Introduction

This book focuses on ideas about statistics. These are ideas that you need to understand thoroughly and be able to use flexibly to be highly effective in your teaching of mathematics in grades 9–12. The book discusses many statistical ideas that are common in high school curricula, and it assumes that you have had a variety of experiences with statistics that have motivated you to delve into—and move beyond—the statistics that you expect your students to learn.

The book is designed to engage you with these ideas, helping you to develop an understanding that will guide you in planning and implementing lessons and assessing your students' learning in ways that reflect the full complexity of statistical concepts. A deep, rich understanding of these concepts will enable you to communicate their influence and scope to your students, showing them how these ideas permeate the statistics that they have encountered—and will continue to encounter—throughout their experiences with statistics.

The understanding of statistics that you gain from this focused study thus supports the vision of *Principles and Standards for School Mathematics* (NCTM 2000): "Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction" (p. 3). This vision depends on classroom teachers who "are continually growing as professionals" (p. 3) and routinely engage their students in meaningful experiences that help them learn statistics with understanding.

Why Statistics?

Like the topics of all the volumes in NCTM's Essential Understanding Series, statistical procedures and concepts compose a major area of school mathematics that is crucial for students to learn but challenging for teachers to teach. Students in grades 9–12 need to understand statistical concepts well if they are to succeed in these grades and in their subsequent experiences with statistics. Learners often struggle with ideas about statistics. For example, they are puzzled about what conclusions can be drawn from a sample when the sample is only a small fraction of a large population. Teachers of statistics in grades 9–12 understand the characteristics of good sampling methods and the concepts of bias and precision and are able to use their understanding to help students value the use of samples in statistical inference.

Your work as a teacher of mathematics in these grades calls for a solid understanding of the statistical concepts that you—and your school, your district, and your state curriculum—expect your students to learn. Your work also requires you to know how these ideas relate to other statistical and mathematical ideas that your students will encounter in the lesson at hand, the current school year, and beyond. A rich understanding of statistical concepts guides teachers' decisions in much of their work, such as choosing tasks for a lesson, posing questions, selecting materials, ordering topics and ideas over time, assessing the quality of students' work, and devising ways to challenge and support their thinking.

Understanding Statistical Concepts

Teachers teach statistics because they want others to understand it in ways that will contribute to success and satisfaction in school, work, and life. Helping your students develop a robust and lasting understanding of statistical concepts requires that you understand statistics deeply. But what does this mean?

It is easy to think that understanding statistics means knowing certain processes for analyzing data and mastering relevant vocabulary. For example, you are expected to be able to calculate a standard deviation, determine a standard error, and construct a confidence interval. You know different sampling strategies, such as simple random sampling and stratified random sampling. You are expected to use terms such as *random assignment*, *confidence interval*, and *precision*.

Obviously, vocabulary and processes are not all that you are expected to know about statistics. For example, in your ongoing work with students, you have undoubtedly discovered that you need not only to know common statistical procedures but also to be able to follow strategies that your students create. You need to make subtle distinctions between terms such as *random assignment* and *random selection*, and *sample distribution* and *sampling distribution*.

It is also easy to focus on a very long list of statistical ideas that all teachers of mathematics in grades 9–12 are expected to know and teach about statistics. Curriculum developers often devise and publish such lists. However important the individual items might be, these lists cannot capture the essence of a rich understanding of the topic. Understanding statistics deeply requires you not only to know important statistical ideas but also to recognize how these ideas relate to one another. Your understanding continues to grow with experience and as a result of opportunities to embrace new ideas and find new connections among familiar ones.

Furthermore, your understanding of statistics should transcend the content intended for your students. Some of the differences between what you need to know and what you expect them to learn are easy to point out. For example, you need not only to know how to specify statistical models by using familiar mathematical notation and to quantify goodness of fit, but also to see how these ideas underlie more complex statistical models, such as multiple regression models.

Other differences between the understanding that you need to have and the understanding that you expect your students to acquire are less obvious, but your experiences in the classroom have undoubtedly made you aware of these differences at some level. For example, how many times have you been grateful to have an understanding of statistics that enables you to recognize the merit in a student's unanticipated question or claim? How many other times have you wondered whether you could be missing such an opportunity or failing to use it to full advantage because of a gap in your knowledge?

As you have almost certainly discovered, knowing and being able to engage in the statistical problem-solving process is not enough when you are in the classroom. You also need to be able to identify and justify or refute novel claims. These claims and justifications might draw on ideas or techniques that are beyond the mathematical experiences of your students and current curricular expectations for them. For example, you need to know more complicated mathematical models than students encounter in algebra and geometry and be able to explain how these models are similar to yet different from statistical models that account for both structure and variation.

Big Ideas and Essential Understandings

Thinking about the many particular ideas that are part of a rich understanding of statistics can be an overwhelming task. Articulating all of those statistical ideas and their connections would require many books. To choose which ideas to include in this book, the authors considered a critical question: What is e*ssential* for teachers of mathematics in grades 9–12 to know about statistics to be effective in the classroom? To answer this question, the authors drew on a variety of resources, including personal experiences, the expertise of colleagues in mathematics, mathematics education, statistics, and statistics education, and the reactions of reviewers and professional development providers, as well as ideas from curricular materials and research on mathematics learning and teaching.

As a result, the content of this book focuses on essential ideas for high school teachers about statistics. In particular, chapter 1 is organized around five big ideas related to this important area. Each big idea is supported by smaller, more specific ideas, which the book calls *essential understandings*.

Benefits for Teaching, Learning, and Assessing

Understanding statistics can help you implement the Teaching Principle enunciated in *Principles and Standards for School Mathematics*. This Principle sets a high standard for instruction: “Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well” (NCTM 2000, p. 16). As in teaching about other critical topics, teaching about statistics requires knowledge that goes “beyond what most teachers experience in standard preservice mathematics courses” (p. 17).

Chapter 1 comes into play at this point, offering an overview of the topic that is intended to be more focused and comprehensive than many discussions that you are likely to have encountered. This chapter enumerates, expands on, and gives examples of the big ideas and essential understandings related to statistics, with the goal of supplementing or reinforcing your understanding. Thus, chapter 1 aims to prepare you to implement the Teaching Principle fully as you provide the support and challenge that your students need for robust learning about statistics.

Consolidating your understanding in this way also prepares you to implement the Learning Principle outlined in *Principles and Standards*: “Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge” (NCTM 2000, p. 20). To support your efforts to help your students learn about the concepts in this way, chapter 2 builds on the understanding that chapter 1 communicates by pointing out specific ways in which the big ideas and essential understandings connect with mathematics that students typically encounter earlier or later in school. This chapter supports the Learning Principle by emphasizing longitudinal connections in students’ learning about statistics. For example, as their statistical experiences expand, students gain a richer understanding of variability. They move from understanding variability in the case of one variable to considering variability in the case of bivariate models. They can understand the idea of data varying about a line or curve by building on a firm understanding of data varying about the mean of a univariate data set. Understanding variability in general will help students as they move to more complicated statistical models and explore variability that involves multiple variables.

The understanding that chapters 1 and 2 convey can strengthen another critical area of teaching. Chapter 3 addresses this area, building on the first two chapters to show how an understanding of statistics can help you select and develop appropriate tasks, techniques, and tools for assessing your students' understanding of statistics. An ownership of the big ideas and essential understandings related to statistics, reinforced by an understanding of students' past and future experiences with related ideas, can help you ensure that assessment in your classroom supports the learning of foundational statistical concepts.

Such assessment satisfies the first requirement of the Assessment Principle set out in *Principles and Standards*: "Assessment should support the learning of important mathematics and furnish useful information to both teachers and students" (NCTM 2000, p. 22). An understanding of statistics can also help you satisfy the second requirement of the Assessment Principle, by enabling you to develop assessment tasks that give you specific information about what your students are thinking and what they understand. For example, in developing questions for class discussion or for an exam, you can move beyond items that require students only to display their computational skills. You are able to use situations and questions that include real-world contexts and that ask students for interpretations of numerical results and demand explanations that require conceptual understanding.

Ready to Begin

This introduction has painted the background, preparing you for the big ideas and associated essential understandings related to statistics that you will encounter and explore in chapter 1. Reading the chapters in the order in which they appear can be a very useful way to approach the book. Read chapter 1 in more than one sitting, allowing time for reflection. Take time also to use a graphing calculator or other computational tools as you consider items that recommend technology use. Absorb the ideas—both big ideas and essential understandings—related to statistics. Appreciate the connections among these ideas. Carry your newfound or reinforced understanding to chapter 2, which guides you in seeing how the ideas related to these operations are connected to the statistics that your students have encountered earlier or will encounter later in school. Then read about teaching, learning, and assessment in chapter 3.

Alternatively, you may want to take a look at chapter 3 before engaging with the statistical ideas in chapters 1 and 2. Having the challenges of teaching, learning, and assessment issues clearly in mind, along with possible approaches to them, can give you a different perspective on the material in the earlier chapters. No matter how you read the book, let it serve as a tool to expand your understanding, application, and enjoyment of statistics.

Two appendixes supplement the text. The first, a glossary of common statistical terms, is intended for quick reference and clarification as readers move through the text. Terms that are included in the glossary are shown in green in the text. The second appendix, a short list of resources for teachers, is designed to point to information that can extend, reinforce, and enrich readers' experience of the book. Both appendixes are also available online at www.nctm.org/more4u.

Chapter
1

Statistics: The Big Ideas and Essential Understandings

Statistics has been a recommended part of the high school mathematics curriculum for many years. *Curriculum and Evaluation Standards* (NCTM 1989) addressed statistics in Standard 10 of the Curriculum Standards for grades 9–12. *Principles and Standards for School Mathematics* (NCTM 2000) reaffirmed statistics as part of the high school mathematics curriculum with the inclusion of the Data Analysis and Probability Standard.

However, widespread implementation of these statistics standards has been elusive, and many schools and districts that have included statistics and data analysis in the curriculum have incorporated them in an ad hoc fashion. As a consequence, students and teachers often see statistics as a loose collection of graphical and numerical methods with no underlying, unifying theory. Although students exposed to statistics in this manner might be able to construct graphical displays and compute numerical summaries of data, they often develop only a superficial understanding of important statistical concepts and fail to build the kind of statistical reasoning skills that have become essential to making informed decisions in today's quantitative world.

See *Developing Essential Understanding of Statistics for Teaching Mathematics in Grades 6–8* (Kader and Jacobbe 2013) for an extended discussion of statistics as a problem-solving process that involves graphical and numerical methods.

The landscape of statistics in the high school mathematics curriculum is changing in a profound way. With the adoption of the Common Core State Standards for Mathematics (CCSSM; Common Core State Standards Initiative 2010), statistics will take a more prominent place in the high school curriculum. Successful implementation of the Common Core State Standards will require that the high school mathematics curriculum include statistics content that goes beyond the mechanical and computational aspects of descriptive statistical methods to focus on the conceptual understanding necessary for the development of sound statistical reasoning.

In this book, the five big ideas that we have identified focus on concepts that illustrate the use of probabilistic reasoning in **statistical inference**. We have chosen to focus on concepts rather than on the mechanical and computational aspects of specific inferential methods for two reasons. First, in a book as brief as this one, it is not possible to offer a textbook-style treatment of introductory statistics. Second, we believe that many high school mathematics teachers are comfortable with the mechanical and computational aspects of data analysis but may be somewhat less comfortable with the concepts that provide a unifying structure supporting the use of probabilistic reasoning in statistical inference. Our hope is that the book will allow teachers who are experienced in teaching algebra 1, geometry, and algebra 2 to successfully meet the challenges that they face in integrating a statistics component into these courses as they implement the Common Core State Standards.

Terms shown in **green** are defined in the glossary that appears in appendix 1, which is also available at www.nctm.org/more4u.

Big Ideas and Associated Understandings

This book focuses on five big ideas, which we summarize briefly below before listing them with all of their associated essential understandings. After this quick overview of the big ideas and the complete list that follows it, we will devote the remainder of chapter 1 to developing each big idea and essential understanding in detail.

The big ideas in brief

The concept of a mathematical model is ubiquitous in the modern mathematics curriculum. Big Idea 1 draws a distinction between mathematical models and statistical models. For example, in looking at weights of U.S. pennies, we might make a mathematical model that specifies the weight of a penny as 2.5 grams (which is what the U.S. Mint claims as the weight). But the weights of individual pennies will differ, if only slightly, from this value, and a statistical model would extend the mathematical model by incorporating the variability about this central value of 2.5 grams.

A mathematical model might also specify or describe a relationship (for example, a linear relationship) between variables in a bivariate situation. Statistical models build on such mathematical models by explicitly including descriptions of random variation. Statistical methods can then be used to acknowledge and to quantify this variability, allowing us to evaluate the usefulness of a particular model in describing data in terms of structure plus variability.

Understanding variability is necessary for collecting, describing, analyzing, and drawing conclusions from data in a sensible way. Thinking about data in terms of distributions and distinguishing among the different ways to use distributions (to describe the values in a population, the values in a sample, or the values of a **statistic** for different possible samples) are key to understanding statistical inference. The idea that distributions describe variability is the fundamental concept in Big Idea 2.

The reasoning involved in carrying out a *hypothesis test* is simple and intuitive, but it is easy to lose sight of this simple logic in the development of a formal (and often mechanical) process for carrying out such a test. Big Idea 3 focuses on the way in which these tests are used to determine whether a particular outcome might have happened by chance.

Statistical methods involve using available, but usually incomplete, information to draw conclusions about a **population** or about the effect of experimental conditions on some response. Because such conclusions are based on incomplete information, they have an associated risk of error, and this potential needs to be acknowledged and quantified. Characteristics of the data collection method (such as **random selection** and **random assignment**) make this calculation possible. In addition, both the type of analysis that is appropriate and the conclusions that can be drawn depend on the type of data and the way in which they are collected. Data collection methods and their implications are the focus of Big Idea 4.

Understanding that a relatively small sample from a population can be used to make accurate estimates of the characteristics of the entire population is critical to understanding sampling and inference. The size of the sample affects the *precision* with which estimates can be made or claims tested; when the sampling plan is properly designed, a larger sample will always produce more *precise* results than a smaller sample. The fact that the size of the population is not an important factor in determining the *accuracy* of estimates runs counter to people's intuition that a larger population requires a larger sample. The key to understanding how sampling and inference are related depends on the sampling method, the sample size, and the interplay between them. The roles of sampling method, precision, and possible bias in the evaluation of an estimator are the focus of Big Idea 5.

The complete list of ideas and understandings

The list of the big ideas and their associated essential understandings that appears below offers a preview of the detailed discussion of each big idea to follow. Additionally, we present the list here for your convenience in referring back to the big ideas and related understandings as you move through the book. Read through them now, but do not think that you must absorb them fully at this point; we will discuss each one in turn in detail.

Big Idea 1. Data consist of structure and variability.

> **Essential Understanding 1*a*.** Mathematical models describe structure.
>
> **Essential Understanding 1*b*.** Statistical models extend mathematical models by describing variability around the structure.
>
> **Essential Understanding 1*c*.** Statistical models are evaluated by how well they describe data and whether they are useful.

Big Idea 2. Distributions describe variability.

> **Essential Understanding 2*a*.** A population distribution describes variability in the values that make up a population.
>
> **Essential Understanding 2*b*.** The population distribution is often unknown but can be approximated by a sample distribution.
>
> **Essential Understanding 2*c*.** The sampling distribution of a sample statistic describes how the value of the statistic varies from sample to sample.
>
> **Essential Understanding 2*d*.** Simulation can be used to approximate sampling distributions.

Big Idea 3. Hypothesis tests answer the question, “Do I think that this could have happened by chance?”

> **Essential Understanding 3*a*.** A hypothesis test involves choosing between two competing hypotheses—the null hypothesis and the alternative hypothesis.
>
> **Essential Understanding 3*b*.** The alternative hypothesis is determined by the statistical question of interest.