics (NCTM). *An Agenda for Action*. Reston, Va.: NCTM, 1980.
- . *Curriculum and Evaluation Standards for School Mathematics*. Reston, Va.: NCTM, 1989.
- . *Principles and Standards for School Mathematics*. Reston, Va.: NCTM, 2000.
- . *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*. Reston, Va.: NCTM, 2006.
- . *Focus in High School Mathematics: Reasoning and Sense Making*. Reston, Va.: NCTM, 2009.
- National Governor's Association (NGA) and Council of Chief State School Officers (CCSSO). *College and Career Readiness Standards for Mathematics: Draft for Review and Comment, September 21, 2009*. <http://www.CoreStandards.org/Standards/index.htm> (accessed September 25, 2009).
- Reeve, William David, ed. *Curriculum Problems in Teaching Mathematics*, Second Yearbook of the National Council of Teachers of Mathematics. New York: Bureau of Publications: Teachers College, Columbia University, 1927.
- . "United States." In *Significant Changes and Trends in the Teaching of Mathematics throughout the World since 1910*, Fourth Yearbook of the National Council of Teachers of Mathematics, edited by William David Reeve, pp. 132–86. New York: Bureau of Publications: Teachers College, Columbia University, 1929.