

CHAPTER 1

An Inquiry-Based Approach: Project-Based Learning

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What Is Project-Based Learning?

A project-based learning (PBL) curriculum engages learners in meaningful problems that are important to them while advancing their creativity and problem-solving abilities. The PBL model is based on the assumption that most academic content is learned best in the context of projects.

PBL is an inquiry-based instructional approach that reflects a learner-centered environment and concentrates on learners' application of disciplinary concepts, tools, experiences, and technologies to research the answers to questions and solve real-world problems (Condcliffe et al. 2017; Larmer, Mergendoller, and Boss 2015). PBL can help increase both the range of learners' interests and their conceptual understanding of mathematics content. Teachers support ways for learners to construct their own understanding and orchestrate conversations in which learners explore complex connections and relationships among ideas.

General core principles and practices of PBL include the following:

- Promoting a professional culture of trust, respect, and responsibility among the learners and the teacher
- Focusing on 21st-century skills and academic standards, such as the Common Core State Standards for Mathematics (CCSSM)
- Improving character education traits such as leadership, civic responsibility, and compassion
- Scaffolding activities that include student-centered instruction to increase relevance and rigor
- Connecting learning to other subject areas
- Infusing technology as a tool for communicating, collaborating, and learning
- Partnering with community institutions, such as higher education, businesses, and nonprofit agencies, so that learners can build relationships with other local stakeholders

In this book, we showcase a number of PBL units that were designed and implemented by elementary school teachers, coupled with tips and narratives to support readers in implementing PBL.

PBL Is Not Just “Doing Projects”

In a traditional mathematics classroom, “doing projects” happens at the end of a unit, after the teacher has presented the content through a series of lessons and learners have completed homework assignments, practice problems, readings, lectures, textbook activities, and class discussions. Learners then demonstrate their understanding of the content in a culminating project (see figure 1.1).

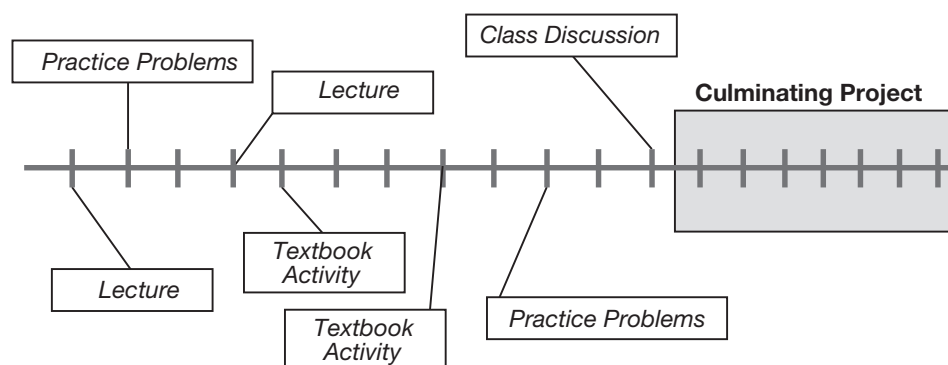


Fig. 1.1. Here is a timeline of “doing projects.”

Because PBL instruction occurs through an “extended inquiry process” (Markham, Larmer, and Ravitz 2003, p. 4), the project does not take place at the end of the unit as a culminating product. Instead, it is given to learners at the beginning of a unit specifically to engage them in the content.

The project is launched with an entry event.

Entry Event: This refers to an activity used to “kick off” the PBL unit (e.g., a letter, video, or other presentation of a real-life problem) to maximize learners’ engagement and inquiry.

A driving question, presenting an authentic problem, then pulls or guides learners through the curriculum, giving them a further incentive to learn the mathematics content (see figure 1.2).

Driving Question: Learners explore an open-ended challenge or problem throughout the project.

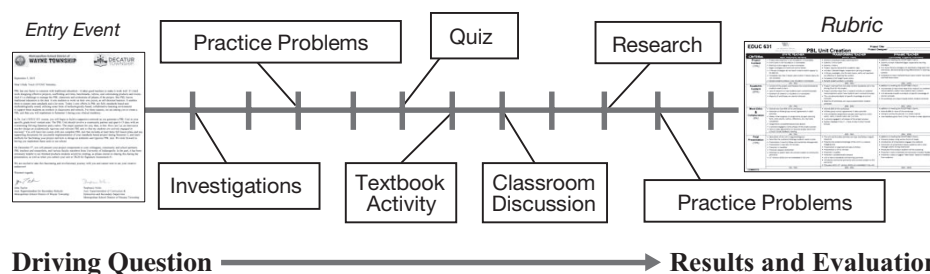


Fig. 1.2. This is a timeline of a PBL unit. Project-based learning pulls or guides learners through the curriculum, giving them a further incentive to learn the mathematics content.

Instruction is integrated into the project as learners need it, thus increasing the relevance of mathematics and the need to learn it.

Many elements found in a traditional classroom—such as practice problems, textbook activities, and class discussions—are integrated into the unit in response to learners' need-to-knows (NTKs), based on the entry event.

Need-to-Knows (NTKs): Learners generate a list of questions after the entry event that pertain to the knowledge and skills needed to successfully complete the project. The list is revisited daily, and more NTKs are added as needed.

However, instead of listening to lectures, learners investigate, build, and research concepts as part of inquiry-based instruction.

Key Features of PBL

Larmer, Mergendoller, and Boss (2015) describe the Gold Standard PBL Model with eight key features:

1. Key knowledge, understanding, and success skills: Students learn important content standards and concepts while exercising critical thinking, problem solving, working as a team, and exhibiting self-management skills and habits of mind.
2. Challenging problem or question: The project or unit starts with a driving question or challenge that addresses authentic concerns for students to investigate, solve, or explore and answer.
3. Sustained inquiry: Students investigate the driving question through problem solving and sustained inquiry by using a variety of resources. In the process, they learn and apply important ideas specific in the discipline.
4. Authenticity: The project represents a real-world scenario and challenges students to use real-world tools. The project has an impact on students and the community.
5. Student voice and choice: Students have input on how to solve the problem. Students have a voice in the resources they use, what questions to ask, how each team member contributes, or the products they produce.
6. Reflection: Students and teachers examine what they are learning, how they are learning, and why they are learning. Reflection includes how to move forward.
7. Critique and revision: Students receive and provide constructive peer feedback by using rubrics, protocols, and models in order to revise and improve the work.
8. Public product: Students create tangible products or presentations that address the driving question and present them to their class and community members.

Essential Tools for Designing Effective PBL Units

A number of elements are essential to the design of an effective PBL unit, including a focus on 21st-century skills, an entry event, a driving question, NTKs, the project planning form, the project calendar, scaffolding instruction, and a project rubric. Each is described in greater detail next.

21st-Century Skills

In today's rapidly changing economy and an increasingly tech-savvy industry, employers seek graduates who can solve problems, think critically, exercise creativity, and work in teams (KnowledgeWorks Foundation 2018; Partnership for 21st-Century Learning 2019). These “soft” skills, which are imperative for jobs in the 21st century, are often not addressed in traditional methods of teaching (Lee and Galindo 2018).

In a PBL environment, learners need to do much more than simply remember information: They need to use higher-order thinking skills, and they must learn to work as a team and contribute to a group effort. They must listen to others, make their own ideas clear when speaking, be able to read a variety of materials, write or otherwise express themselves in various modes, and give effective presentations. These skills, competencies, and habits of mind are known as 21st-century skills, which include communication, creativity, use of technology, group process and collaboration, problem solving and critical thinking, and task- and self-management (Boss, Larmer, and Mergendoller 2013). Teaching and assessing these skills helps prepare learners to thrive in today's global economy, life, and citizenship.

Learners also work independently and take responsibility when they are asked to make choices. Opportunities to make choices and to express their learning in their own voice also help increase learners' educational engagement.

The project planning forms showcased in this book allow readers to see which 21st-century skills are encouraged from the project work or explicitly taught and assessed. (This form is discussed in more detail later.)

Entry Event

PBL units are most often launched with an entry event that contextualizes the problem and motivates learners to engage in the content. Entry events may be letters, documents, videos, presentations, or any other activity that engages learners by presenting an authentic problem.

An entry event can be launched in several ways. Some examples follow:

- A community member poses a problem to the learners and asks for their help.
- Learners watch a video that provides a context for the problem.
- Learners read a letter from someone who outlines the expectations for the project and will later evaluate their products.

Ideally, the problem presented is an authentic one, and a representative of the company or organization in question can present the challenge, either virtually or in person.

Relationships Tip: Some teachers have found it expedient to draft the entry event letter themselves and then have the community partner approve the draft before signing it. In most cases, partners were happy with this arrangement and did not request any changes.

An entry event should accomplish four things:

1. Hook the learners
2. Allow learners to discern their roles
3. Lay out the project or problem to be completed or solved
4. Provide information that motivates the learners to ask questions and seek answers

Each PBL unit in this book showcases how teachers used an entry event to launch their project and describes how the entry event was presented to the learners.

Driving Question

The driving question is an open-ended challenge or problem that focuses learners' work and deepens their learning by centering on a significant real-life question, problem, issue, or debate. It requires the teacher to articulate a scenario that can be meaningful to learners.

Once learners are engaged by the entry event, they should be able to articulate the driving question, with the teacher's guidance. (It is also OK to present the driving question to the students, particularly for teachers doing PBL for the first time.) Teachers can guide learners to define the problem stated within the driving question by having them reflect on the following framework (Hallermann, Larmer, and Mergendoller 2011, p. 39): *How do we, as . . . (learner's role), create/research/develop . . . (task) so that . . . (desired outcome)?* For example, the driving question presented in "Yes, We Can: Help the Hungry in Our Community through CANstruction" (chapter 6) is *How can we, as engaged citizens of our community, help those who are hungry?* The learners for this PBL unit are in a fourth-grade class. Framing their role as engaged citizens gives learners a dual layer of authenticity and adult connections. Students engage and tackle a problem that is situated in the real world, and they develop meaningful relationships with members of the community.

Need-to-Knows (NTKs)

The learners' next task is to generate and record a list of items they will "need to know" or understand to answer the driving question and ultimately complete the project successfully. It may be more appropriate for upper elementary students (rather than younger elementary students) to identify what they think they need to know. Creating this list is an essential problem-solving skill that allows learners to have a voice in how they will receive the content and what kinds of instruction they will need, thus empowering them to take charge of their learning and creating relevance for their learning throughout the project.

A teacher can use a variety of resources to organize the NTKs, including sticky notes, flip charts, Google Docs™, and a shared digital wall (such as Padlet®, Wallwisher®, Aww App, Backchannel Chat, Trello®, etc.). The NTK list becomes a living document for the duration of the project: Learners revisit the list daily and add to or revise it to assess their progress.

Before launching the unit, the teacher should anticipate some likely NTKs that the learners will have. Each project showcased in this book contains a list of NTKs that teachers anticipated their learners would raise, how they planned to support learners in addressing or answering them, and how this learning would be assessed.

Note: The NTKs listed in each PBL unit are not exhaustive; they are merely examples to indicate the level of detail required in the PBL unit-planning process.

Template 1.1 (see p. 13): Scaffolding NTKs: Activity and Assessment Planning (see figure 1.3) is a tool for teachers to list likely NTKs, possible assignments or activities for learners to demonstrate the NTKs, possible assessments to confirm that learners understand the NTKs, and what learning outcomes will be addressed in the unit (Larmer, Ross, and Mergendoller 2009).

SCAFFOLDING NEED-TO-KNOWS (NTKs): Activity and Assessment Planning			
Anticipated Knowledge and Skills Students Need (NTKs)	Assignment or Activity to Address NTKs	How Assignment or Activity Will Be Assessed	Learning Outcomes Addressed in Assignment/Activity

Fig. 1.3. A snapshot of Template 1.1.

Although lessons and NTKs are generally planned by the project teacher, they may also be provided by other teachers, experts, mentors, or community members, depending on the context of the project. For example, if students are engaging in a service-learning project, the community partner will likely need to cover some important content. Partners may also contribute to how students' work will be assessed—what students will need to do to successfully complete the culminating products and performances.

Note: Templates can be found at the end of this chapter. Electronic copies of each template are also available on the NCTM website (www.nctm.org/more4u).

Project Planning Form

Many logistics must be considered when planning a PBL unit. Template 1.2 (see pp. 14–15): Project Planning Form (adapted from Hallerman, Larmer, and Mergendoller 2011, pp. 127–28) can guide teachers in laying out their thinking—about the big picture as well as some important details that go into a PBL unit, including its rigor, relevance, and relationships. This form can also help ensure that the entry event elicits the NTKs that teachers anticipate from learners. When the NTKs are in alignment with the targeted standards and skills, learners are encouraged to take a deeper dive into their project research.

Each PBL unit showcased in this book includes a completed project planning form.

Project Calendar

The project calendar helps teachers plan the scope and sequence of what mathematics is taught and what learning opportunities learners will engage themselves in throughout the unit. However, because PBL learning is a dynamic process, teachers must be flexible about revising the calendar as

needed—sometimes daily! PBL teachers often allot one or two “buffer days” in the calendar that are used to answer unanticipated NTKs and to provide extra work time as needed.

Each PBL unit in this book includes a completed project calendar (see Template 1.3 Project Calendar).

Scaffolding Instruction

The ways in which the teacher supports learners’ learning (e.g., practice problems, textbook activities, class discussions, investigations, research) and the activities that support the problem-solving process are referred to as *scaffolding techniques*. Scaffolds are integrated into the instruction of the unit as learners need the information so that their learning is both authentic and relevant. Using the project calendar can help teachers visualize how they are scaffolding instruction. PBL teachers often review the entry event, project planning form, and project rubric (see below) to ensure that the standards and skills addressed are consistent and that there are no surprises in the expectations across documents for learners.

Balancing learning about the context of the project with learning about its content is important. For example, in “Empowering Our Youngest Learners: Designing a Sensory ‘Recess Path’ Using Early Elementary Math” (chapter 4), learners design an indoor sensory path (context) and also master the concepts of counting and cardinality, measurement and data, weather forecasting and patterns, and information writing (content).

Because PBL involves an “extended, student-influenced inquiry process” (Hallermann, Larmer, and Mergendoller 2011, p. 5), it is important that learners have opportunities to construct something new—an idea, an interpretation, or a new way of displaying what they have learned. In a PBL mathematics classroom, learners’ actual creation of the project’s products and the process of creating the products are equally important. Mathematics learners should practice problem-solving skills in which they “make sense of problems and persevere in solving them,” which is one of the eight Standards for Mathematical Practice (NGA Center and CCSSO 2010). Furthermore, NCTM (2016, 2017) advocates instructional approaches that make reasoning and sense making foundational to the mathematics content, because such approaches best prepare learners for citizenship, the workplace, and further study. During the PBL design phase, it is important to ask oneself, *What opportunities am I providing for my learners to problem solve, persevere, and make sense and reason? How am I scaffolding instruction so they are engaging in these practices?*

Project Rubric

Rubrics help learners understand the expectations of the project and prepare them for how they will demonstrate their learning for public scrutiny and critique. PBL unit rubrics are designed so that learners demonstrate not only content mastery but also “soft” skills (i.e., 21st-century skills).

Rigor Tip: Even though a number of soft skills may be encouraged throughout a unit, Larmer, Ross, and Mergendoller (2009) recommend that novice PBL practitioners identify no more than two that are explicitly taught and assessed as project outcomes.

Each PBL project in this book includes the rubric created and used by the teacher.

Roots of PBL

Dewey, Piaget, Vygotsky, and other progressive educators laid the curricular and psychological foundations for PBL instruction. Dewey (1902) observed that children must be guided and provided with appropriate learning experiences if they are to develop a habit of critical examination and inquiry. Piaget (1970) and Vygotsky (1978) further strengthened this method of instruction by focusing on student-centered learning and construction of knowledge through practice and reflection. Student-centered learning, hands-on learning, and guided learning are some of the core values of PBL instruction.

Building on the work of these educators, Krajcik and Blumenfeld (2006) proposed four major learning-science ideas that describe the curricular and psychological foundations for PBL instruction: (1) active construction, (2) situated learning, (3) social interactions, and (4) cognitive tools.

Active Construction

According to Krajcik and Blumenfeld (2006), learners must actively construct meaning based on their experiences and interactions in the world for deep understanding to occur. Development of understanding is an iterative process in which learners reconstruct what they know from prior experiences and apply it to new experiences and ideas. Thus, rather than passively take in information from the teacher, learners “actively build knowledge as they explore the surrounding world, observe and interact with phenomena, take in new ideas, make connections between new and old ideas, and discuss and interact with others” (p. 319). In a PBL setting, learners construct their knowledge by engaging in real-world activities similar to the kinds of activities that are demanded of experts in the field, such as solving problems and developing artifacts.

While learners actively construct their solutions in PBL settings, the teacher scaffolds content and activities to enhance learners’ skills and capabilities. Hence, the teacher serves as a facilitator or “metacognitive coach” and designs units so that the learning environment supports and challenges the learners’ thinking (Hmelo-Silver and Barrows 2015). The teacher plays a crucial role in monitoring and assessing each group’s or learner’s progress during PBL activities. During each stage of the problem-solving process, the teacher may interject with additional resources to help learners in their pursuit of a solution to the problem.

Situated Learning

In situated learning, the context of learning is inextricably tied to the situation or context in which the learners are placed (Brown, Collins, and Duguid 1989; Lave and Wenger 1991). Rather than present information that learners may or may not be able to use to solve the stated problem, situated learning emphasizes that knowledge should be presented in context, preferably in a problem-solving scenario. In addition, if teachers want learners to solve complex real-world problems, they need to provide learning opportunities in those contexts (Dabbagh and Dass 2013; Marra et al. 2014).

Situated learning allows learners to acquire information in a meaningful context and relate it to their prior knowledge and experiences so that they can form connections between new information and their prior knowledge. In a PBL setting, teachers design units that anchor all learning activities to a

larger task or problem. Contextualizing the learning in this way enables learners to easily see the value and purpose of the tasks and activities they are asked to do (Moallem, Hung, and Dabbagh 2019).

Social Interaction

Vygotsky (1962) noted that learning is a social activity that takes place within the context of a unit. In PBL, teachers, learners, and community members work together to construct a shared understanding of the activity (Krajcik and Blumenfeld 2006). The use of collaborative groups, then, is inherent in a PBL classroom. Learners are able to develop a more linked conceptual understanding between new information and prior knowledge by sharing, challenging, and expanding on the principles and ideas of others (Fonteijn and Dolmans 2019). Refining one another's ideas and challenging one another's understandings also helps foster a community of learners. Learners learn firsthand what it means to function as part of a community.

Cognitive Tools

Cognitive tools—such as visual aids, graphic organizers, computer software, and manipulatives—can amplify and build on what learners learn. These tools can help learners understand a conceptual idea, expand the range of questions that learners can investigate, and offer learning experiences that might not otherwise be possible. In a PBL classroom, the teacher uses cognitive tools as scaffolds to support learning and to assess learners' understanding of key concepts.

Designing and Implementing a PBL Unit: The Six A's

An exemplary PBL unit ensures that both instruction and content enable learners to master core competencies. The Six A's (Markham, Larmer, and Ravitz 2003, p. 34) can be used as a guide for designing a mathematics PBL unit:

1. **Authenticity:** The project is situated in the real world—other professionals are tackling the same problem or question addressed by the project. In addition, the problem has meaning and relevance to learners, and there is an appropriate audience to view learners' products.
2. **Academic Rigor:** The driving question is well-defined and tightly integrated into the content standards. The project also demands breadth and depth of both specific knowledge and central concepts. Learners develop habits that are indicative of efficient and effective problem solvers, such as questioning and posing problems, applying past knowledge to new situations, employing precision of language and thought, and maintaining persistence.
3. **Applied Learning:** Learners use multiple high-performance work organization skills, such as working in teams, communicating ideas, applying new knowledge to the problem, and organizing and analyzing information. Learners are able to identify and apply the self-management skills needed to improve their group's performance.
4. **Active Exploration:** Learners conduct field-based activities, such as interviewing experts, surveying groups, and exploring worksites. Learners gather information from various sources and use appropriate methods to obtain the needed data.

Rigor Tip: Active Exploration can be challenging for mathematics educators. Because the Six A's criteria are not discipline-specific, teachers may overlook important criteria that are specific to a mathematics classroom. For example, although gathering information from a variety of sources and using a variety of methods to solve the problem—such as interviews, model building, and online research—are exemplary characteristics of active exploration (Markham, Larmer, and Ravitz 2003), teachers are often unsure how these might translate in a mathematics-focused lesson. “Tips for Teachers from a PBL Mathematics Educator” (chapter 14) discusses in more detail how teachers can encourage their learners to actively explore mathematics content.

5. **Adult Connections:** Learners are provided with mentorship opportunities, where they work alongside adults at a worksite relevant to the project. Learners develop meaningful relationships with members of the community who have expertise and experience in a particular field.
6. **Assessments:** Various formal and informal assessments occur intermittently throughout the project, and learners are given timely feedback from both peers and teachers. The project requires multiple products, all of which are aligned with the project's ultimate goal. The project culminates in an exhibition or presentation for an informed audience.

Assessment

Principles to Actions: Ensuring Mathematical Success for All (NCTM 2014, pp. 91–92) describes several principles of assessments that support effective teaching and learning:

- Assessment is an ongoing process that is embedded in instruction to support student learning and to allow teachers to adjust their instruction as needed.
- Mathematical understanding and processes can be measured through the use of a variety of assessment strategies and tasks.
- Multiple data sources are needed to provide an accurate picture of both teacher and student performance.
- Assessment is a process that should help students become better judges of their own work, assist them in recognizing high-quality work when they produce it, and support them in using evidence to advance their own learning.

Meaningful assessments provide information on learners' mastery of key mathematical ideas and enable educators to draw conclusions about their own progress in achieving the learning goals of the unit. Rigorous math PBL units must include learners' mastery of mathematical ideas in the project rubric.

When designing and implementing a PBL unit, alignment of key mathematical ideas, learning objectives, instructional tasks and methods, and assessment practices is crucial. NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) summarizes this issue clearly: “The degree to which meaningful inferences can be drawn from [assessments] depends on the degree to which the assessment methods and tasks are aligned or are in agreement with the curriculum” (p. 193).

Author's Biography

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